Modulus stabilization in Gauge-Higgs Unification

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N. Maru & Y.S., JHEP04 (2010) 100
Y.S., arXiv:1009.5353

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Introduction

Beyond the Standard model

- SUSY → Light Higgs ($m_H \lesssim 130$ GeV)

  (Experimental bound: $m_H \geq 114$ GeV)

- Strong dynamics at TeV scale

  (Little Higgs, Composite Higgs, …)

- Extra dimensions

  (Universal extra dim., 5D Higgsless model,

  Gauge-Higgs unification, …)

Gauge-Higgs unification model

[N. Manton, NPB158 (1979)141, D.B. Fairlie, PLB82(1979)97,
Y. Hosotani, PLB126 (1983) 309, ...]

Higher-dimensional gauge field

$$A_M = (A_\mu, A_y)$$

A_y || Higgs

Higher-dim. gauge symmetry protects the Higgs mass against the radiative corrections.

⟹ **Solution to the gauge hierarchy problem**

Wilson line phase

$$\theta_H \equiv g_5 \int_0^L dy\ A_y(y) \quad (\Leftrightarrow \text{Higgs VEV})$$

The effective potential $V_{\rm eff}(\theta_H)$ is generated at the quantum level.

$\theta_H \neq 0 \pmod{\pi}$

→ **Electroweak symmetry is broken.**

GHU in 5D flat spacetime

[Hall, Nomura, Tucker-Smith, NPB639 (2002) 307, ...]

Mass spectrum

$$m_W = \frac{\theta_H}{L}. \quad (L\text{ : Size of the extra dimension.})$$

$$m_{\rm KK} = \frac{\pi}{L} = \frac{\pi}{\theta_H} m_W$$

→ θ_H must be small.

$$m_H \simeq \frac{g_4 \sqrt{\mathcal{O}(10)} m_W}{4\pi} = \mathcal{O}(10 \text{ GeV}).$$

→ A lot of extra matter fields are necessary.

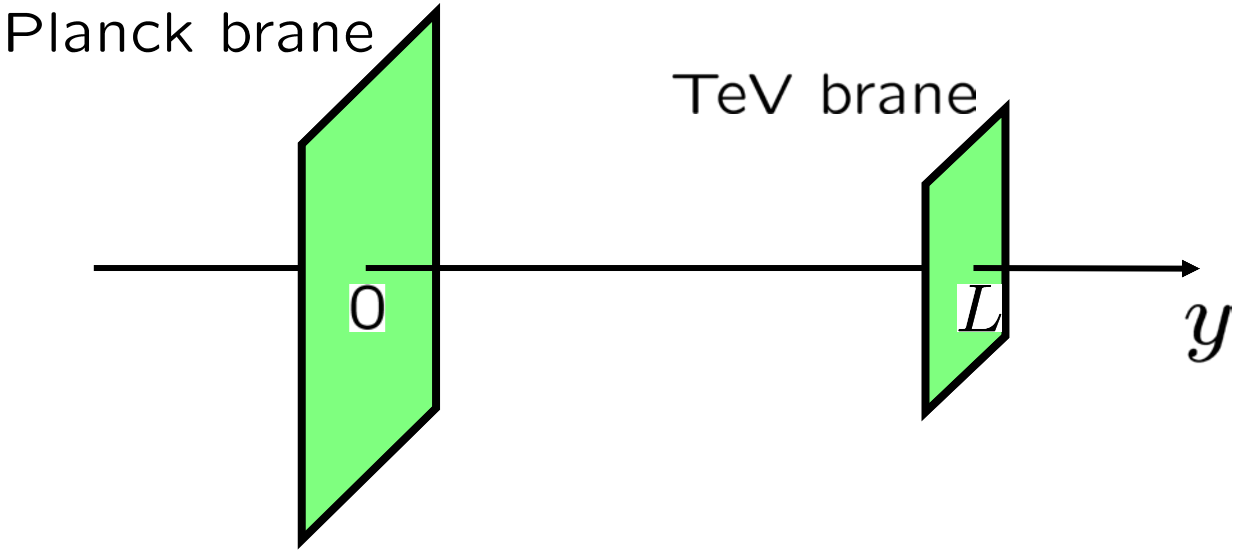
Warped spacetime on S^1/Z_2

[Randall, Sundrum, PRL83 (1999) 3370]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$(0 \le y \le L)$$

(In the original RS, $e^{kL} \sim \frac{M_{\rm Pl}}{\rm TeV} \sim 10^{15}$)



Properties of GHU in warped spacetime

[Hosotani & Y.S., PLB645 (2007) 442; ,...]

