Flavor & CP Violations in Gauge-Higgs Unification

Nobuhito Maru
(Chuo University)

1/7/2011 Workshop
"Physics beyond the Standard Model & Predictable Observables"@Kobe

References

“Flavor Mixing in Gauge-Higgs Unification”
Y.Adachi, N.Kurahashi, C.S.Lim & N.M.
JHEP1011 150 (2010)

“CP Violation due to Compactification”
C.S.Lim, N.M. & K.Nishiwaki
PRD81 076006 (2010)

“Neutron Electric Dipole Moment in the Gauge-Higgs Unification”
Y.Adachi, C.S.Lim & N.M.
PRD80 055024 (2010)

PLAN

- Introduction
- Flavor Violation
  - Mechanism, $K^0 - \bar{K}^0$ mixing
- CP Violation
  - by Higgs VEV
  - by Compactification
- Summary

Introduction

Unsettled problems in Higgs sector of the SM

- ☆ *Hierarchy problem*
- ☆ *Origin of fermion mass hierarchy & flavor mixings*
- ☆ *Origin of CP violation*

We now discuss "Gauge-Higgs unification" scenario as an attractive candidate of New Physics, which is expected to shed some lights on these problems by higher dimensional gauge symmetry

Gauge-Higgs Unification

Zero mode of the extra component of higher dimensional gauge field = SM Higgs

Manton (79), Fairlie (79),
Hosotani(83)

↓

- Higgs mass is forbidden at tree level
- Radiatively generated Higgs mass is finite & cutoff independent

due to higher dim. gauge symmetry

This fact opens up an avenue to solve the hierarchy problem w/o SUSY

Hatanaka, Inami & Lim (98)

Explicit calculations of Higgs mass

- D-dim QED on S^1@1-loop — Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2@1-loop — Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2@1-loop — Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2@1-loop — Lim, N.M. & Hasegawa (2006)
- 5D QED on S^1@2-loop — N.M. & Yamashita (2006); Hosotani, N.M., Takenaga & Yamashita (2007)
- 5D Gravity on S^1 (GGH) — Hasegawa, Lim & N.M. (2004)

…

In gauge-Higgs unification, the following issues are nontrivial

1: Realizing Yukawa hierarchy

2: Accommodating the flavor mixing

3: CP violation

∵ Yukawa coupling is originated
from the gauge coupling
which is real and universal for all flavors

In this talk,
we discuss how these issues are incorporated
in the gauge-Higgs unification scenario

Flavor Violation

Adachi, Kurahashi, Lim & Maru,
JHEP1011 015 (2010)

As a new feature of higher dimensional models with Z_2 orbifold, Z_2-odd bulk masses are allowed

$$M_i \varepsilon(y) \bar{\psi}_i \psi_i \quad (\varepsilon(y): \text{sign function})$$

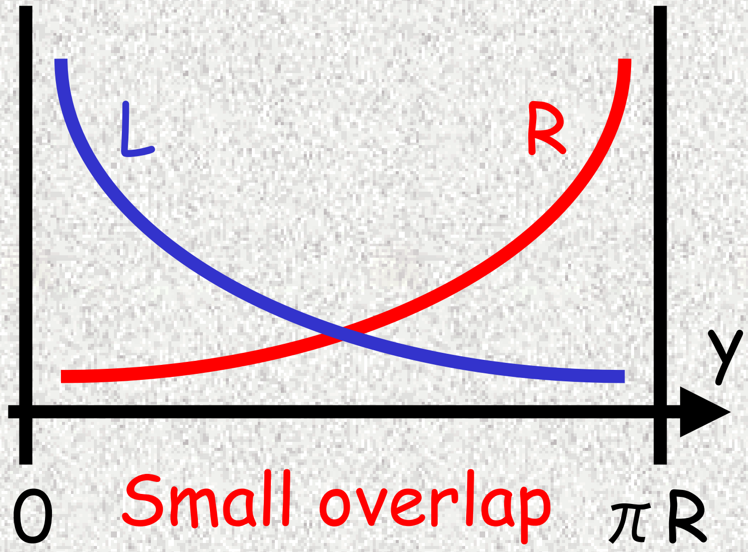
with M_i being different depending on each flavor

↓

New source of flavor violation specific to higher dimensional models

The bulk mass controls the location of 0 mode fermions localization

$$f_L^{(0)}(y) = \sqrt{\frac{M_i}{1-e^{-2\pi M_i R}}}e^{-M_i|y|}, \quad f_R^{(0)}(y) = \sqrt{\frac{M_i}{e^{2\pi M_i R}-1}}e^{M_i|y|}$$



4D effective Yukawa coupling

$$Y = g_4 \int_{-\pi R}^{\pi R} dy f_L^{(0)}(y) f_R^{(0)}(y) \approx 2\pi M R g_4 e^{-\pi M R} \le g_4$$

No need of unnatural fine-tuning for 5D parameters due to exponential suppression

Getting a viable top mass is not trivial, but possible Cacciapaglia, Csaki & Park (2005)

At first glance,
the bulk masses can be off-diagonal in flavor space,
which seems to generate flavor mixing

Unfortunately, it is not the case:
For each representation R of the gauge group,
a general form of bulk mass terms

$$M(R)_{ij}\,\varepsilon(y)\bar{\psi}(R)_i\,\psi(R)_j$$

can be always diagonalized
by a suitable unitary transformation,
leaving the kinetic term invariant

We are led to introduce
brane localized mass terms,
which are the sources of flavor mixing &
are also necessary to make exotics heavy
as will be seen below

Model

5D SU(3) model compactified on S^1/Z_2

N-generations of bulk fermion are introduced

$$\psi^i(3) = Q_3^i \oplus d^i$$

$$\psi^i(\overline{6}) = \Sigma^i \oplus Q_6^i \oplus u^i$$

(i = 1, …, N)

Need to eliminate the redundant quark doublets (Q) and exotics (Σ)

→ **Brane localized mass terms**

Burdman & Nomura (2003)

$$\mathcal{L} = \bar{\psi}_3^i \left(i \not{D} - M^i \varepsilon(y) \right) \psi_3^i + \bar{\psi}_{\bar{6}}^i \left(i \not{D} - M^i \varepsilon(y) \right) \psi_{\bar{6}}^i$$

"generation dependent" bulk masses

$$+ \delta(y) \sqrt{2\pi R} \bar{Q}_R^i(x) \left[\eta_{ij} Q_{3L}^j(x,y) + \lambda_{ij} Q_{\bar{6}L}^j(x,y) \right] + \cdots$$

Brane localized fields

Brane mass matrices (off-diagonal elements are generically allowed)

⇒ "Flavor mixing"

$$\mathcal{L}_{BM}^{Q} \sim \delta(y) \bar{Q}_R [\eta \;\; \lambda] \begin{bmatrix} Q_3 \\ Q_6 \end{bmatrix}_L = \delta(y) \bar{Q}'_R \left[m_{diag} \;\; 0 \right] \begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L$$

$$\begin{bmatrix} Q_3 \\ Q_6 \end{bmatrix}_L = \begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix} \begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L, \quad U^{\bar{Q}} Q_R = Q'_R$$

"2Nx2N unitary matrix"

Yukawa coupling

$$\mathcal{L}_{Yukawa} = g_5 A_y^6 \bar{d}^i Q_3^i + g_5 A_y^6 \bar{u}^i Q_6^i$$

$$\rightarrow g_5 \langle A_y^6 \rangle \left(\bar{d}_R^{i(0)} Y_d^{ii} U_3^{ij} Q_{SM}^{j(0)} + \bar{u}_R^{i(0)} Y_u^{ii} U_4^{ij} Q_{SM}^{j(0)} \right)$$

Yukawa coupling with flavor mixing

$$Y^{ii} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

4D effective Yukawas $Y_u U_3$, $Y_d U_4$ are diagonalized in a usual way

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger Y_d U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger Y_u U_4 V_{uL} \end{cases} \quad V_{CKM} = V_{uL}^\dagger V_{dL} \quad \left(U_3^\dagger U_3 + U_4^\dagger U_4 = 1_{N\times N} \right)$$

