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Appendix to a review article

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1. Parameters

Parameter	Description	Value
S	Susceptible population (unvaccinated)	-
E	Exposed population (unvaccinated)	-
I	Infectious population (unvaccinated)	-
R	Removed population (unvaccinated)	-
S_1	Susceptible population (vaccinated once)	-
E_1	Exposed population (vaccinated once)	-
I_1	Infectious population (vaccinated once)	-
R_1	Removed population (vaccinated once)	-
S_2	Susceptible population (vaccinated twice)	-
E_2	Exposed population (vaccinated twice)	-
I_2	Infectious population (vaccinated twice)	-
R_2	Removed population (vaccinated twice)	-
S_3	Susceptible population (vaccinated more than 3 times)	-
E_3	Exposed population (vaccinated more than 3 times)	-
I_3	Infectious population (vaccinated more than 3 times)	-
R_3	Removed population (vaccinated more than 3 times)	-
t	Time	-
τ	Class age (time elapsed since the recovery)	-
a	Class age (time elapsed since the vaccination)	-
β	Infection rate	Estimated using data in [9]
ε	Onset rate	0.2 (incubation period $1/\varepsilon = 5$ days) [4]
γ	Removal rate	0.1 (infection period $1/\gamma = 10$ days) [1]
λ	Force of infection	Equation (1)
$1-\sigma$	Efficacy of the first vaccination	0.46 [6]
v_n	Vaccination rate (for n-th)	Estimated using data in [7]
T	Duration between the vaccination	150 days or 180 days
$1-p(a)$	Efficacy of full vaccination at class age a	$0.8e^{-0.003a}$ (estimated using data in [6])
δ	Detection rate	0.5 (estimated using data in [5])
N	Total population in each prefecture	[8]
$\psi(a)$	Waning rate of natural immunity	$0.65\left(1-e^{-20e^{-0.01a}}\right)$ (assumed from [2])

See [3] for the details of how to estimate each parameter.

2. Model

Before vaccination policy (January 14, 2020 - February 16, 2021).

$$
\begin{aligned}
&S'(t) = -\beta S(t)I(t),\\
&E'(t) = \beta S(t)I(t) - \varepsilon E(t),\\
&I'(t) = \varepsilon E(t) - \gamma I(t),\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}\right) R(t,\tau) = 0, \quad R(t,0) = \gamma I(t).
\end{aligned}
$$

Under vaccination policy (February 17, 2021 - present).

- Unvaccinated population:

$$
\begin{aligned}
&S'(t) = -\lambda(t)S(t) - v_1 S(t),\\
&E'(t) = \lambda(t)S(t) - (\varepsilon + v_1)E(t),\\
&I'(t) = \varepsilon E(t) - (\gamma + v_1)I(t),\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial \tau}\right) R(t,\tau) = -v_1 R(t,\tau), \quad R(t,0) = \gamma I(t).
\end{aligned}
$$

- Vaccinated once:

$$
\begin{aligned}
&S_1'(t) = v_1 S(t) - \sigma\lambda(t)S_1(t) - v_2 S_1(t),\\
&E_1'(t) = v_1 E(t) + \sigma\lambda(t)S_1(t) - (\varepsilon + v_2)E_1(t),\\
&I_1'(t) = v_1 I(t) + \varepsilon E_1(t) - (\gamma + v_2)I_1(t),\\
&R_1'(t) = v_1 \int_0^\infty R(t,\tau)d\tau + \gamma I_1(t) - v_2 R_1(t).
\end{aligned}
$$

- Vaccinated more than twice ($n = 2, 3$):

$$
\begin{aligned}
&S_n(t,0) = \begin{cases} v_2 S_1(t), & n = 2,\\ v_3 \int_T^\infty S_2(t,a)da + v_4 \int_T^\infty S_3(t,a)da, & n = 3, \end{cases}\\
&E_n(t,0) = \begin{cases} v_2 E_1(t), & n = 2,\\ v_3 \int_T^\infty E_2(t,a)da + v_4 \int_T^\infty E_3(t,a)da, & n = 3, \end{cases}\\
&I_n(t,0) = \begin{cases} v_2 I_1(t), & n = 2,\\ v_3 \int_T^\infty I_2(t,a)da + v_4 \int_T^\infty I_3(t,a)da, & n = 3, \end{cases}\\
&R_n(t,0) = \begin{cases} v_2 R_1(t), & n = 2,\\ v_3 \int_T^\infty R_2(t,a)da + v_4 \int_T^\infty R_3(t,a)da, & n = 3, \end{cases}
\end{aligned}
$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right)S_n(t,a)=-p(a)\lambda(t)S_n(t,a)-q_n(a)S_n(t,a),$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right)E_n(t,a)=p(a)\lambda(t)S_n(t,a)-[\varepsilon+q_n(a)]E_n(t,a),$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right)I_n(t,a)=\varepsilon E_n(t,a)-[\gamma+q_n(a)]I_n(t,a),$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right)R_n(t,a)=\gamma I_n(t,a)-q_n(a)R_n(t,a),$$