Mass spectrum

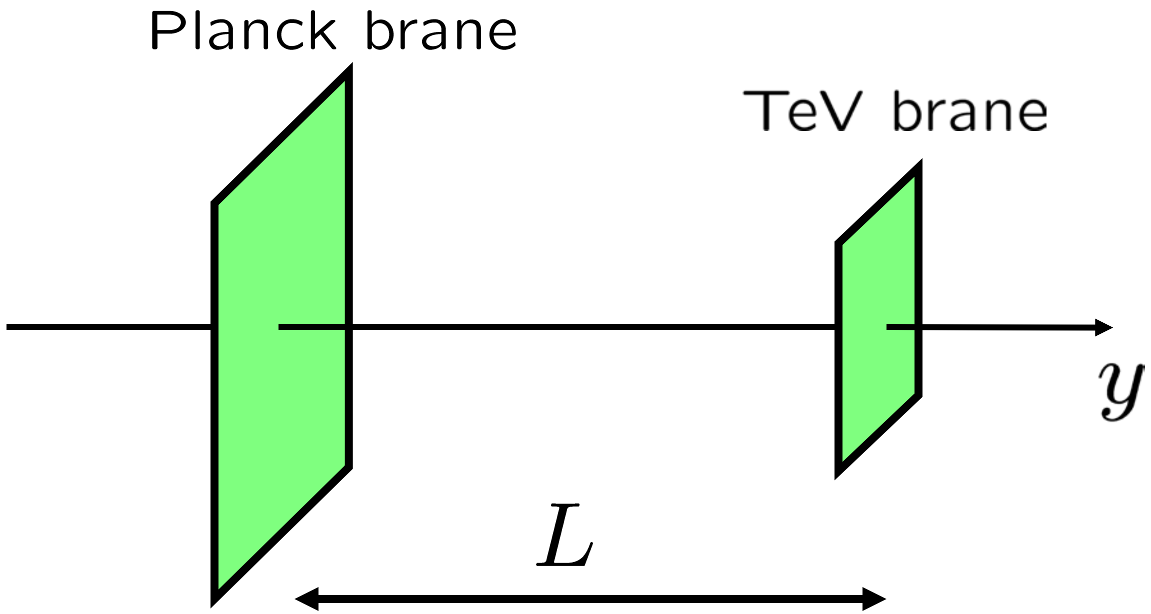
$$\frac{m_{\mathrm{KK}}}{m_W} \simeq \frac{\pi\sqrt{kL}}{\sin\theta_H} \gg 1.$$

$$m_W \simeq \frac{k\sin\theta_H}{e^{kL}\sqrt{kL}}, \quad m_Z \simeq \frac{k\sin\theta_H}{e^{kL}\sqrt{kL}\cos\theta_W}, \cdots$$

Higgs couplings

$$\lambda_{WWH} \simeq g m_W \cos\theta_H,$$
$$\lambda_{ZZH} \simeq \frac{g m_Z}{\cos\theta_W}\cos\theta_H, \cdots$$

Modulus stabilization



L is also a dynamical d.o.f., i.e. **Radion.**

It must be stabilized to a finite value.

Stabilization by a bulk scalar field

[Goldberger & Wise, PRL83 (1999) 4922]

- Modulus is stablized at tree-level.
- An additional 5D scalar field needs to be introduced.

Stablization by Casimir energy

[Garriga, Pujolas, Tanaka, NPB605 (2001) 192;
Goldberger & Rothstein, PLB491 (2000) 339, ...]