$M_{3,6} \propto 1$ ($Y_{u,d} \propto 1$) case (flavor symmetry restored)

$$\begin{cases} \hat{Y}_d \sim V_{dR}^\dagger U_3 V_{dL} \rightarrow \hat{Y}_d^\dagger \hat{Y}_d \sim V_{dL}^\dagger U_3^\dagger U_3 V_{dL} \\ \hat{Y}_u \sim V_{uR}^\dagger U_4 V_{uL} \rightarrow \hat{Y}_u^\dagger \hat{Y}_u \sim V_{uL}^\dagger U_4^\dagger U_4 V_{uL} \end{cases} \xrightarrow{U_3^\dagger U_3 + U_4^\dagger U_4 = 1} V_{uL} \propto V_{dL}$$

$$\Rightarrow V_{CKM} = V_{uL}^\dagger V_{dL} \propto V_{dL}^\dagger V_{dL} = 1 \text{(No mixing)}$$

Lesson

To get flavor mixing,

we need non-degenerate bulk masses

as well as the off-diagonal brane masses

(characteristic to gauge-Higgs unification)

Natural flavor conservation

FCNC has played crucial roles
in the discussion of the viability of New Physics

We ask if "natural flavor conservation" is satisfied,
i.e. if FCNC processes at tree level are forbidden

Glashow-Weinberg condition

"Fermions with the same electric charges, chirality
should have the same isospin (I_3)"

Glashow & Weinberg (1977)

The condition is satisfied in the down sector relevant to K-Kbar mixing @tree level

Parity assignment

$$\mathbf{3} = \begin{cases} \mathbf{2}_{L1/6}\,(Q)\,(+) + \mathbf{1}_{L-1/3}\,(-) \\ \mathbf{2}_{R1/6}\,(-) + \mathbf{1}_{R-1/3}\,(d_R)\,(+) \end{cases}$$

$$\mathbf{6}^* = \begin{cases} \mathbf{3}_{L-1/3}\,(-) + \mathbf{2}_{L1/6}\,(Q)\,(+) + \mathbf{1}_{L2/3}\,(-) \\ \mathbf{3}_{R-1/3}\,(+) + \mathbf{2}_{R1/6}\,(-) + \mathbf{1}_{R2/3}\,(u_R)\,(+) \end{cases}$$

"exotic"

$$3_{\uparrow R}(2/3) \oplus 3_{0R}(-1/3) \oplus 3_{\downarrow R}(-4/3)$$

Same quantum number as down quark

"Not" the end of the story

$K^0 - \bar{K}^0$ mixing

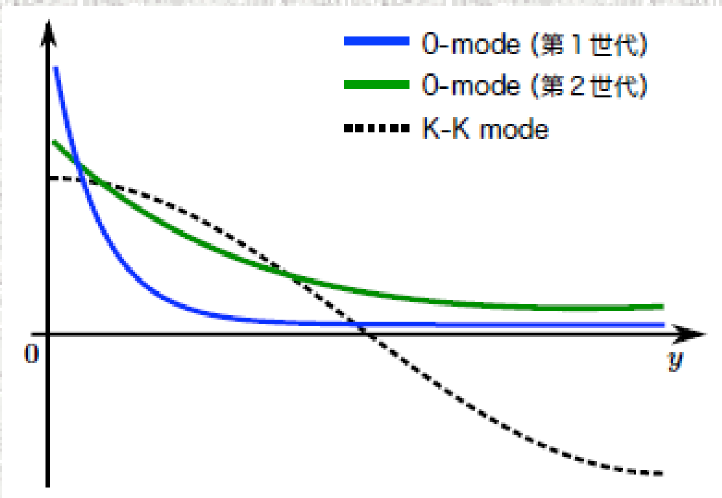
FCNC at tree level even in QCD sector

$$\mathcal{L}_{strong} \supset \frac{g_s}{\sqrt{2\pi R}} G_\mu^{a(0)} \left(\bar{d}_R^{i(0)} \gamma^\mu \lambda^a d_R^{i(0)} + \bar{d}_L^{i(0)} \gamma^\mu \lambda^a d_L^{i(0)} \right)$$
$$+ g_s G_\mu^{a(n)} \bar{d}_R^{i(0)} \gamma^\mu \lambda^a d_R^{j(0)} \left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{ij}$$
$$+ g_s G_\mu^{a(n)} \bar{d}_L^{i(0)} \gamma^\mu \lambda^a d_L^{j(0)} \left[V_{dL}^\dagger U_3^\dagger I_{LL}^{(0n0)} U_3 V_{dL} + V_{dL}^\dagger U_4^\dagger I_{LL}^{(0n0)} U_4 V_{dR} \right]_{ij}$$

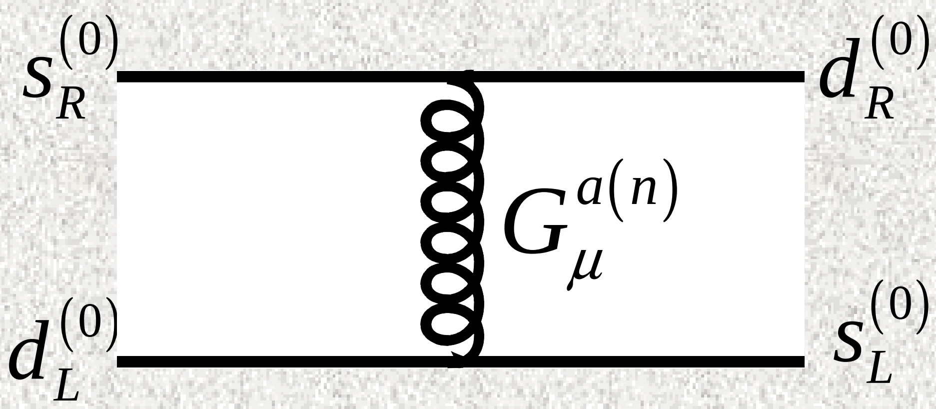
0 mode sector: No mixing O.K.

Nonzero KK gluon couplings induce nontrivial flavor mixing

$\Rightarrow K^0 - \bar{K}^0$ mixing@tree level



Dominant process of $K^0 - \bar{K}^0$ mixing



Chiral enhancement factor ~ 25

$$\Delta m_K \sim -\pi^2 \alpha_S C R^3 \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

$< 5.8 \times 10^{-13}$ MeV (exp)

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \left(f_R^i(y) \right)^2 \cos\left(\frac{n}{R} y \right)$$

$$\Rightarrow \quad \frac{1}{R} > \mathcal{O}(10)\,\text{TeV}$$

"GIM-like" mechanism in GHU

The lower bound for the compactification scale is smaller than that from naïve order estimate

$$\frac{(\sin\theta_c \cos\theta_c)^2}{M_c^2} \le \frac{1}{(10^5 TeV)^2} \Rightarrow M_c \ge 300\ TeV$$

This apparent discrepancy can be understood since the "GIM-like" mechanism works in GHU

i.e. FCNC process is automatically suppressed for light generation of quarks

Light quark masses are obtained from the large bulk masses through the factor exp[-πRM]

In the large bulk mass limit (πRM >> 1), the KK mode sum can be approximated as follows

$$S_{KK} \equiv \pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$\simeq -\frac{\pi^2}{2} \left(e^{-2\pi R M^1} + e^{-2\pi R M^2} \right)$$

$$-\frac{\pi}{2R} \frac{(M^1)^2 - M^1 M^2 + (M^2)^2}{M^1 M^2 (M^1 - M^2)} \left(e^{-2\pi R M^1} - e^{-2\pi R M^2} \right) \quad (\pi R M^i \gg 1)$$

exponential suppression!!

$$e^{-2\pi R M^i} \Leftrightarrow \frac{m_{q^i}^2}{m_W^2}$$

similar to GIM suppression

$$\frac{m_c^2 - m_u^2}{m_W^2}$$

More intuitive understanding of "GIM-like" suppression

FCNC is controlled by the factor

$$\left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)}\right)^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \frac{M^i}{e^{2\pi R M^i} - 1} e^{2M^i y} \cos\left(\frac{n}{R} y\right)$$

In $\pi MR \gg 1$ limit & consider only for small mode index n

Width "1/M" of 0 mode function $\ll$ Period $2\pi R/n$ of KK gluon mode function

Almost flat KK gluon mode function for fast exponential dumping 0 mode fermions

⇒ Almost flavor universal (similar to 0 mode sector, RS-GIM)

CP Violation

Adachi, Lim & Maru, PRD80 055024 (2010)
Lim, Maru & Nishiwaki, PRD81 076006 (2010)

