Universal Extra Dimension Model on an interval

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Based on the collaboration with
Kin-ya Oda (Osaka University)
arXiv:1011.0405 [hep-ph]
and work in progress

2011 workshop
Physics Beyond the Standard Model and
Predictable Observables at Kobe University

- A 5D field = infinite 4D KK modes.

  (On S^1/Z_2)

  - ★ Dirichlet: $\Phi|_{bd} = 0$,

    - ✳ For A_y,

    - ✳ $\Phi(x,y) = \sum_{n=1} \Phi_n(x) \underline{\sin(ny/R)}$,

    - ✳ $m_{KK} = 1/R, 2/R, ...$

  - ★ Neumann: $\partial_y\Phi|_{bd} = 0$,

    - ✳ For (bulk) SM fields (containing Higgs)

    - ✳ $\Phi(x,y) = \sum_{n=0} \Phi_n(x) \underline{\cos(ny/R)}$,

    - ✳ $m_{KK} = 0, 1/R, 2/R, ...$

- Dark matter candidate = Lightest KK particle
- A few new parameters
- Loose constraint on m_{KK}

How EWSB Occurs in Ordinary UED

$$S = \int d^4 x \int_0^L dy \left(-|\partial_\mu \Phi|^2 - |\partial_y \Phi|^2 - V(\Phi) \right)$$

[Higgs potential]

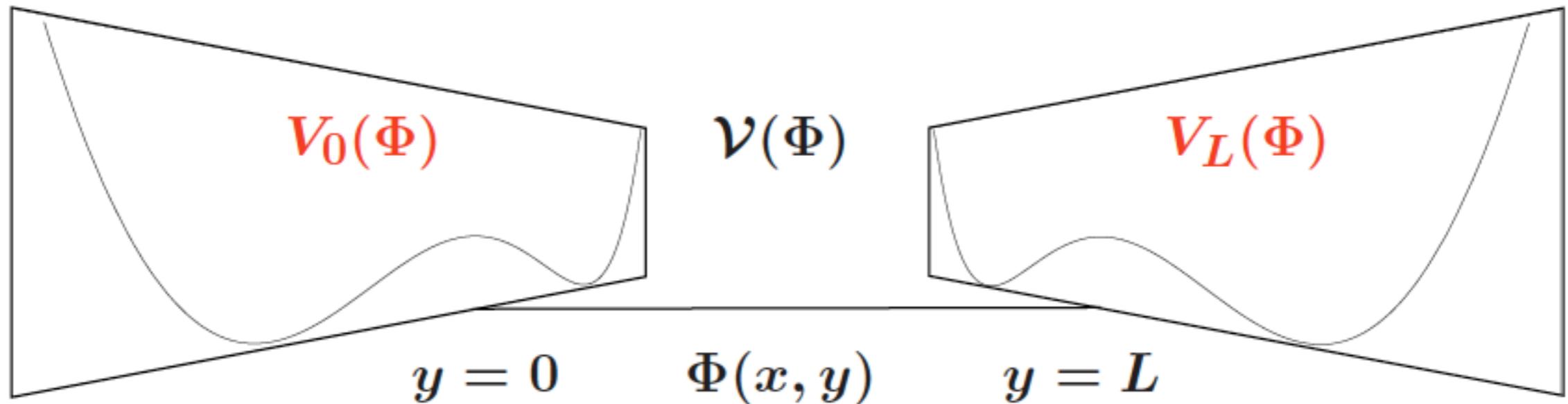$$V(\Phi) = -m^2 |\Phi|^2 + \lambda |\Phi|^4$$

Ordinary Higgs potential

What is the origin of EWSB?
(the source of negative mass squared?)

Bulk Higgs with Brane Potentials on an interval

$$S = \int d^4x \int_0^L dy[-|\partial_M \Phi|^2 - \mathcal{V}(\Phi)$$

[For Scalar]