- Modulus is stabilized at quantum level.
- Bulk gauge and fermion fields are enough for the stabilization.

No need to introduce extra fields!

Set-up

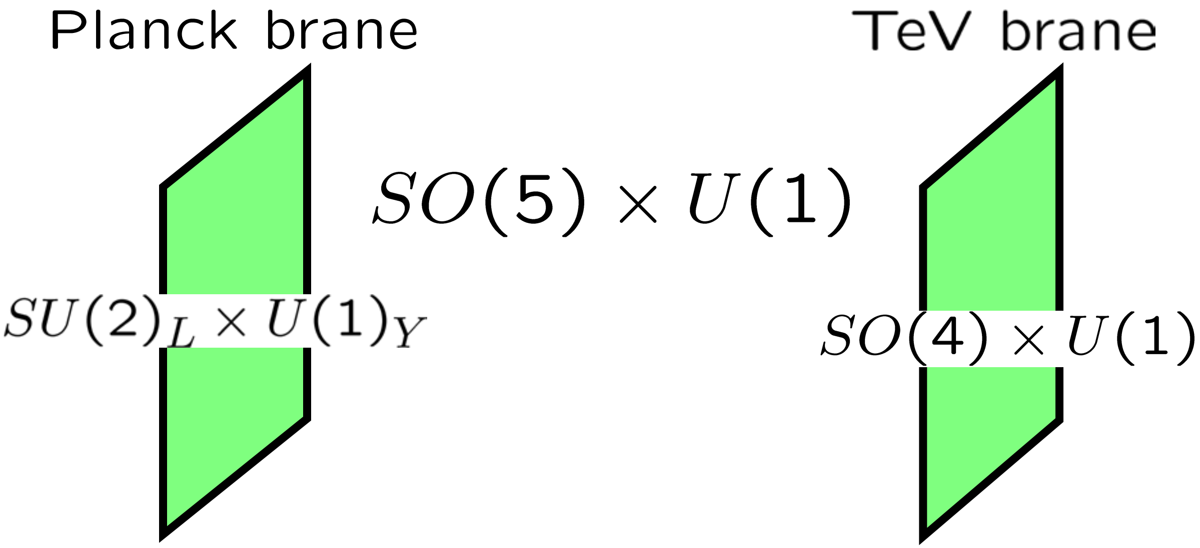
- GHU in warped spacetime
- Extra dim. is stabilized by Casimir energy

Purpose

Evaluate the Higgs and radion masses, and clarify their dependence on e^{kL} and θ_H.

SO(5)xU(1) model

[Agashe, Contino, Pomarol, NPB719 (2005) 165]



Field content

[Hosotani, Oda, Ohnuma, Y.S., PRD78 (2008) 096002]

Gauge field

G_μ : $SU(3)_C$ gauge field

A_μ : $SO(5)$ gauge field

B_μ : $U(1)$ gauge field

Spinor field (quark sector)

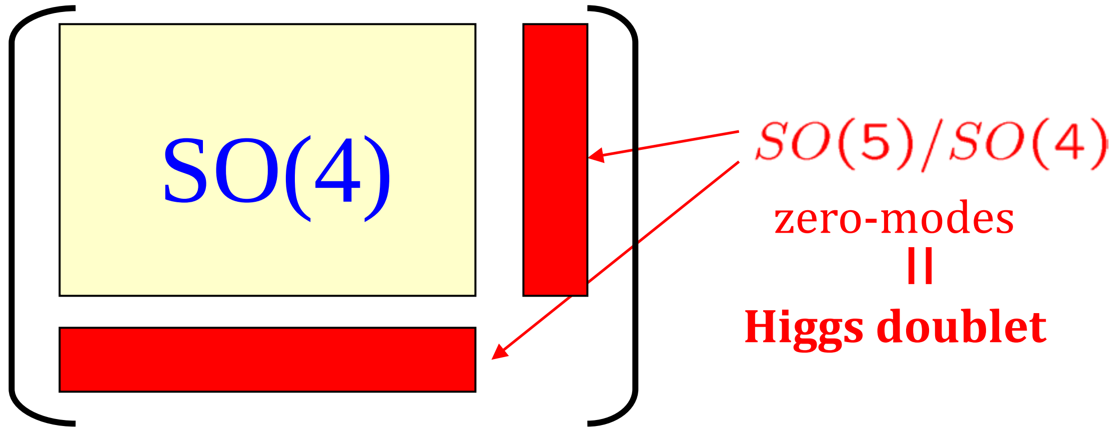
Ψ_1, Ψ_2: $SO(5)$ vector rep.