In gauge-Higgs unification,
Yukawa coupling is provided by real gauge coupling

↓

The theory is CP invariant, no explicit breaking

The two mechanisms of "spontaneous" CP violation are now discussed

1: CP violation by $\langle A_y \rangle$

2: CP violation due to Compactification

CP violation by Higgs VEV

Adachi, Lim & Maru (2010)

CP can be broken due to the VEV of CP odd Higgs

Consider 5D theory

5D CP trf = 4D CP trf $\quad \mathcal{CP}: \psi \to i\gamma^0\gamma^2\psi^*$

Accordingly, CP trf for the space-time coordinate & the gauge field are found as

$$\mathcal{CP}: \left(x^\mu, y\right) \to \left(x_\mu, y\right),\ \left(A^\mu, A^y\right) \to \left(-A_\mu, -A^y\right)^T$$

$\langle A^y \rangle$ is CP odd and leads to CP violation

Comment

In the case of vanishing bulk mass,
the coupling of A_y can be the scalar coupling
by a chiral rotation of matter fermion

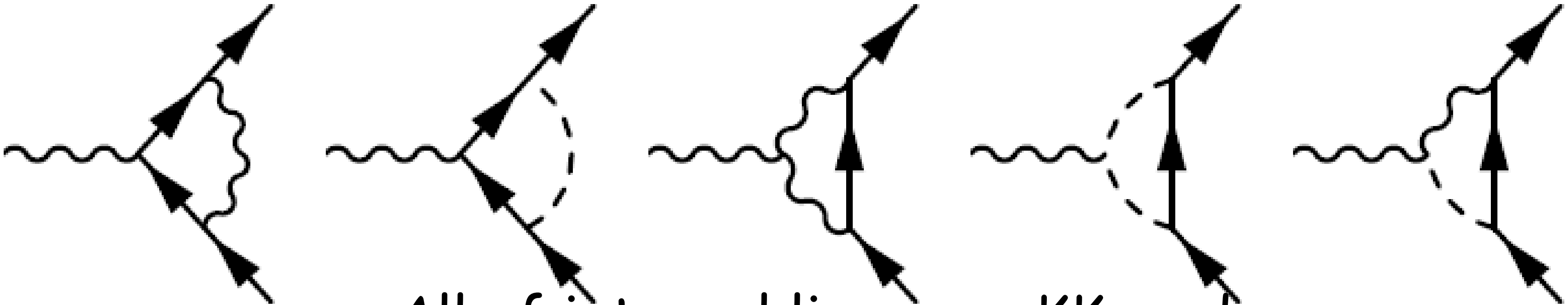
↓

A_y is CP even

Lesson

To get physical CP violation,
we need both bulk masses & Higgs VEV

To check this expectation,
neutron EDM (P & CP violating quantity) is calculated
in 5D SU(3) gauge-Higgs model
with a fermion in the **3**-dim. representation



All of internal lines are KK modes

$$|d_N| \simeq 4.6\times10^{-6} g_5 \left(\frac{eMR}{\pi}\right)^3 R^2 \langle A_y \rangle < 2.9\times10^{-26} (e\cdot cm)$$

→ $M_c > 2.6\,\mathrm{TeV}$

!! Already generate at 1-loop (e.g. at least 3-loop in SM)
& even with one generation (e.g. at least 3 gen. in SM)

CP violation due to Compactification

Lim, Maru & Nishiwaki (2010)

If the extra space has a complex structure,
CP can be broken due to the geometry of compactified space

Consider the simplest case: 6D

Gamma matrices:

$$\Gamma^\mu = \gamma^\mu \otimes I_2 = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \Gamma^y = \gamma^5 \otimes i\sigma_1 = \begin{pmatrix} 0 & i\gamma^5 \\ i\gamma^5 & 0 \end{pmatrix}, \Gamma^z = \gamma^5 \otimes i\sigma_2 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix}$$

Charge conjugation matrix satisfying $C^\dagger \Gamma^M C = -\left(\Gamma^M\right)^T$

is $C = \underbrace{i\gamma^0\gamma^2}_{C_4} \otimes \sigma_2$

4D case is not reproduced

Mixing of ψ & Ψ $\Psi_6 = (\psi, \Psi)^T$

Modified C, P transformation for 6D Dirac fermion

$$\Psi_6 = (\psi, \Psi)^T$$

$$\mathcal{P} : \Psi_6 \to (\gamma^0 \otimes \sigma_3)\Psi_6, \quad \mathcal{C} : \Psi_6 \to (C_4 \otimes \sigma_3)\bar{\Psi}_6^T$$

Lim (1991)

C, P transformation for extra coordinates

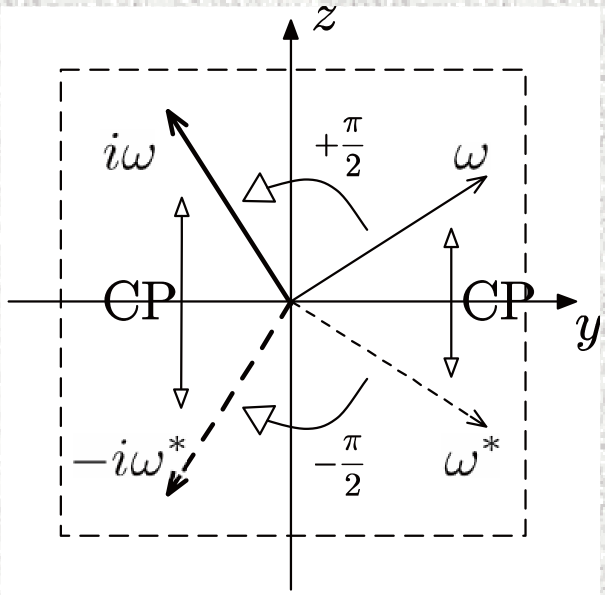
$$\mathcal{P} : (y, z) \to (y, z), \quad \mathcal{C}, \mathcal{CP} : (y, z) \to (y, -z)$$

Introducing a complex coordinate $\omega = y + iz$

CP transformation becomes complex conjugation

$$\text{CP:} \ \omega \to \omega^*$$

Consider an orbifold compactification T^2/Z_4



CP symmetry is incompatible with Z4 orbifold condition

(If Z2 instead of Z4 is considered, CP is not broken ∵ $\pi = -\pi$)

To check this expectation,
we have shown in a model of 6D U(1) gauge-Higgs on T^2/Z4

- CP phase remained in the interaction vertex after the re-phasing of fields
- Nonzero Jarskog-type parameter
- CP is broken even with one generation
  → New mechanism different from that of KM

Summary

- Gauge-Higgs unification predicts finite Higgs mass and is an attractive scenario for the physics beyond the SM
- As Yukawa coupling is universal, Yukawa hierarchy and flavor mixing are challenging issues
- Yukawa hierarchy is realized through exp[-MR] by the fermion localization at different points
- Non-degenerate bulk masses are new sources of flavor violation beyond the Glashow-Weinberg argument & lead to FCNC at tree level
- In the case of $K^0 - \bar{K}^0$ mixing, nonzero KK gluon exchange at tree level yields the amplitude suppressed by the compactification scale and the data put its lower bound like O(10TeV)

- "GIM-like" mechanism works in GHU as well
- As Yukawa coupling is real,
  CP violation is also a challenging issue
- Two new mechanisms of spontaneous CP violation
  by the VEV of Higgs & by the compactification
  were proposed
- These mechanisms of CP violation are beyond that of KM,
  & might be relevant for Baryon asymmetry of the universe

- "GIM-like" mechanism works in GHU as well
- As Yukawa coupling is real,
  CP violation is also a challenging issue
- Two new mechanisms of spontaneous CP violation
  by the VEV of Higgs & by the compactification
  were proposed
- These mechanisms of CP violation are beyond that of KM,
  & might be relevant for Baryon asymmetry of the universe

Thank you very much for your attention!!