$$-\delta(y - L)V_L(\Phi) - \delta(y)V_0(\Phi)]$$

$L:$ Compactification length

$V_0(\Phi)$ $\mathcal{V}(\Phi)$ $V_L(\Phi)$

$y = 0$ $\Phi(x, y)$ $y = L$

(From Takahashi-san's slide)   Haba, Oda, Takahashi, 09, 10   4D

Φ = Φᶜ + Φ�q

classical part(VEV)   quantum fluctuation around VEV

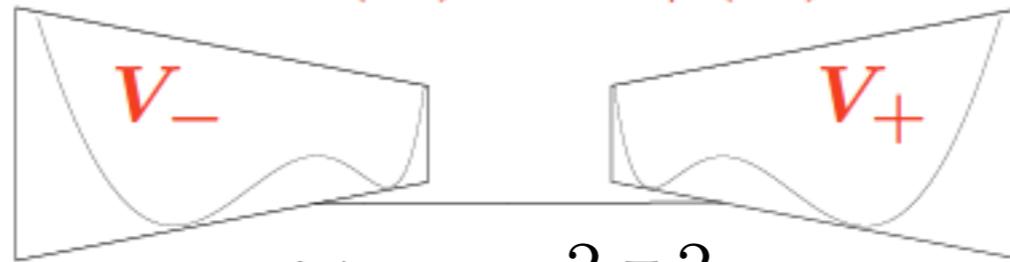Classical part is determined firstly by variational principle.

An Example

$$V_-(\Phi) = V_+(\Phi)$$

$$V_-(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2$$

V_-

V_+

$$\mathcal{V} = m^2 \Phi^2$$

$$V_+(\Phi) = \frac{\lambda}{4}(\Phi^2 - v^2)^2$$

(From Takahashi-san's slide)

VEV profile:

n-th KK mode function:

$$\Phi^c(y) = A\cosh(my) + B\sinh(my)$$
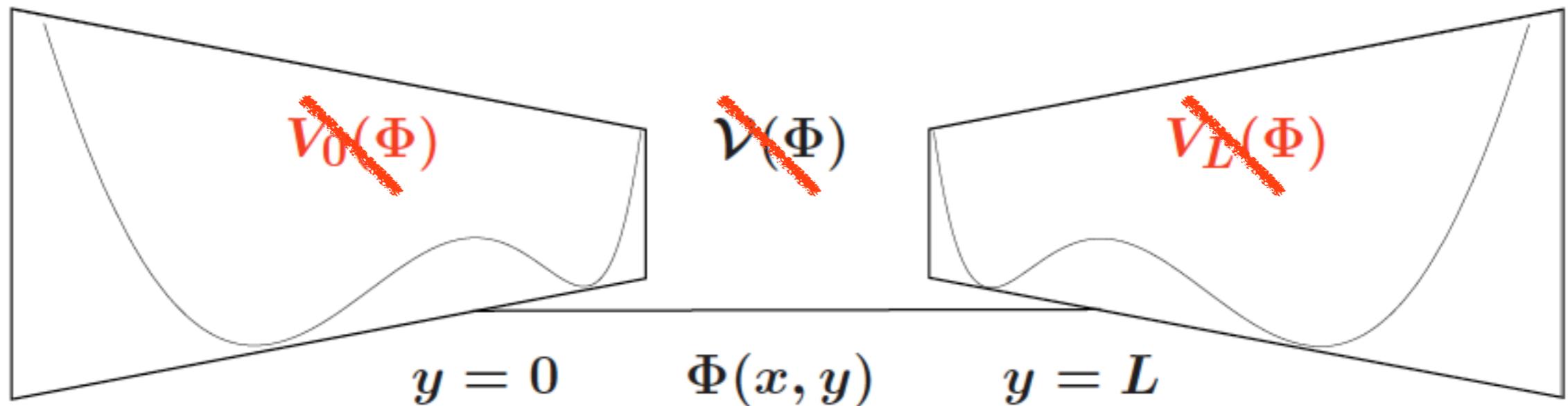
$$f_n(y) = \alpha_n \cos(k_n y) + \beta_n \sin(k_n y)$$

$(k_n{}^2 = m_{KK(n)}{}^2 - m^2)$

- EWSB without negative mass squared
- Position dependent VEV ($\rightarrow$ deviation at Yukawa couplings)

Very Simple Configuration

$$S = \int d^4x \int_0^L dy [-|\partial_M \Phi|^2 - \cancel{\mathcal{V}(\Phi)}$$
$$\cancel{-\delta(y-L)V_L(\Phi)} - \cancel{\delta(y)V_0(\Phi)}]$$

$L:$ Compactification length

$\cancel{V_0(\Phi)}$ $\cancel{\mathcal{V}(\Phi)}$ $\cancel{V_L(\Phi)}$

$y = 0$ $\Phi(x,y)$ $y = L$

(From Takahashi-san's slide)

4D

Surface term: $\int d^4x \left[-\delta\Phi^c \partial_y \Phi^{c\dagger} \right]_{y=0}^{y=L}$

Dirichlet: $\delta\Phi^c|_{bd}$ = 0 rather than $\Phi^c|_{bd}$ = 0!!