Color triplets

Gauge symmetry :

$$SO(5) \times U(1) \rightarrow SU(2)_L \times U(1)_Y$$

$$A_y^{SO(5)} =$$



Wilson line phase: $\theta_H = g_5 \int_0^L dy\ A_y \neq 0$

$$\Rightarrow SU(2)_L \times U(1) \rightarrow U(1)_{\text{EM}}$$

Components of spinors

$$SO(5) \supset SO(4) = SU(2)_L \times SU(2)_R$$

$$\Psi_1 = \left[\begin{pmatrix} T \\ B \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, t' \right],$$

$$\Psi_2 = \left[\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} X \\ Y \end{pmatrix}, b' \right].$$

$$Q_{\mathrm{EM}} = \tfrac{5}{3} : T$$

$$Q_{\mathrm{EM}} = \tfrac{2}{3} : B, t, t', U$$ → top quark

$$Q_{\mathrm{EM}} = -\tfrac{1}{3} : b, D, X, b'$$ → bottom quark

$$Q_{\mathrm{EM}} = -\tfrac{4}{3} : Y$$

Effective potential

We promote $kL \to \varphi(x), \quad \theta_H \to \theta_H(x).$

radion Higgs

In this model,

$$V_{\rm eff}(\varphi, \theta_H) \simeq V_0(\varphi) + V_2(\varphi)\cos^2\theta_H$$

Stationary condition for θ_H

$$\sin 2\theta_H = 0 \quad \Longrightarrow \quad \theta_H = 0, \frac{\pi}{2}$$

Due to the top quark contribution,

$V_{\rm eff}$ has a minimum at $\theta_H = \frac{\pi}{2}$. ⟹ **EW breaking**

Radion mass

The radion mass is calculated by

$$m_{\rm rad}^2 = \frac{k(e^{2kL}-1)}{3M_5^3}\,\partial_\varphi^2 V_{\rm eff}\Big|_{\varphi=kL,\theta_H=\frac{\pi}{2}}$$

In our model, the modulus is not stabilized due to a large negative contribution of the gluon loop to $m_{\rm rad}^2$.

This problem can be solved by introducing brane kinetic terms for the gauge fields on the TeV brane.

[Garriga & Pomarol, PLB560 (2003) 91]

$$\mathcal{L}_{\rm brane} = 2\sqrt{-g}\left[-\frac{\kappa}{4k}{\rm tr}\{F_{\mu\nu}^{(G)}F^{(G)\mu\nu}\} - \frac{\kappa}{4k}{\rm tr}\{F_{\mu\nu}^{(A)}F^{(A)\mu\nu}\} - \frac{\kappa}{4k}F_{\mu\nu}^{(B)}F^{(B)\mu\nu}\right]\delta(y-L)$$

At the vacuum $\theta_H = \dfrac{\pi}{2}$,

the WWH, ZZH, Yukawa couplings vanish.

$$\left(\begin{aligned} \lambda_{WWH} &\simeq g m_W \cos\theta_H , \\ \lambda_{ZZH} &\simeq \frac{g m_Z}{\cos\theta_W} \cos\theta_H , \cdots \end{aligned} \right)$$

⇓

Higgs boson becomes stable, and can be a dark matter.

[Hosotani, Ko, Tanaka, PLB680 (2009) 179]

LEP constraint

$$S \lesssim 0.3$$

This is translated into

$$\sin\theta_H \lesssim 0.3\text{-}0.5$$

[Agashe, Contino, Pomarol, NPB719 (2005) 165]

This condition can be satisfied by introducing additional fermion fields.