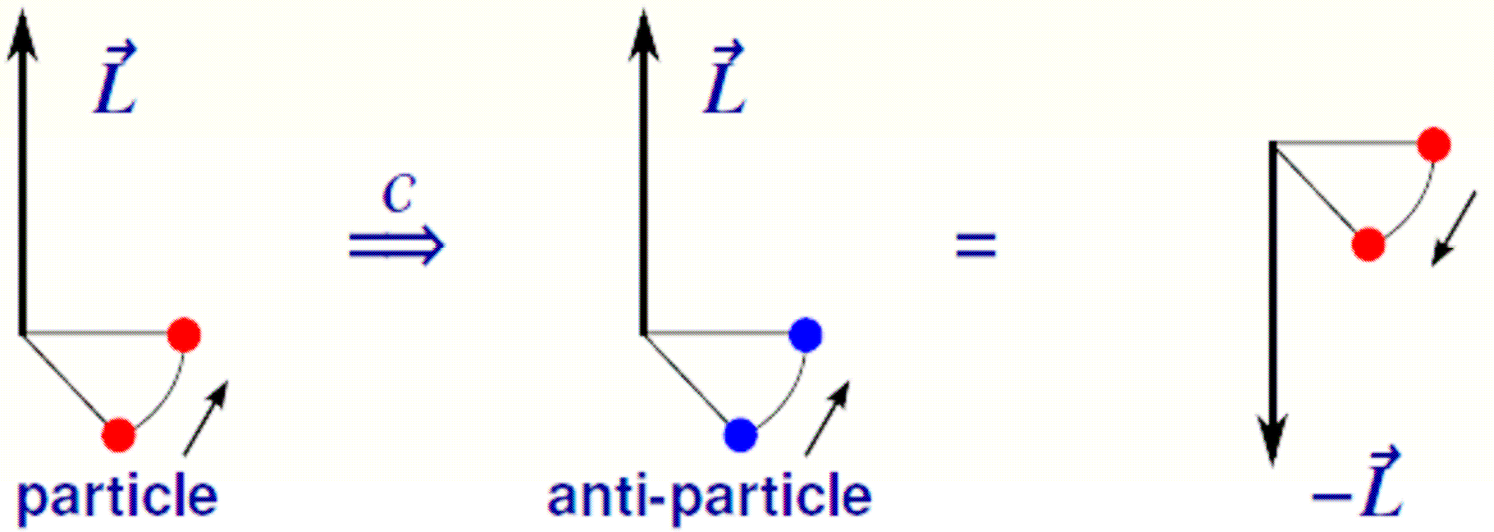
Backup Slides

- $\mathcal{C}, \mathcal{P}$ **transformation of electric field**

$$\vec{E} \propto \vec{\nabla}\phi \xrightarrow{\mathcal{P}} (-\vec{\nabla})\phi \xrightarrow{\mathcal{C}} (-\vec{\nabla})(-\phi)$$

- $\mathcal{C}, \mathcal{P}$ **transformation of spin**

$$\text{spin} \;:\; \vec{\sigma} \xrightarrow{\mathcal{P}} \vec{\sigma} \xrightarrow{\mathcal{C}} -\vec{\sigma}$$



Charge conjugation changes the sign of angular momentum

Electric Dipole Moment (EDM)

EDM $\cdots$ interaction between spin $\vec{\sigma}$ and electric field $\vec{E}$

$$H_{\rm int} = d\vec{\sigma}\cdot\vec{E}, \quad \text{operator} = \langle H^\dagger\rangle \bar{\psi}_R \sigma^{\mu\nu}\psi_L F_{\mu\nu}$$

d : electric dipole moment

EDM : $\mathcal{P}$ and $\mathcal{CP}$ odd quantity

$$\begin{cases} \vec{\sigma} \xrightarrow{\mathcal{P}} \vec{\sigma} \xrightarrow{\mathcal{C}} -\vec{\sigma} \\ \vec{E} \xrightarrow{\mathcal{P}} \underbrace{-\vec{E}}_{\mathcal{P}\ \text{odd}} \xrightarrow{\mathcal{C}} \underbrace{\vec{E}}_{\mathcal{CP}\ \text{odd}} \end{cases}$$

$$\begin{cases} \mathcal{CP}(\bar{\psi}\sigma^{\mu\nu}\gamma^5\psi F_{\mu\nu}) = \bar{\psi}\sigma_{\mu\nu}\gamma^5\psi(-F^{\mu\nu}) \\ \mathcal{P}(\bar{\psi}\sigma^{\mu\nu}\gamma^5\psi F_{\mu\nu}) = -\bar{\psi}\sigma_{\mu\nu}\gamma^5\psi F^{\mu\nu} \end{cases}$$

$\mathcal{C},\mathcal{P}$ transformation

- $\mathcal{P}$ transformation

$$\begin{cases} \mathcal{P} : (x^\mu, y) = (x_\mu, -y), \\ \mathcal{P} : \psi(x^\mu, y) = \gamma^0 \psi(x_\mu, -y), \\ \mathcal{P} : (A^\mu, A^y)(x^\mu, y) = (A_\mu, -A^y)(x_\mu, -y). \end{cases}$$

$$\begin{aligned} [\bar{\psi} i \not{D} \psi] =& \bar{\psi}[i\partial_\mu \gamma^\mu + i\partial_y \gamma^y + g_5(A_\mu \gamma^\mu + A_y \gamma^y)]\psi \\ &\overset{\mathcal{P}}{\Rightarrow} \bar{\psi}[i\partial_\mu \gamma_\mu + i\partial_y \gamma_y + g_5(\mathcal{P} A_\mu \mathcal{P}^{-1} \gamma_\mu + \mathcal{P} A_y \mathcal{P}^{-1} \gamma_y)]\psi \end{aligned}$$

- C transformation

$$\begin{cases} C : (x^\mu, y) = (x^\mu, -y), \\ C : \psi(x^\mu, y) = i\gamma^2\psi^*(x^\mu, -y), \\ C : (A^\mu, A^y)(x^\mu, y) = (-A^\mu, A^y)^{\mathrm{T}}(x^\mu, -y). \end{cases}$$

$$[\bar{\psi} i \not{D} \psi] = \bar{\psi}[i\partial_\mu\gamma^\mu + i\partial_y\gamma^y + g_5(A_\mu\gamma^\mu + A_y\gamma^y)]\psi$$

$$\overset{C}{\Rightarrow} \bar{\psi}[i\partial_\mu\gamma^\mu - i\partial_y\gamma^y + g_5(-CA_\mu C^{-1}\gamma^\mu + CA_y C^{-1}\gamma^y)]\psi$$

No reason to choose the same bulk masses for different representations, 3 & 6*

Natural choice if we have some GUT where 3 & 6* are embedded into a single representation of the GUT group

$$Sp(8) \to Sp(6) \times SU(2) \to SU(3) \times U(1) \times SU(2)$$

$$\begin{aligned} 36 &\to (1, 3) + (21, 1) + (6, 2) \\ &\to (1, 3) + (1 + 6 + 6^* + 8, 1) + (3 + 3^*, 2) \end{aligned}$$

3 & 6* of SU(3) can be embedded into an adjoint representation 36 of Sp(8)

Lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + i\bar{\Psi}\Gamma^M D_M \Psi \qquad \Gamma^M = \left(\gamma^\mu, i\gamma^5\right)$$

$$F_{MN} = \partial_M F_N - \partial_N F_M - ig_{D+1}\left[A_M, A_N\right] \left(M, N = 0,1,2,3,5\right)$$

$$D_M = \partial_M - ig_5 A_M \left(A_M = A_M^a \lambda^a/2 : \lambda^a : \text{Gell-Mann matrices}\right)$$

$$\Psi = \left(\psi_1, \psi_2, \psi_3\right)^T$$

Boundary conditions:

$$A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_{D+1} = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \Psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}$$

SU(3) → SU(2) x U(1)

Higgs

Chiral fermions

4D fermion effective Lagrangian in terms of mass eigenbasis

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\left[\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} i\gamma^{\mu}\partial_{\mu} - m_n & 0 & 0 \\ 0 & i\gamma^{\mu}\partial_{\mu} - (m_n + m) & 0 \\ 0 & 0 & i\gamma^{\mu}\partial_{\mu} - (m_n - m) \end{pmatrix}\begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}\right.$$

$$\left. + \frac{g}{2}\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} W_{\mu}^{3} + \frac{B_{\mu}}{\sqrt{3}} & W_{\mu}^{+} & W_{\mu}^{+} \\ W_{\mu}^{-} & -\frac{W_{\mu}^{3}}{2} - \frac{B_{\mu}}{2\sqrt{3}} & -\frac{W_{\mu}^{3}}{2} + \frac{B_{\mu}}{2\sqrt{3}} \\ W_{\mu}^{-} & -\frac{W_{\mu}^{3}}{2} + \frac{B_{\mu}}{2\sqrt{3}} & -\frac{W_{\mu}^{3}}{2} - \frac{B_{\mu}}{2\sqrt{3}} \end{pmatrix}\gamma^{\mu}\begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}\right]$$

$$+ i\bar{t}_L\gamma^{\mu}\partial_{\mu}t_L + \bar{b}\left(i\gamma^{\mu}\partial_{\mu} - m\right)b + \frac{g}{\sqrt{2}}\left(\bar{t}\gamma^{\mu}LbW_{\mu}^{+} + \bar{b}\gamma^{\mu}LtW_{\mu}^{-}\right) + \frac{g}{2}\left(\bar{t}\gamma^{\mu}Lt + \bar{b}\gamma^{\mu}Lb\right)W_{\mu}^{3}$$

$$+ \frac{\sqrt{3}g}{6}\left(\bar{t}\gamma^{\mu}Lt + \bar{b}\gamma^{\mu}Lb - 2\bar{b}\gamma^{\mu}Rb\right)B_{\mu}$$

$$\begin{pmatrix} \tilde{\psi}_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}$$

$$L \equiv \frac{1}{2}(1-\gamma_5),\ R \equiv \frac{1}{2}(1+\gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\,(= M_W)$$

