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Appendix to a review article for the Tokyo Foundation for Policy Research

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Main result

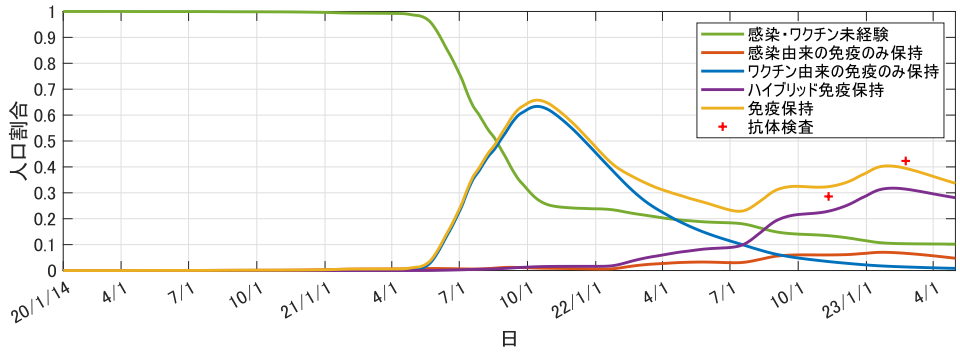
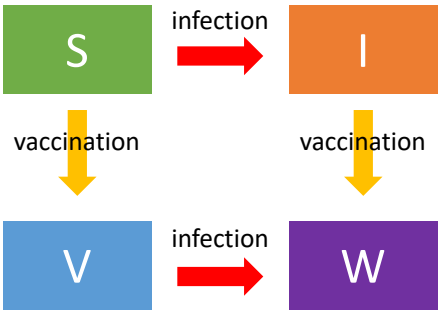


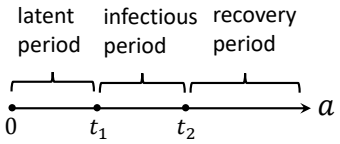
Figure 1: The level of herd immunity induced by infection (red), vaccine (blue) and hybrid of them (purple). The yellow curve represents the sum of them, and the green curve represents the proportion of unimmunized population. The red crosses represent the results of the antibody test [3].

Meaning of symbols

- t: time (day)
- a: class age (day)
- $S(t)$: susceptible population
- $I(t,a)$: infected population
- $V(t,a)$: vaccinated susceptible population
- $W(t,a)$: vaccinated infected population
- v_1: vaccination rate for the first dose
- v_2: vaccination rate for the n-th dose ($n \geq 2$)
- $\lambda(t)$: force of infection
- $\beta(a)$: infection rate
- $\delta(a)$: efficacy of natural immunity ($0 \leq \delta(a) \leq 1$)
- $\sigma(a)$: efficaty of vaccine immunity ($0 \leq \sigma(a) \leq 1$)
- $\omega(a)$: efficaty of hybrid immunity ($0 \leq \omega(a) \leq 1$)
- T: length of time span

Model diagram





Model equations

$$\lambda(t) = \int_0^T \beta(a)\,[I(t,a) + W(t,a)]\,da,$$

$$S'(t) = -\lambda(t)S(t) - v_1 S(t),$$

$$(\partial_t + \partial_a)\,I(t,a) = -\,[1-\delta(a)]\,\lambda(t)I(t,a) - v_1 I(t,a),$$

$$I(t,0) = \lambda(t)\left\{S(t) + \int_0^T [1-\delta(a)]\,I(t,a)da\right\},$$

$$(\partial_t + \partial_a)\,V(t,a) = -\,[1-\sigma(a)]\,\lambda(t)V(t,a) - v_2 V(t,a),$$

$$V(t,0) = v_1 S(t) + v_2 \int_0^T V(t,a)da,$$

$$(\partial_t + \partial_a)\,W(t,a) = -[1-\omega(a)]\lambda(t)W(t,a) - v_2 W(t,a),$$

$$W(t,0) = v_1 \int_0^T I(t,a)da + v_2 \int_0^T W(t,a)da$$
$$+\lambda(t)\left\{\int_0^T [1-\sigma(a)]V(t,a)da + \int_0^T [1-\omega(a)]\,W(t,a)da\right\}.$$

Parameters

The unit time is 1 day, and the simulation time span is from January 14, 2020 to May 1, 2023 ($T = 1203$). The infection rate $\beta(a)$ is assumed to be

$$\beta(a) = \begin{cases} 0, & a \in [0,t_1) \cup (t_2,T], \\ \kappa, & a \in [t_1,t_2], \end{cases} \qquad t_1 = 3, \quad t_2 = 10,$$

and κ is estimated as a time-varying parameter fitting

$$Y(t) = p\lambda(t)\left\{S(t) + \int_0^T [1-\delta(a)]\, I(t,a)da + \int_0^T [1-\sigma(a)]V(t,a)da + \int_0^T [1-\omega(a)]\, W(t,a)da\right\} N$$

to the daily number of newly reported cases in Japan [5], where $p = 0.5$ and $N = 1.26\times10^8$ are the detection rate and the total population in Japan, respectively. The fitted curve is plotted in Figure 2.