Models with smaller θ_H

Model I Introduce Ψ_3 : $SO(5)$ spinor rep.
(color triplet)

$$V_{\rm eff}(\varphi,\theta_H) \simeq V_0(\varphi) + V_1(\varphi)\cos\theta_H + V_2(\varphi)\cos^2\theta_H$$

$$\Rightarrow \quad \sin\theta_H = 0, \qquad \cos\theta_H = -\frac{V_1}{2V_2}$$

Model II Introduce $\Phi_1,\ \Phi_2$: $SO(5)$ vector rep.
and boundary mass terms.
[Contino, Da Rold, Pomarol, PRD75 (2007) 055014;
Medina, Shah, Wagner, PRD76 (2007) 095010]

$$V_{\rm eff}(\varphi,\theta_H) \simeq V_0(\varphi) + V_2(\varphi)\cos^2\theta_H + V_4(\varphi)\cos^4\theta_H$$

$$\Rightarrow \quad \sin\theta_H = 0, \qquad \cos^2\theta_H = -\frac{V_2}{2V_4}$$

In both models,

$$\begin{cases} m_{\rm rad} \sim \dfrac{e^{kL}\sqrt{kL+\kappa}\, m_W^2}{4\sqrt{6}\pi M_{\rm Pl} \sin^2\theta_H} \times \mathcal{O}(10), \\ m_H \sim \dfrac{g_4\sqrt{kL+\kappa}\, m_W}{4\sqrt{2}\pi} \times \mathcal{O}(10), \end{cases}$$

and **the radion-Higgs mixing is negligible**.

Consistency condition

$$(\text{Curvature scale}) < (\text{Fundamental scale})$$

$$|\mathcal{R}_5| = 20k^2 < M_5^2 \quad \Longleftrightarrow \quad \zeta \equiv \frac{M_5}{k} > 4.5$$

By using $M_5^3 \simeq 2kM_{\text{Pl}}^2, \quad m_W \simeq \dfrac{k \sin\theta_{\text{H}}}{e^{kL}\sqrt{kL+\kappa}},$

$$e^{kL}\sqrt{kL+\kappa} \simeq \frac{M_{\text{Pl}}}{m_W}\sqrt{\frac{2}{\zeta^3}}\sin\theta_H \lesssim 4.5\times 10^{15}\sin\theta_H$$

In the following, we focus on a parameter region:

$$0.06 \le \sin\theta_H \le 0.3$$

$$\Longrightarrow \quad e^{kL} \lesssim 5\times 10^{13}$$

When $e^{kL} = 5 \times 10^{13}$,

$$\begin{cases} m_{\text{rad}} \sim \dfrac{\mathcal{O}(0.1)\ \text{GeV}}{\sin^2\theta_H}, \\ m_H \sim \mathcal{O}(100)\ \text{GeV}. \end{cases}$$

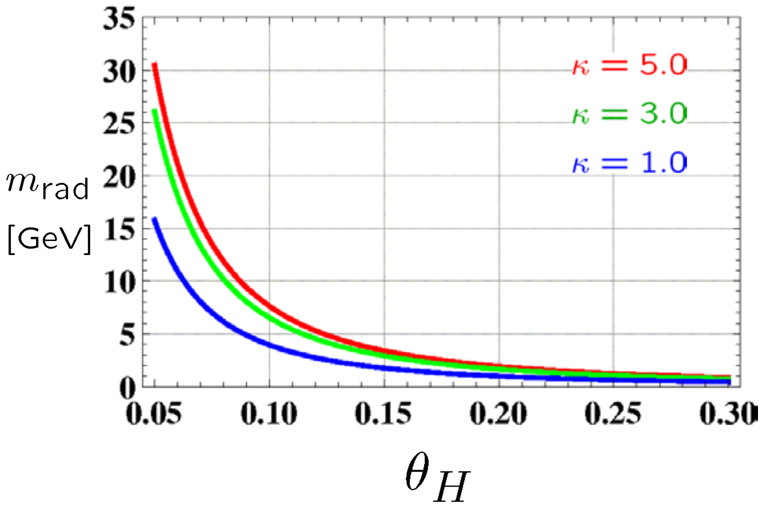
c.f.) The typical KK mass scale is given by