Derivation of chiral suppression

4-Fermi operator

$$\sum_a \sum_{\alpha,\beta,\alpha',\beta'} \bar{s}_\alpha \left(\lambda^a\right)_{\alpha\beta} \gamma^\mu L d_\beta \bar{s}_{\alpha'} \left(\lambda^a\right)_{\alpha'\beta'} \gamma_\mu R d_{\beta'} = -\frac{1}{6}\bar{s}_\alpha \gamma^\mu L d_\alpha \bullet \bar{s}_\beta \gamma_\mu R d_\beta + \frac{1}{2}\bar{s}_\alpha \gamma^\mu L d_\beta \bullet \bar{s}_\beta \gamma_\mu R d_\alpha$$

$$\lambda^a_{ij}\lambda^a_{kl} = -\frac{1}{6}\delta_{ij}\delta_{kl} + \frac{1}{2}\delta_{il}\delta_{jk}$$

Hadronic matrix element

$$\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \bullet \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \middle| K \right\rangle$$

$$= \sum_n \left[2\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \middle| n \right\rangle \left\langle n \middle| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \middle| K \right\rangle - 2\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^{b'}_\beta \middle| n \right\rangle \left\langle n \middle| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^b_\alpha \middle| K \right\rangle \right]$$

$$\approx 2\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \middle| 0 \right\rangle \left\langle 0 \middle| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \middle| K \right\rangle - 2\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^{b'}_\beta \middle| 0 \right\rangle \left\langle 0 \middle| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^b_\alpha \middle| K \right\rangle$$

$$= -\frac{1}{2}\left\langle \bar{K} \middle| \bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha \middle| 0 \right\rangle \left\langle 0 \middle| \bar{s}_\beta \gamma_\mu \gamma^5 d_\beta \middle| K \right\rangle - 4\left\langle \bar{K} \middle| \bar{s}^a_\alpha \left(R\right)_{ab'} d^{b'}_\beta \middle| 0 \right\rangle \left\langle 0 \middle| \bar{s}^{a'}_\beta \left(L\right)_{a'b} d^b_\alpha \middle| K \right\rangle$$

"vacuum saturation"

$$= -\frac{1}{2}\left[-\frac{f_K^2}{2m_K}\left(p^\mu_K\right)^2 \right] + \frac{\delta_{ab}}{3}\frac{\delta_{ab}}{3}\left\langle \bar{K} \middle| \bar{s}_\alpha \gamma^5 d_\alpha \middle| 0 \right\rangle \left\langle 0 \middle| \bar{s}_\beta \gamma^5 d_\beta \middle| K \right\rangle$$

Fierz transformation

$$= \frac{1}{4} f_K^2 m_K + \frac{1}{6}\left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K \approx \frac{1}{6}\left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K$$

$$\langle 0| j_5^\mu |K\rangle = \langle 0|\bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha |K\rangle = \frac{f_K}{\sqrt{2m_K}} p_K^\mu, \langle 0|\bar{s}_\alpha \gamma^\mu \gamma^5 d_\beta |K\rangle = \frac{1}{3}\delta_{\alpha\beta}\langle 0|\bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha |K\rangle$$

$$\langle 0|\bar{s}_\alpha \gamma^5 d_\alpha |K\rangle = -\frac{f_K}{\sqrt{2m_K}} \frac{m_K^2}{m_d + m_s}, \langle 0|\bar{s}_\alpha \gamma^5 d_\beta |K\rangle = \frac{1}{3}\delta_{\alpha\beta}\langle 0|\bar{s}_\alpha \gamma^5 d_\alpha |K\rangle$$

The relevant hadronic matrix elements are written by use of the "bag parameters" B_4, B_5, which denote the deviation from the approximation of vacuum saturation and whose numerical results are obtained by lattice calculations $B_4 = 0.81$, $B_5 = 0.56$ [18]:

$$\langle \bar{K}|\bar{s}_\alpha \gamma^\mu L d_\alpha \cdot \bar{s}_\beta \gamma_\mu R d_\beta |K\rangle \approx \frac{B_5}{6}\left(\frac{m_K}{m_d + m_s}\right)^2 f_K^2 m_K, \tag{4.9}$$

$$\langle \bar{K}|\bar{s}_\alpha \gamma^\mu L d_\beta \cdot \bar{s}_\beta \gamma_\mu R d_\alpha |K\rangle \approx \frac{B_4}{2}\left(\frac{m_K}{m_d + m_s}\right)^2 f_K^2 m_K, \tag{4.10}$$

$$\begin{aligned} &\frac{1}{4}\langle \bar{K}|\bar{s}\lambda^a \gamma^\mu L d \cdot \bar{s}\lambda^a \gamma_\mu R d|K\rangle \\ &= -\frac{1}{6}\langle \bar{K}|\bar{s}_{\alpha L}\gamma^\mu d_\alpha \cdot \bar{s}_{\beta R}\gamma_\mu d_{\beta R}|K\rangle + \frac{1}{2}\langle \bar{K}|\bar{s}_{\alpha L}\gamma^\mu d_{\beta L} \cdot \bar{s}_{\beta R}\gamma_\mu d_{\alpha R}|K\rangle \\ &\approx \left(\frac{B_4}{4} - \frac{B_5}{36}\right)\left(\frac{m_K}{m_d + m_s}\right)^2 f_K^2 m_K . \end{aligned}$$

Now, we focus on 2 generation case

Parameterize rotation angles as (CP invariance is assumed)

$$U_4 = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad U_3 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix}$$

with satisfying a unitarity condition $U_3^\dagger U_3 + U_4^\dagger U_4 = 1$

Yukawa couplings

$$\hat{Y}_d = V_{dR}^\dagger I_{RL}^{(00)} U_3 V_{dL} = V_{dR}^\dagger \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} V_{dL}$$

$$\hat{Y}_u = V_{uR}^\dagger I_{RL}^{(00)} U_4 V_{uL} = V_{uR}^\dagger \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix} V_{uL}$$

$$I_{RL}^{(00)} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

Parameter fitting (6 parameters - 5 observables = 1 parameter)

$$\begin{cases} \hat{m}_u \hat{m}_c = \det \hat{Y}_u ,\ \hat{m}_d \hat{m}_s = \det \hat{Y}_d ,\ \hat{m}_i \equiv m_i / m_W \\ \hat{m}_u^2 + \hat{m}_c^2 = Tr \hat{Y}_u \hat{Y}_u^\dagger ,\ \hat{m}_d^2 + \hat{m}_s^2 = Tr \hat{Y}_d \hat{Y}_d^\dagger ,\ \theta_c = \theta_{dL} - \theta_{uL} \end{cases}$$

$$\hat{m}_u^2 \hat{m}_c^2 = a^2 b^2 c^2 d^2 ,\ \hat{m}_d^2 \hat{m}_s^2 = \left(1-a^2\right)\left(1-b^2\right) c^2 d^2$$

$$\hat{m}_u^2 + \hat{m}_c^2 = a^2 c^2 + b^2 d^2 - \left(a^2 - b^2\right)\left(c^2 - d^2\right) \sin^2 \theta'$$

$$\hat{m}_d^2 + \hat{m}_s^2 = \left(1-a^2\right) c^2 + \left(1-b^2\right) d^2 - \left(a^2 - b^2\right)\left(c^2 - d^2\right) \sin^2 \theta$$

$$\tan 2\theta_c = \frac{\tan 2\theta_{dL} - \tan 2\theta_{uL}}{1 + \tan 2\theta_{dL} \tan 2\theta_{uL}}$$

$$\begin{cases} \tan 2\theta_{dL} = \dfrac{2\sqrt{\left(1-a^2\right)\left(1-b^2\right)}\left(d^2 - c^2\right) \sin\theta \cos\theta}{\left(1-a^2\right)\left(c^2 \cos^2\theta + d^2 \sin^2\theta\right) - \left(1-b^2\right)\left(c^2 \sin^2\theta + d^2 \cos^2\theta\right)} \\ \tan 2\theta_{uL} = \dfrac{2ab\left(d^2 - c^2\right) \sin\theta' \cos\theta'}{a^2\left(c^2 \cos^2\theta' + d^2 \sin^2\theta'\right) - b^2\left(c^2 \sin^2\theta' + d^2 \cos^2\theta'\right)} \end{cases}$$