$\Phi^c|_{bd}$ = const. suffices!! (EWSB by boundary condition)

EWSB by Non-Zero Dirichlet B.C.

- Our proposal:

  ★ Put non-zero Dirichlet b.c. on Higgs! (No SM Higgs)

    ✳ (with KK-parity being assumed)   Haba, Oda, Takahashi, 09, 10

$$\Phi\big|_{y=0,L} = \begin{bmatrix} \phi_D^1 \\ \phi_D^2 \end{bmatrix} \xrightarrow{\text{field red.}} \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}$$

$$\frac{v}{\sqrt{2}} := \sqrt{|\phi_D^1|^2 + |\phi_D^2|^2}$$

Classical(VEV) profile

$$V(\Phi) = -m^2|\Phi|^2 + \frac{\hat{\lambda}}{4}|\Phi|^4$$

- Merit:

  ★ No need of <u>negative mass-squared</u>, nor <u>quartic coupling</u>.

  ★ Fewer number of free parameters.

  ★ Deviation in Higgs interaction (interesting phenomenologically).

But, there remain two questions...

1. EWSB by b.c. $\underline{\varphi^c|_{bd} = Const}$ looks explicit (well-defined as quantum theory?).

2. Who unitarizes the $W_L W_L$-scattering?

To answer the first question

1. EWSB by b.c. $\varphi^c|_{bd}$ = Const looks explicit (well-defined as quantum theory?).

2. Who unitarizes the $W_L W_L$-scattering?

Background Field Method

$$\Phi = \Phi^c + \Phi^q$$

classical part(VEV)   quantum fluctuation around VEV

Two (gauge) transformations:

Redefinition of VEV

✳ Background gauge transformation

$$\delta\Phi^c = ig\varepsilon\Phi^c \ , \ \delta\Phi^q = ig\varepsilon\Phi^q \ , \ \text{(others)}$$

✳ True gauge transformation

$$\delta\Phi^c = 0 \ , \ \delta\Phi^q = ig\varepsilon(\Phi^c+\Phi^q) \ , \ \text{(others)}$$

The bulk action is invariant under the transformations without new surface term.

→BRST

(By use of $\Phi^c = Const$ and $\Phi^q|_{bd} = 0.$)

Ordinary Dirichlet BC

Our Proposal

True gauge transformation → BRST

Gauge fixing term:

$$S_{\rm GF} = \int d^4x \int_0^L dy \left[-\frac{1}{2\xi} f^a f^a \right],$$

$$f^a = \partial_\mu A^{a\mu} + \xi \partial_5 A^{a5} + \xi \left(ig \left(\Phi^q \right)^\dagger T^a \Phi^c + {\rm h.c.} \right)$$

$$s\Phi^c = 0 \ , \ s\Phi^q = ig\omega(\Phi^c + \Phi^q) \ , \ \text{(others)}$$

(s:BRST operator, ω:ghost)

We can proof that

$$s(S + S_{\rm GF} + S_{\rm ghost}) = 0.$$

This model is well defined as quantum theory!

BRST Transformation for scalar

☑ In 5D (Bulk) picture (Abelian case)

$$s^2\Phi^q = i(s\omega)(\Phi^c + \Phi^q) - i\omega(s\Phi^q)$$

$$= -\omega^2(\Phi^c + \Phi^q) + \omega^2(\Phi^c + \Phi^q) = 0$$

Nilpotent!

$$s\Phi^q = ig\,\omega(\Phi^c + \Phi^q)$$

$$s\Phi^q|_{bd} = ig\omega(\Phi^c + \Phi^q)|_{bd} = ig\,\omega\Phi^c|_{bd}$$

Neumann-like

☑ In 4D (KK) picture (Abelian case)

$$s\Phi^q_n(x) = ig\Phi^c \sum_\ell \omega_\ell(x) \boxed{CS_{\ell,n}} + ig \sum_{\ell,m} \omega_\ell(x)\Phi^q_m(x) \boxed{CSS_{\ell,mn}}$$

$s^2\Phi^q_n = 0$
Nilpotent!

just numbers
(overlap integrals of mode functions)

● $CS_{l,(n=0)}$, $CSS_{l,m(n=0)} = 0$ → No Φ^q_0 fluctuation.