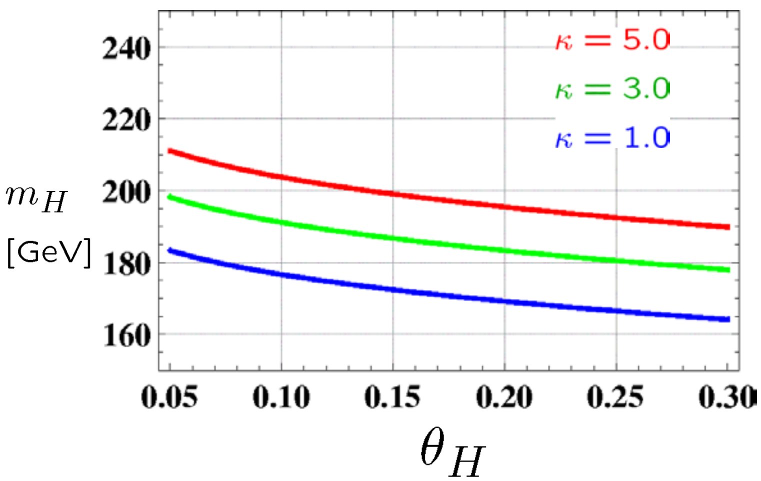
$$m_{\text{KK}} \equiv \frac{k\pi}{e^{kL}-1} \simeq \frac{\pi\sqrt{kL+\kappa}\, m_W}{\sin\theta_H} \simeq \frac{1.4\text{TeV}}{\sin\theta_H},$$

for $\kappa = 1.0$.

Scalar masses (in model I)

$(e^{kL} = 5 \times 10^{13})$



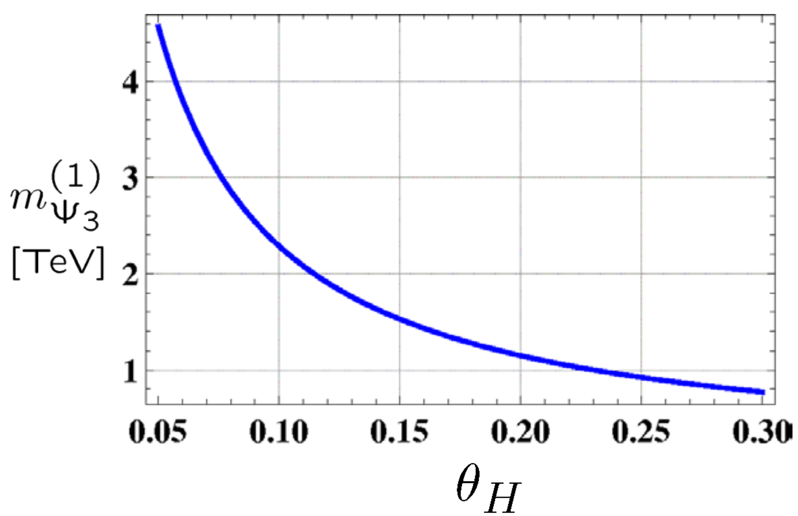


$$\begin{cases} m_{\rm rad} \sim \dfrac{0.05\,{\rm GeV}}{\sin^2\theta_H}, \\ m_H \sim 150\,{\rm GeV} + |\mathcal{O}(10 \ln \sin\theta_H)|\,. \end{cases}$$

Kaluza-Klein mass

The lightest KK mode comes from Ψ_3.
(when $\kappa < 20$.)

$$m_{\Psi_3}^{(1)} \simeq \frac{\sqrt{kL+\kappa}}{2\sin\theta_H} m_W \simeq \frac{230\text{GeV}}{\sin\theta_H}, \quad \text{for } \kappa = 1.0.$$



Radion couplings

$$m_I(kL) \to m_I\left(kL + \frac{r(x)}{\Lambda_r}\right), \qquad (I = W, Z, t, b, \cdots)$$

where $\Lambda_r \equiv \sqrt{\frac{3}{2}}\frac{M_{\mathsf{Pl}}}{\mathsf{sinh}\, kL}$.