Mixing again

$$\tan 2\theta_c = \frac{\tan 2\theta_{dL} - \tan 2\theta_{uL}}{1+\tan 2\theta_{dL}\tan 2\theta_{uL}} \propto \left(d^2 - c^2\right)$$

$$\begin{cases} \tan 2\theta_{dL} = \dfrac{2\sqrt{\left(1-a^2\right)\left(1-b^2\right)}\left(d^2-c^2\right)\sin\theta\cos\theta}{\left(1-a^2\right)\left(c^2\cos^2\theta + d^2\sin^2\theta\right) - \left(1-b^2\right)\left(c^2\sin^2\theta + d^2\cos^2\theta\right)} \\ \tan 2\theta_{uL} = \dfrac{2ab\left(d^2-c^2\right)\sin\theta'\cos\theta'}{a^2\left(c^2\cos^2\theta' + d^2\sin^2\theta'\right) - b^2\left(c^2\sin^2\theta' + d^2\cos^2\theta'\right)} \end{cases}$$

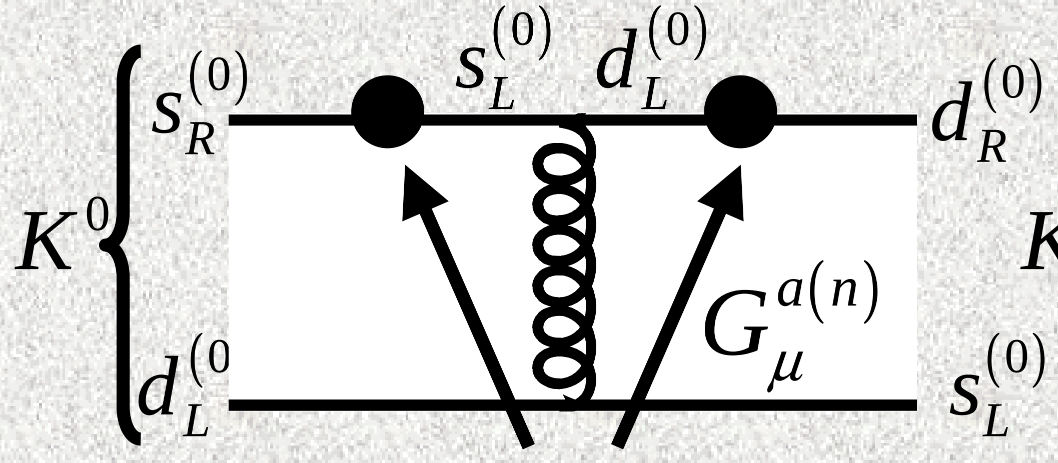
Check

Universal bulk mass limit (c=d)

→ $\tan 2\theta_c = 0$ i.e. $\theta_c = 0$

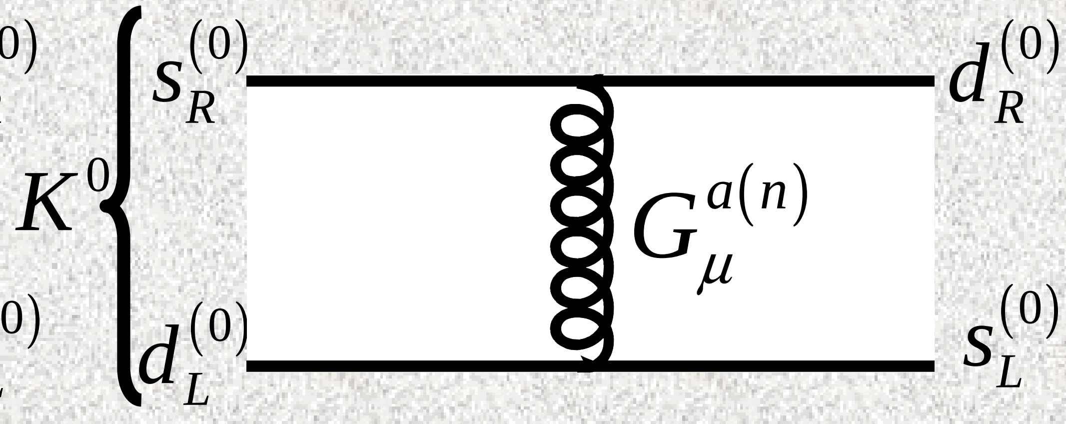
3 types of amplitudes should be considered

Left-Left type



Mass insertion is necessary

Left-Right type



$K^0 \sim d\gamma^5\overline{s}$

"Dominant"

"Chiral suppression"

$$\left(\frac{m_d + m_s}{m_K}\right)^2 \approx \left(\frac{106MeV}{497MeV}\right)^2 \approx \left(\frac{1}{5}\right)^2$$

(Right-Right type is also suppressed similarly)

K_L-K_S mass difference

We have calculated K_L-K_S mass difference from the dominant left-right type amplitude

$$\Delta m_K\,(\text{KK modes}) = 2\left\langle \bar{K} \middle| \mathcal{L}_{eff}^{\Delta S=2} \middle| K \right\rangle$$

$$\sim -2\pi^2 \alpha_S C R^3 \left(B_4 - \frac{B_5}{9} \right) \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

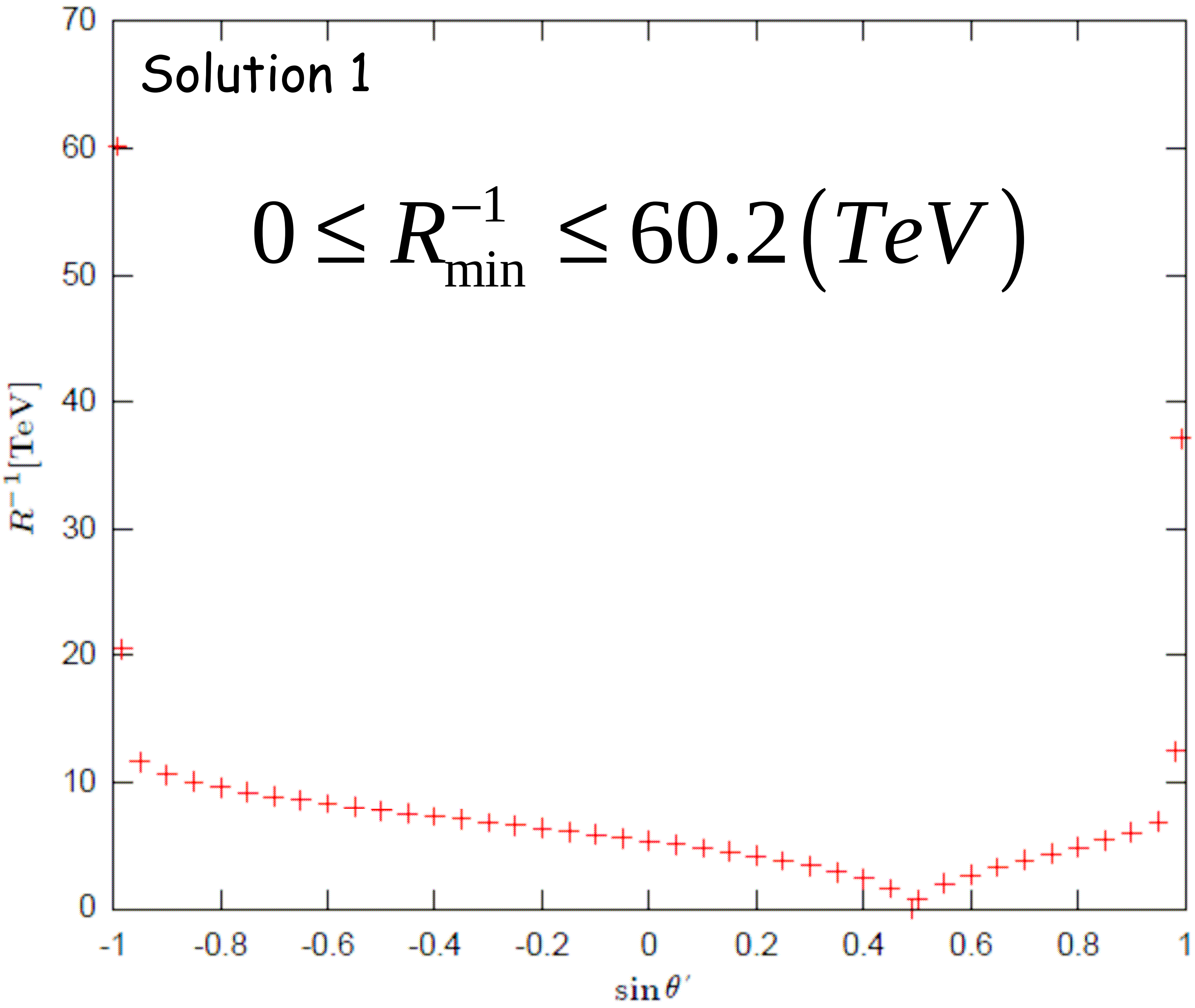
$$\sim 7.5\times 10^4 \left(R f_\pi \right)^2 C \pi R \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