● $\Phi^c\omega_l$ performs like Neumann.
  → On an interval, mode functions act as non-orthonormal.

  $$\int_0^L dy \cos\left(\frac{n\pi y}{L}\right) \neq 0 \ (n : \text{odd})$$

  (On S^1/Z_2, this term vanishes.)
  (This fact is important for unitarization.)

  Natural properties on an interval.
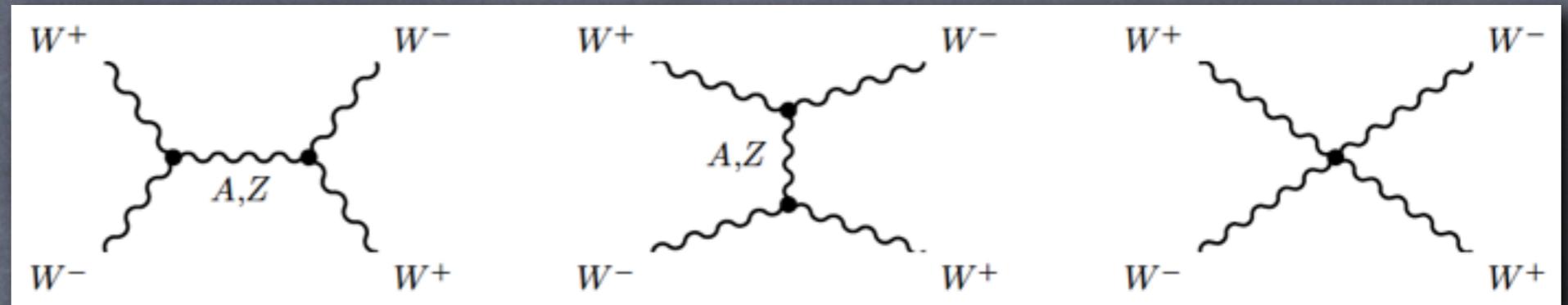
To answer
the second question

1. EWSB by b.c. $\varphi|_{bd}$ = Const looks explicit (well-defined as quantum theory?).

2. Who unitarizes the $W_L W_L$-scattering?

W$_L$W$_L$–Scattering in SM

Unitarity violation occurs if M grows as: <u>M ∝ s.</u>
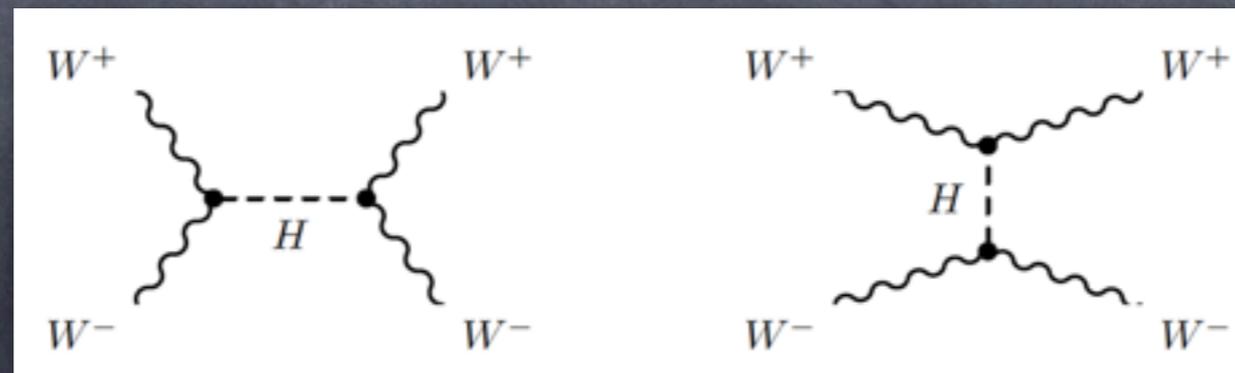(M: scattering amplitude, s: (total energy)^2)

Gauge boson contribution:

Unitarity Violation!