The radion couplings are obtained by expanding them in terms of r/Λ_r.

$$(\text{Radion coupling}) = \mathcal{O}(1) \times \frac{v}{\Lambda_r}(\text{Higgs coupling})$$

$$(v = 246 \text{ GeV})$$

In our case, $\frac{v}{\Lambda_r} \simeq 2 \times 10^{-3}$

No constraints on m_{rad} from the collider experiments.

FCNC bound

If the fermion mass hierarchy is realized by the wave function localization,

$$\Lambda_r m_{\rm rad} > 2.3 a_{ds}\ \text{TeV}^2,\ \text{where } 0.03 < a_{ds} < 0.12.$$

[Azatov, Toharia, Zhu, PRD80 (2009) 031701]

The most stringent bound on $m_{\rm rad}$ is

$$\begin{cases} m_{\rm rad} > 2.7\ \text{GeV} & \text{for } e^{kL} = 5.0 \times 10^{13}, \\ m_{\rm rad} > 10\ \text{GeV} & \text{for } e^{kL} = 2.3 \times 10^{14}. \end{cases}$$

$$\left(\sin\theta_H = 0.3, \quad \frac{M_5}{k} = 4.5\right)$$

This can restrict the allowed region of θ_H in some cases.

Summary

- Modulus stabilization by Casimir energy is economical in the GHU scenario.
- Brane kinetic terms of the gauge fields are necessary for stabilization.
- The radion-Higgs mixing is negligible.
- The radion mass is sensitive to e^{kL} and θ_H, while the Higgs mass is not.
- The radion couplings are too weak to detect the radion at colliders.

Mass eigenvalues

$$\begin{cases} m_{\text{rad}} \sim \dfrac{e^{kL}\sqrt{kL+\kappa}\, m_W^2}{4\sqrt{6}\pi M_{\text{Pl}} \sin^2\theta_H} \times \mathcal{O}(10), \\ m_H \sim \dfrac{g_4\sqrt{kL+\kappa}\, m_W}{4\sqrt{2}\pi} \times \mathcal{O}(10), \\ m_{\text{KK}} \sim \dfrac{\pi\sqrt{kL+\kappa}\, m_W}{\sin\theta_H} \times \mathcal{O}(1). \end{cases}$$

When $e^{kL} = 5 \times 10^{13}$, (model I)

$$\begin{cases} m_{\text{rad}} \sim \dfrac{0.05\text{GeV}}{\sin^2\theta_H}, \\ m_H \sim 150\text{GeV} + \left|\mathcal{O}(10 \ln \sin\theta_H)\right|, \\ m_{\Psi_3}^{(1)} \simeq \dfrac{230\text{GeV}}{\sin\theta_H}. \end{cases}$$

Nonlinear VEV-dependence of masses

$$\mathcal{L} = -\frac{1}{2}\left(W_\mu, W'_\mu\right)^\dagger \begin{pmatrix} g^2\phi^2 & h^2\phi^2 \\ h^2\phi^2 & M^2 + g'^2\phi^2 \end{pmatrix} \begin{pmatrix} W^\mu \\ W'^\mu \end{pmatrix} + \cdots$$

$$= -\frac{1}{2}\left(W_\mu, W'_\mu\right)^\dagger \begin{pmatrix} g^2v^2 & h^2v^2 \\ h^2v^2 & M^2 + g'^2v^2 \end{pmatrix} \begin{pmatrix} W^\mu \\ W'^\mu \end{pmatrix}$$

$$- \left(W_\mu, W'_\mu\right)^\dagger \begin{pmatrix} g^2 & h^2 \\ h^2 & g'^2 \end{pmatrix} \begin{pmatrix} W^\mu \\ W'^\mu \end{pmatrix} \tilde{\phi} + \cdots ,$$

where $\phi = v + \tilde{\phi}$.