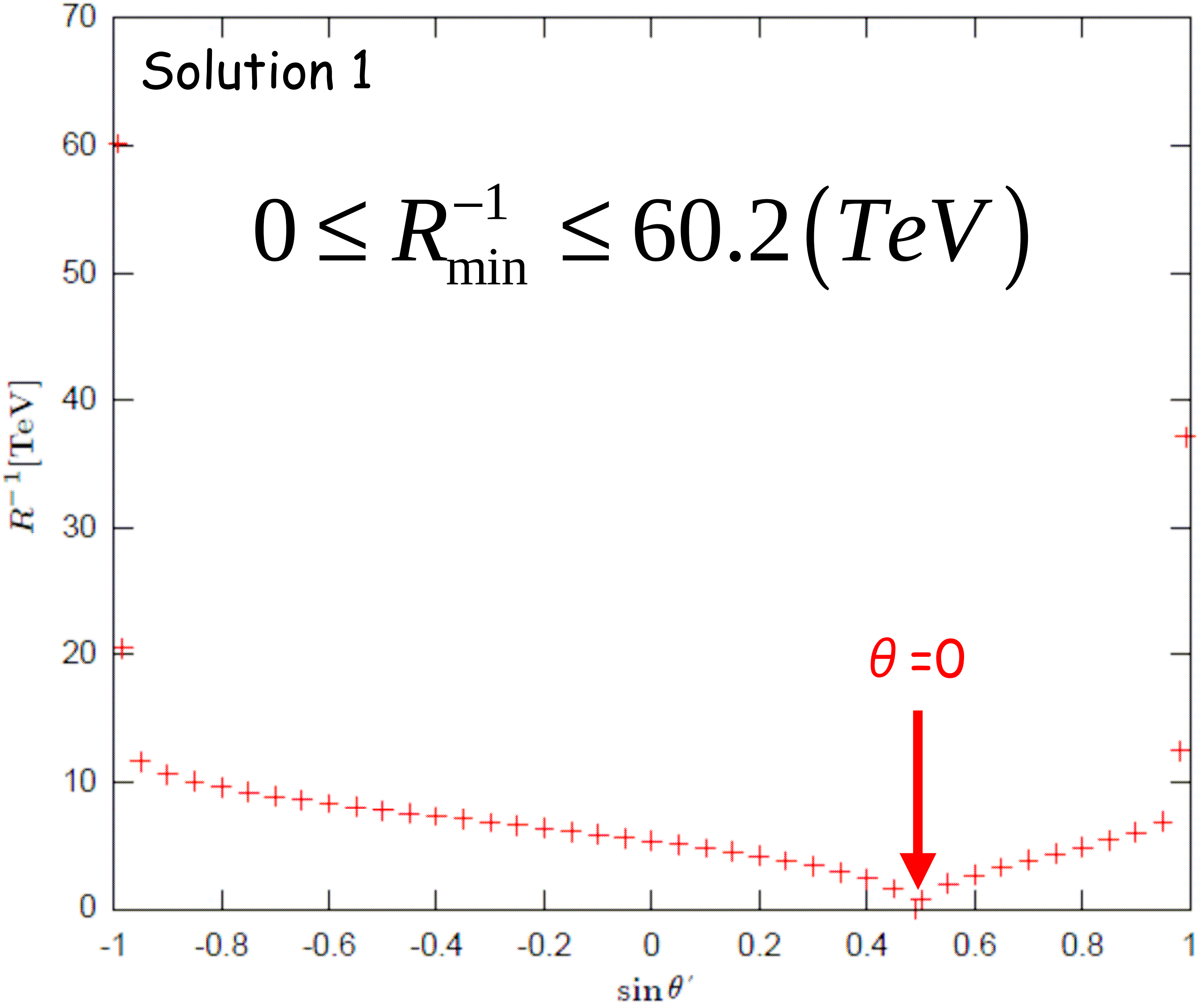
$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \left(f_R^i(y) \right)^2 \cos\left(\frac{n}{R} y \right)$$

$$C = -\frac{1}{2}\left(1-a^2\right)\sin 2\theta_{dL}\cos^2\theta + \frac{1}{2}\left(1-b^2\right)\sin 2\theta_{dL}\sin^2\theta - \frac{1}{2}\sqrt{\left(1-a^2\right)\left(1-b^2\right)}\cos 2\theta_{dL}\sin 2\theta$$

$$-\frac{1}{2}a^2 \sin 2\theta_{dL}\cos^2\theta' + \frac{1}{2}b^2\sin 2\theta_{dL}\sin^2\theta' - \frac{1}{2}ab\cos 2\theta_{dL}\sin 2\theta'$$

$B_{4,5}$: Bag parameters (B_4=0.81, B_5=0.56)


Solution 1
$0 \le R^{-1}_{\min} \le 60.2 \left(TeV \right)$
R^{-1}[TeV]
$\sin\theta'$
70
60
50
40
30
20
10
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Solution 1
$0 \le R_{\min}^{-1} \le 60.2(TeV)$
θ =0
R^{-1}[TeV]
$\sin\theta'$
70
60
50
40
30
20
10
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Comment on a point 1/R=0

$$U_3 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} \xrightarrow{\theta=0} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix}$$

$$\hat{Y}_d = V_{dR}^{\dagger} \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} V'_{dL},$$

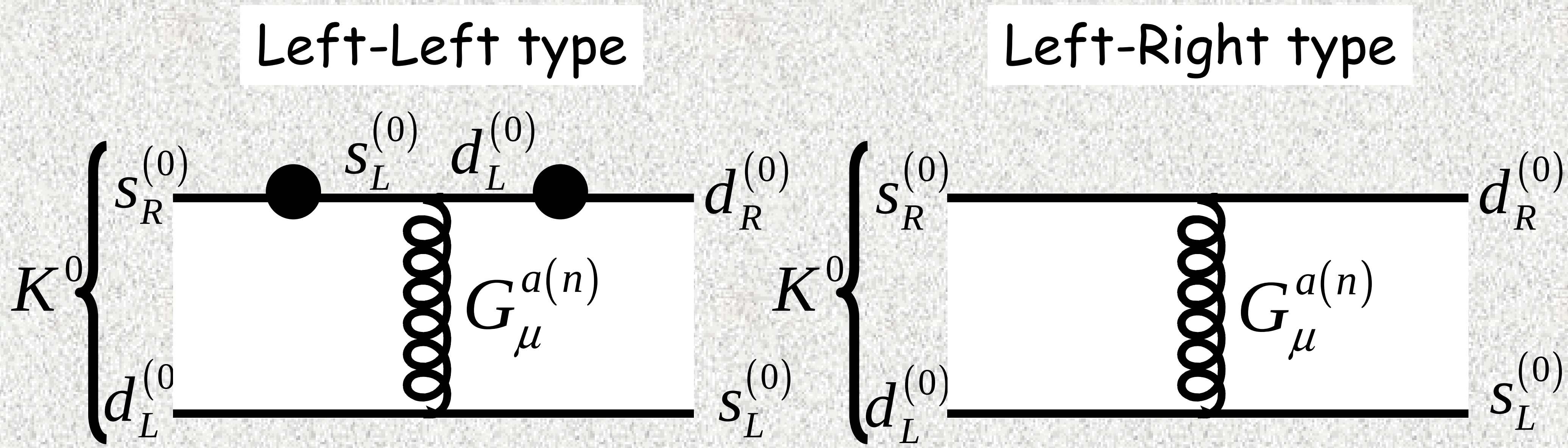
Vanishing mixing in down sector

$$\therefore \hat{Y}_d \hat{Y}_d^{\dagger} = V_{dR}^{\dagger} \begin{bmatrix} (1-a^2)\,c^2 & 0 \\ 0 & (1-b^2)\,d^2 \end{bmatrix} V_{dR} \Rightarrow V_{dR} = 1, \;\; i.e.\; \theta_{dR} = 0$$

$$\Delta m_K\,(\text{KK modes}) \sim \alpha_S R^2 f_K^2 m_K \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2 = 0$$