$$\mathcal{M}^{\text{SM gauge only}}_{W_L^+ W_L^- \to W_L^+ W_L^-} = \frac{s(1 - \cos\theta)}{2v_{\text{EW}}^2} + \mathcal{O}(s^0),$$
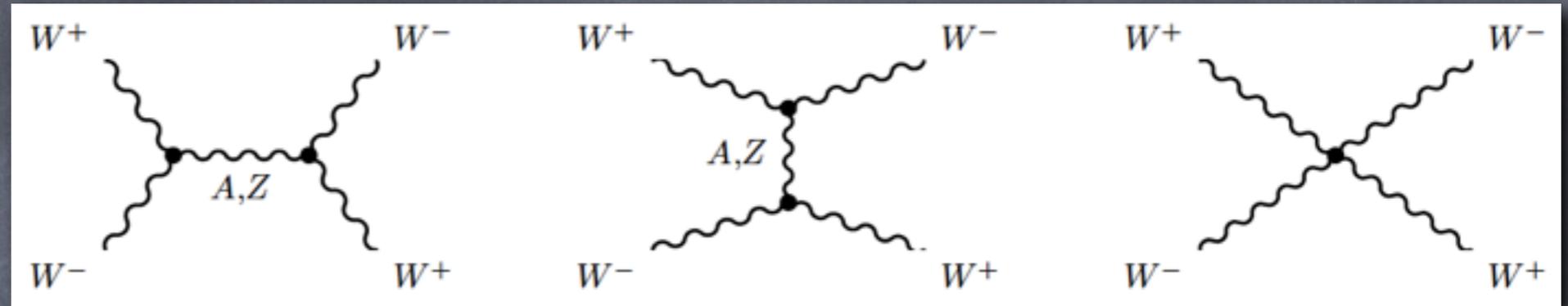
Higgs contribution:

In the SM, Unitarity Violation do not occur because of Higgs.
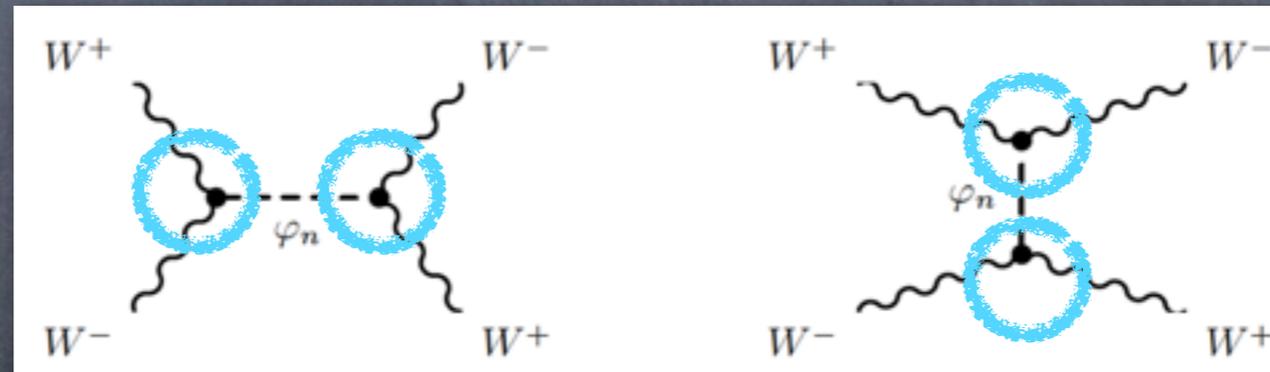
$W_L W_L$–Scattering in Our Model

Gauge boson contribution:

$$\mathcal{M}^{\text{SM gauge only}}_{W_L^+ W_L^- \to W_L^+ W_L^-} = \frac{s\,(1 - \cos\theta)}{2v_{\text{EW}}^2} + \mathcal{O}(s^0),$$

There is no (Zero-mode) Higgs contribution...

KK Higgs contribution:

KK number violating

- Only n=1,3,5,... contributes. (∵KK-parity)
- $g(\varphi_n WW)/g_{\text{SM}} = 2\sqrt{2}/n\pi = 0.9/n$

5D Nature Appears

In High energy limit,

$$\mathcal{M}^{\text{KK Higgs exchange}}_{W_L^+ W_L^- \to W_L^+ W_L^-} \to -\frac{s(1+\cos\theta)}{2v_{\text{EW}}^2} - \frac{\sqrt{2s}}{v_{\text{EW}}^2 \pi R}\left(\sqrt{2}+\sqrt{1-\cos\theta}\right) + \mathcal{O}(s^0)$$

Cancels with SM-gauge contribution

In our model, $M \propto \sqrt{s}$.