The lightest mass eigenvalu is

$$m_0^2 = \frac{1}{2}\left\{g^2v^2 + M^2 + g'^2v^2 - \sqrt{\left\{(M^2 + g'^2v^2) - g^2v^2\right\}^2 + 4h^4v^4}\right\}$$

$$\left(h = 0 \quad \Longrightarrow \quad m_0 = gv \right)$$

Constraints from Higgs search at LEP

The following mass ranges are excluded.

$$90\ \text{GeV} < m_{\text{rad}} < 110\ \text{GeV, if } (v/\Lambda_r)^2 > 0.1,$$

$$12\ \text{GeV} < m_{\text{rad}} < 90\ \text{GeV, if } (v/\Lambda_r)^2 > 0.01.$$

[Barate *et al*., PLB565 (2003) 61]

$$m_{\text{rad}} < 12\ \text{GeV, if } (v/\Lambda_r)^2 > 0.1.$$

[Acton *et al*., PLB268 (1991) 122]

Radion decay

$$\Gamma_{r\to f\bar{f}} \;\sim\; \left(\frac{v}{\Lambda_r}\right)^2 \Gamma_{H\to f\bar{f}}\Big|_{m_H=m_{\rm rad}}$$

When $m_{\rm rad} = 5$ GeV and $e^{kL} = 5\times 10^{13}$,

$$\Gamma_{H\to\tau\bar{\tau}}|_{m_H=m_{\rm rad}} \sim \frac{m_{\rm rad} m_\tau^2}{8\pi\Lambda_r^2}\left(1-\frac{4m_\tau^2}{m_{\rm rad}^2}\right)^{3/2} \sim 10^{-2}\ \text{eV}.$$

Thus, the life-time of the radioin is estimated as

$$\tau_r \lesssim 10^{-14}\ \text{s}$$

Parameters

$$M_5,\ k,\ e^{kL},\ g_A,\ g_B,\ M_1,\ M_2,\ M_3,\ \kappa$$

Input parameters

$$e^{kL} = 5 \times 10^{13}, \quad M_{\mathrm{Pl}}^2 \simeq \frac{M_5^3}{2k}, \quad g_4 \simeq \frac{g_A\sqrt{k}}{\sqrt{kL+\kappa}},$$

$$\sin\theta_W \simeq \frac{g_B^2}{g_A^2 + 2g_B^2}, \quad m_W \simeq \frac{k\sin\theta_H}{e^{kL}\sqrt{kL+\kappa}},$$

$$m_t \simeq m_t(M_1), \quad m_b \simeq m_b(M_1, M_2)$$

Free parameters

$$\left(\frac{M_3}{k},\ \ \kappa\right) \quad \text{or} \quad (\theta_H,\ \ \kappa)$$

Mass spectrum $(m_n = k\lambda_n)$

written in terms of Bessel functions

$$\rho^I(\lambda_n, \theta_H) \equiv \rho_0^I(\lambda_n) + \rho_2^I(\lambda_n)\cos^2\theta_H = 0.$$

All mass spectra are periodic functions of θ_H with π.

e.g.)

$$m_W \simeq \frac{k|\sin\theta_H|}{e^{kL}\sqrt{kL}}, \quad m_Z \simeq \frac{k|\sin\theta_H|}{e^{kL}\sqrt{kL}\cos\theta_W}, \cdots$$

Stationary condition for φ

$$-4\tau_{\text{IR}} + \partial_\varphi \widehat{V}(kL, \langle\theta_H\rangle) - 4\widehat{V}(kL, \langle\theta_H\rangle) = 0$$

τ_{IR} is determined for a given value of kL.

(τ_{UV} is determined so that the 4D cosmological constant vanishes.)

Canonical normalization

$$\begin{cases} r(x) \equiv \sqrt{\dfrac{3M_5^3}{k(e^{2kL}-1)}}\tilde{\varphi}(x), \\ h(x) \equiv \sqrt{\dfrac{4k}{g_5^2(e^{2kL}-1)}}\tilde{\theta}_H(x). \end{cases}$$