No constraint of 1/R from the left-right type…

But, we have to notice that the L-L type or R-R type dominates over the L-R one at some value of sin θ '



$$K^0 \sim \bar{s}\gamma^5 d$$

Chiral suppression

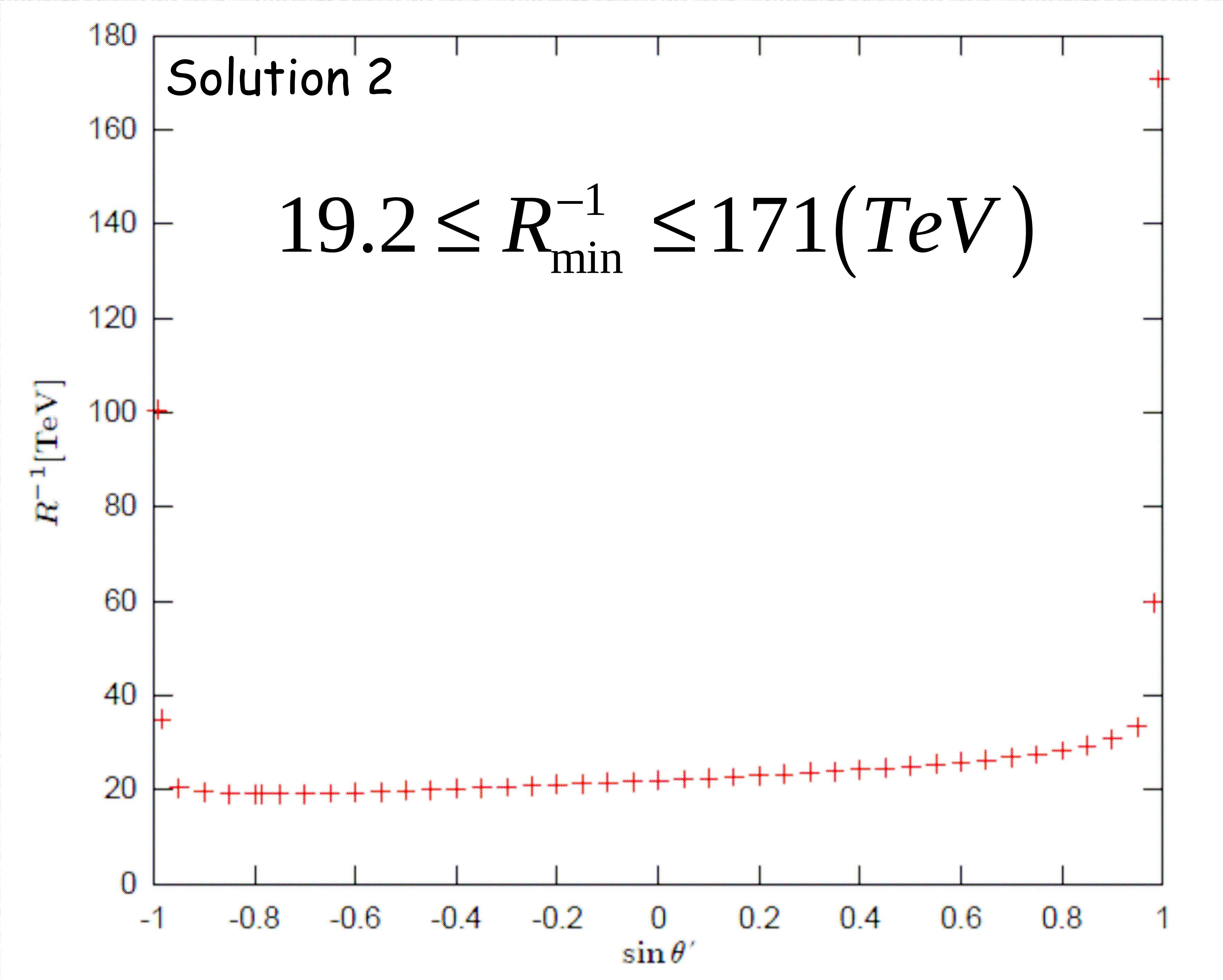
$$\left(\frac{m_d + m_s}{m_K}\right)^2 \approx \left(\frac{106MeV}{497MeV}\right)^2 \approx \left(\frac{1}{5}\right)^2$$

(Right-Right type is also suppressed similarly)

Most stringent lower bound of 1/R from LL

~ 60TeV x 1/5

~ O(10TeV)



Solution 2
$19.2 \le R_{\min}^{-1} \le 171(TeV)$
R^{-1}[TeV]
$\sin\theta'$
0
20
40
60
80
100
120
140
160
180
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

In the extreme case of $|\sin\theta'| \sim 1$
the bulk mass of 2nd generation
happens to be relatively small
(see plot, $d \sim 1$)
↓
"GIM-like" mechanism does not work
↓
Severe lower bound for the compactification scale

$\sin\theta'$	a^2	b^2	c^2	d^2	$\sin\theta$
0.9999	0.0000578	0.99999	4.18×10^{-9}	1.000000	0.000341
0.9	0.0526	0.9986	4.28×10^{-9}	0.001074	0.008414
0.8	0.0927	0.9974	4.37×10^{-9}	0.000597	0.008562
0.7	0.1236	0.9964	4.46×10^{-9}	0.000439	0.006756
0.6	0.1473	0.9956	4.55×10^{-9}	0.000362	0.003876
0.5	0.1652	0.9950	4.63×10^{-9}	0.000317	0.000345
0.4909	0.1665	0.9950	4.64×10^{-9}	0.000314	0.000000
0.4	0.1781	0.9945	4.72×10^{-9}	0.000289	-0.003569
0.3	0.1868	0.9942	4.80×10^{-9}	0.000271	-0.007677
0.2	0.1919	0.9940	4.88×10^{-9}	0.000259	-0.011827
0.1	0.1935	0.9940	4.96×10^{-9}	0.000253	-0.015892
0.0	0.1919	0.9940	5.04×10^{-9}	0.000251	-0.019758
-0.1	0.1873	0.9942	5.12×10^{-9}	0.000253	-0.023314
-0.2	0.1797	0.9945	5.20×10^{-9}	0.000259	-0.026451
-0.3	0.1691	0.9949	5.29×10^{-9}	0.000271	-0.029052
-0.4	0.1555	0.9954	5.38×10^{-9}	0.000290	-0.030985
-0.5	0.1387	0.9959	5.47×10^{-9}	0.000319	-0.032090
-0.6	0.1186	0.9966	5.57×10^{-9}	0.000367	-0.032156
-0.7	0.0950	0.9974	5.67×10^{-9}	0.000450	-0.030871
-0.8	0.0676	0.9982	5.77×10^{-9}	0.000620	-0.027690
-0.9	0.0360	0.9991	5.88×10^{-9}	0.001140	-0.021342
-0.9999	0.0000402	0.99999	6.00×10^{-9}	1.000000	-0.000747

In the extreme case of $|\sin\theta'|\sim 1$
the bulk mass of 2nd generation
happens to be relatively small
(see plot, $d\sim 1$)
↓
"GIM-like" mechanism does not work
(No exponential suppression)
↓
Severe lower bound for the compactification scale