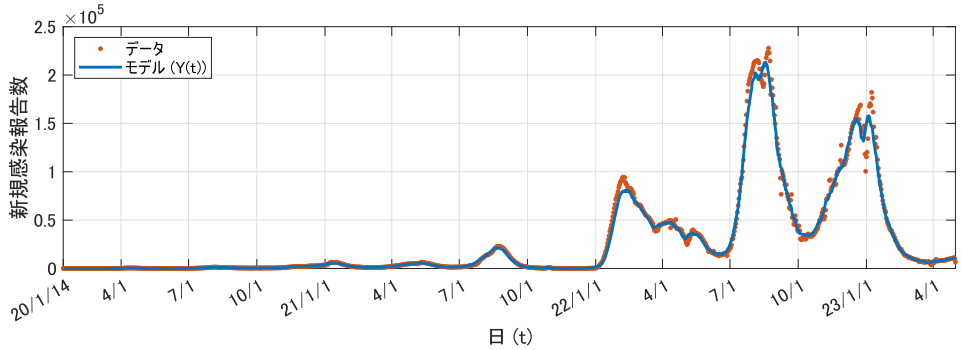


Figure 2: The data collected from [5] and the fitted curve $Y(t)$ of the newly reported cases of COVID-19 in Japan.

The vaccination rates v_1 and v_2 are time-varying parameters determined by

$$v_1 = \frac{V_1(t)}{\left[S(t) + \int_0^T I(t,a)da\right] N}, \quad v_2 = \frac{V_2(t)}{\left[\int_0^T V(t,a)da + \int_0^T W(t,a)da\right] N},$$

where $V_1(t)$ and $V_2(t)$ are the number of first and other vaccination doses at time t, which are collected from the data [4].

The parameters $\delta(a)$, $\sigma(a)$ and $\omega(a)$ are assumed to be given as in Figure 3. Following [1, 2], we assume that $\delta(a)$ and $\sigma(a)$ decrease to its half after 6 months passed, and $\omega(a)$ decreases to its half after 1 year passed.

References

[1] COVID-19 Forecasting Team, Past SARS-CoV-2 infection protection against re-infection: a systematic review and meta-analysis, The Lancet 401 (2023) 833-842.

[2] J. Malato, Stability of hybrid versus vaccine immunity against BA.5 infection over 8 months, The Lancet Infectious Diseases 23 (2023) 148-150.

[3] MHLW Japan, `https://www.mhlw.go.jp/content/10906000/001070846.pdf`, accessed on May 1, 2023.

[4] VRS, `https://info.vrs.digital.go.jp/opendata/`, accessed on May 7, 2023.

[5] WHO, `https://covid19.who.int/`, accessed on May 7, 2023.

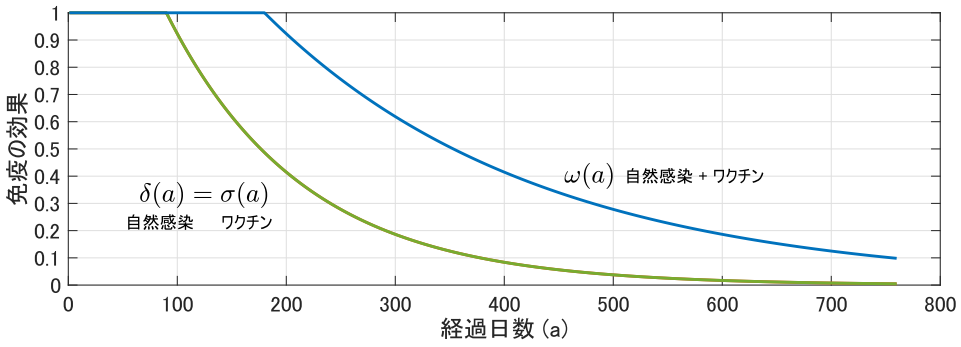


Figure 3: Parameters $\delta(a)$, $\sigma(a)$ and $\omega(a)$ for the efficacy of natural immunity, vaccine immunity and hybrid immunity, respectively.