where

$$q_n(a)=\begin{cases}0, & a<T,\\ v_{n+1}, & \text{otherwise.}\end{cases}$$

- Force of infection:

$$\lambda(t)=\beta\left[I(t)+I_1(t)+\sum_{n=2}^{3}\int_0^\infty I_n(t,a)da\right]. \tag{1}$$

- Let

$$M_0(t):=E(t)+I(t)+\int_0^\infty \psi(\tau)R(t,\tau)d\tau,\quad M_1(t):=E_1(t)+I_1(t)+R_1(t),$$
$$M_n(t):=\int_0^\infty [E_n(t,a)+I_n(t,a)+R_n(t,a)]da,\quad n\geq 2.$$

- Natural infection (with waning): $\sum_{n=0}^{3}M_n(t)$.
- Vaccine (with waning): $(1-\sigma)S_1(t)+\sum_{n=2}^{3}\int_0^\infty[1-p(a)]S_n(t,a)da$.
- Natural infection (with waning) + vaccine (with waning): $\sum_{n=0}^{3}M_n(t)+(1-\sigma)S_1(t)+\sum_{n=2}^{3}\int_0^\infty[1-p(a)]S_n(t,a)da$.
- Partial immunity: $1-S(t)$.

How to estimate $\beta=\beta(t)$ and δ

See [3].

How to estimate the vaccination rates

Note that $v_1\times[S(t)+E(t)+I(t)+R(t)]\times N$ is the number of the first vaccination at time t. Hence, we estimate $v_1=v_1(t)$ as

$$v_1(t)=\frac{(number\ of\ the\ first\ vaccination\ at\ time\ t)}{[S(t)+E(t)+I(t)+\int_0^\infty R(t,\tau)d\tau]\times N}.$$

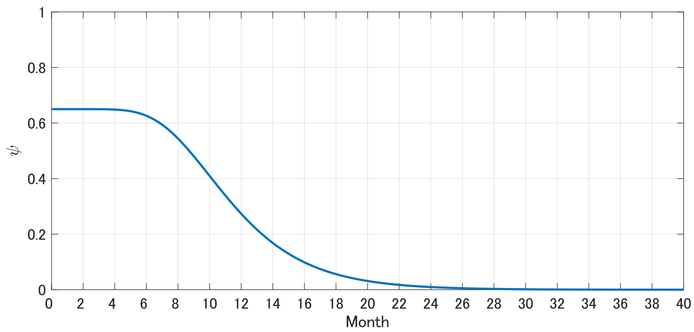
In a similar manner, we estimate $v_n = v_n(t)$ $(n \geq 2)$ as

$$v_n(t) = \begin{cases} \dfrac{(number\ of\ the\ second\ vaccination\ at\ time\ t)}{[S_1(t) + E_1(t) + I_1(t) + R_1(t)] \times N}, & n = 2, \\ \dfrac{(number\ of\ the\ n{-}th\ vaccination\ at\ time\ t)}{\int_T^\infty [S_{n-1}(t,a) + E_{n-1}(t,a) + I_{n-1}(t,a) + R_{n-1}(t,a)] da \times N}, & n \geq 3. \end{cases}$$

How to predict

We fixed the infection rate and vaccination rates using the latest 1 week data.

Waning rate of natural immunity



References

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[2] H. Chemaitelly et al., Duration of immune protection of SARS-CoV-2 natural infection against reinfection in Qatar, medRxiv. `https://doi.org/10.1101/2022.07.06.22277306`

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[6] NIID 国立感染症研究所, 新型コロナワクチンの有効性を検討した症例対照研究の暫定報告（第三報）, `https://www.niid.go.jp/niid/ja/2019-ncov/2484-idsc/10966-covid19-71.html`, accessed on March 7, 2022.

[7] `https://info.vrs.digital.go.jp/opendata/`, accessed on October 4, 2022.

[8] `https://www.stat.go.jp/data/nihon/02.htm`, accessed on October 4, 2022.

[9] `https://www3.nhk.or.jp/news/special/coronavirus/data/`, accessed on October 4, 2022.