- This is five-dimensional effect. $[g_5] = -1/2$
  (Naive Dimensional Analysis: $M \sim g_5^2 \sqrt{s}$)

- Unitarity limit is lower than that of the SM.
  (Natural Result)

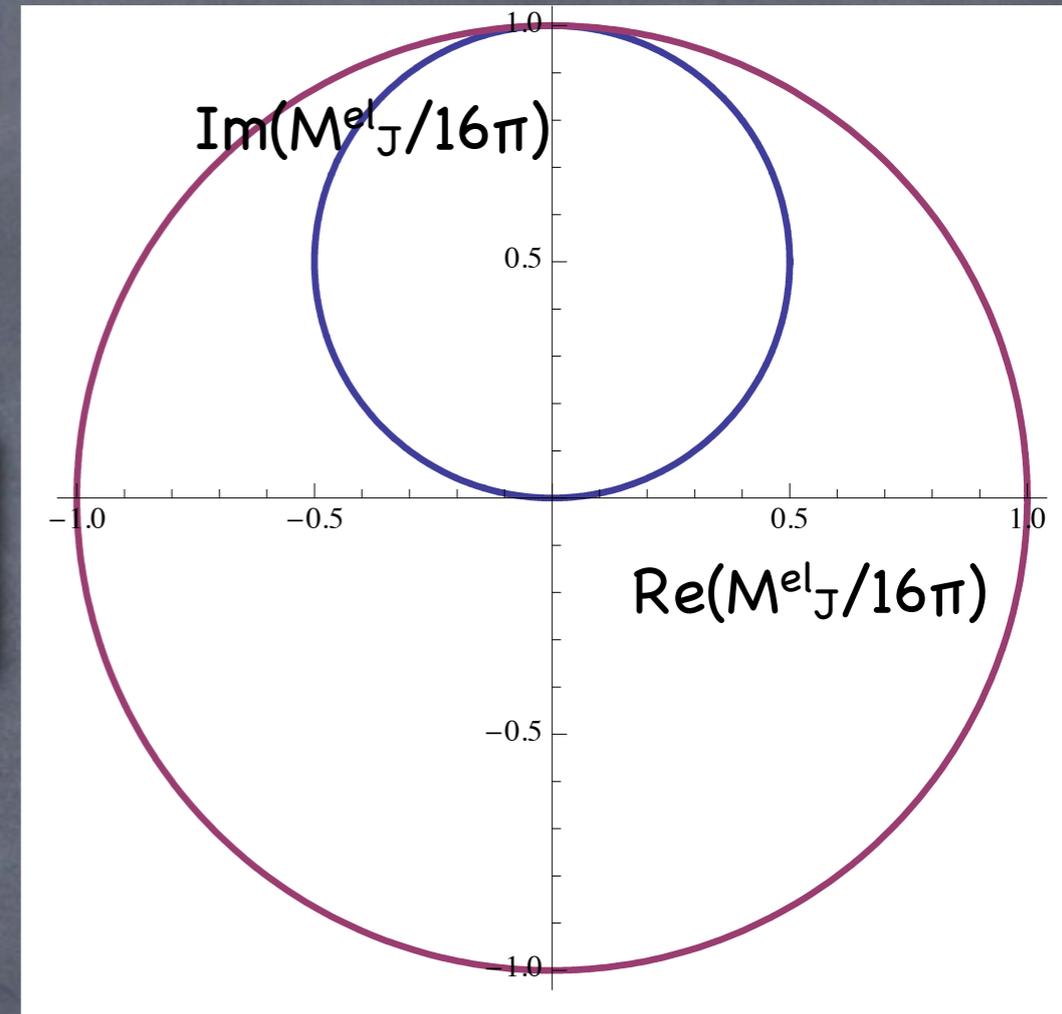# Partial Wave Unitarity

We consider J-th partial wave amplitude.

$$2\text{Im}(T) = |T|^2 > |T^{el}|^2$$

$$\downarrow$$

$$\text{Im}\,\mathcal{M}_J^{el} \geq \frac{1}{16\pi}\left|\mathcal{M}_J^{el}\right|^2$$

$$\xrightarrow{\text{loose condition}}$$

$$1 \geq \frac{1}{16\pi}\left|\mathcal{M}_J^{el}\right|$$

(Tree level) partial wave unitarity condition:

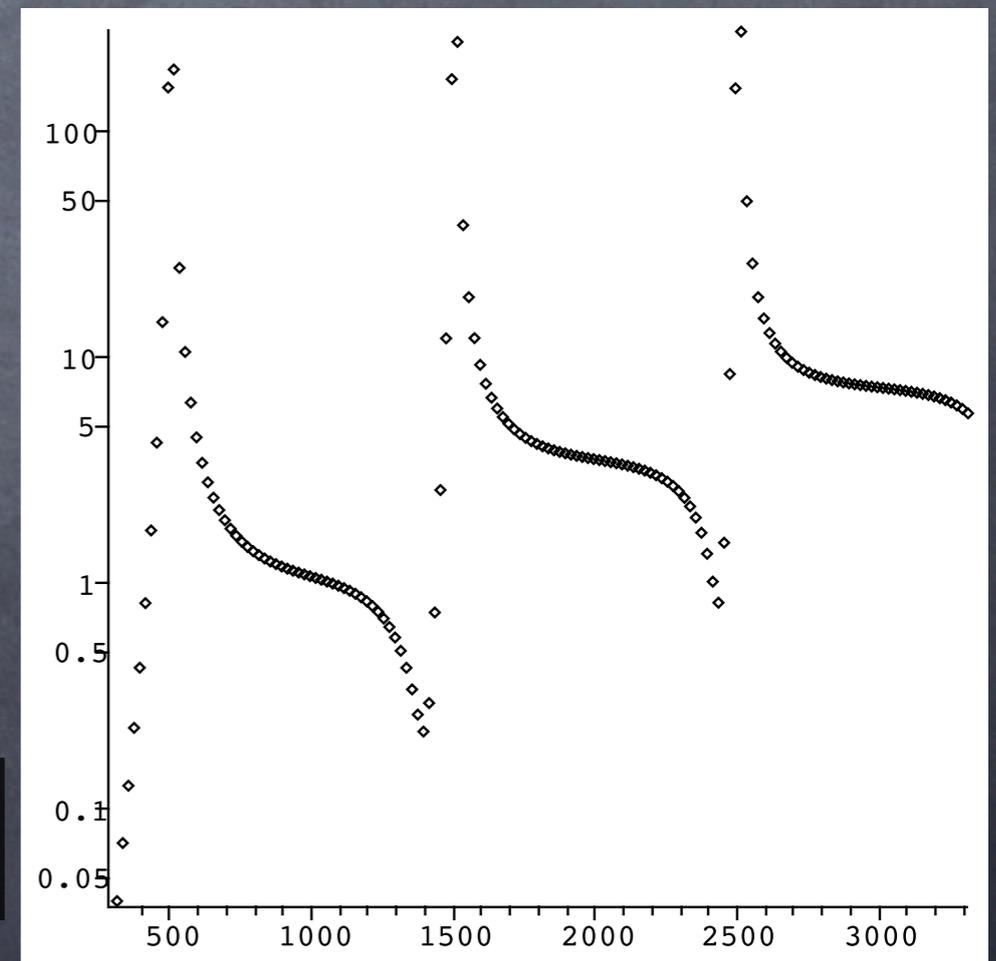$$\sqrt{s} < \frac{24\pi^2 v_{\text{EW}}^2}{5 m_{\text{KK}}} =: \Lambda$$

Haba, Oda, Takahashi, 09, 10

For m_{KK} = 430–500GeV(S-T favored),
we get: Λ = 6.7–5.7TeV.

(Enough room for weekly-coupled 5D theory!)

About first KK Higgs

- 430~500 GeV (S-T favored)

- A candidate for ``Higgs Impostor''
- Coupling to SM particle multiplied by ~0.9

- No self-interaction
- Visible at WW→H→WW?

Preliminary & in arbitrary unit

Summary

- EWSB without Higgs potential

    ★ Non-zero Dirichlet theory is consistent: BRST symmetry

    ★ KK scale at 500GeV

- WW unitarized by infinite KK modes

    ★ 5D gauge, perturbative up to 5~6TeV

- ``Higgs'' impostor and deviation in Higgs potential

    ★ We can that see at the LHC or the ILC?

Thank You
For
Your Attention