INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Status of Horava-Lifshitz gravity

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ref. Horava-Lifshitz Cosmology: A Review
arXiv: 1007.5199 [hep-th]

Contents of this talk

- Basic idea
- Cosmological implications
- Analogue of Vainshtein effect
- Caustic avoidance
- List of future works

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**
  t → b t (E → b^{-1}E)
  x → b x
  ϕ → b^s ϕ
  1+3-2+2s = 0
  s = -1

- Renormalizability

$$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  t → b^z t (E→b^{-z}E)

  x → b x

  ϕ → b^s ϕ

  z+3-2z+2s = 0

  s = -(3-z)/2

- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

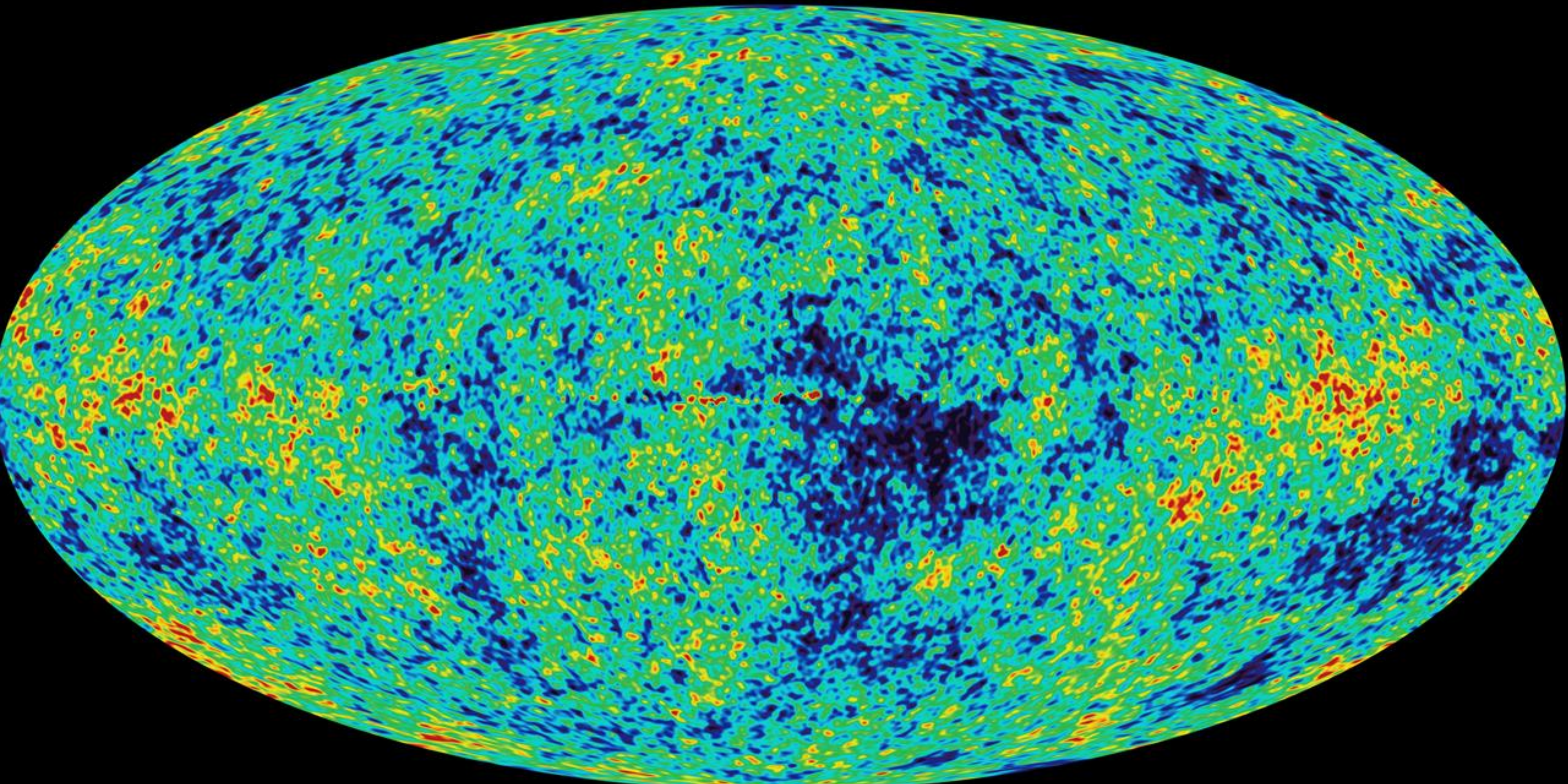
$$\propto E^{-(z+3+ns)/z}$$

- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Scale-invariant cosmological perturbations from Horava-Lifshitz gravity without inflation

arXiv:0904.2190 [hep-th]

c.f. Basic mechanism is common for “Primordial magnetic field from non-inflationary cosmic expansion in Horava-Lifshitz gravity”, arXiv:0909.2149 [astro-th.CO] with S.Maeda and T.Shiromizu.



Usual story with z=1

- $\omega^2 >> H^2$: oscillate

  $\omega^2 << H^2$: freeze

  oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

- Scaling law

  $t \to b\, t$ ($E \to b^{-1} E$)

  $x \to b\, x$

  $\phi \to b^{-1} \phi$

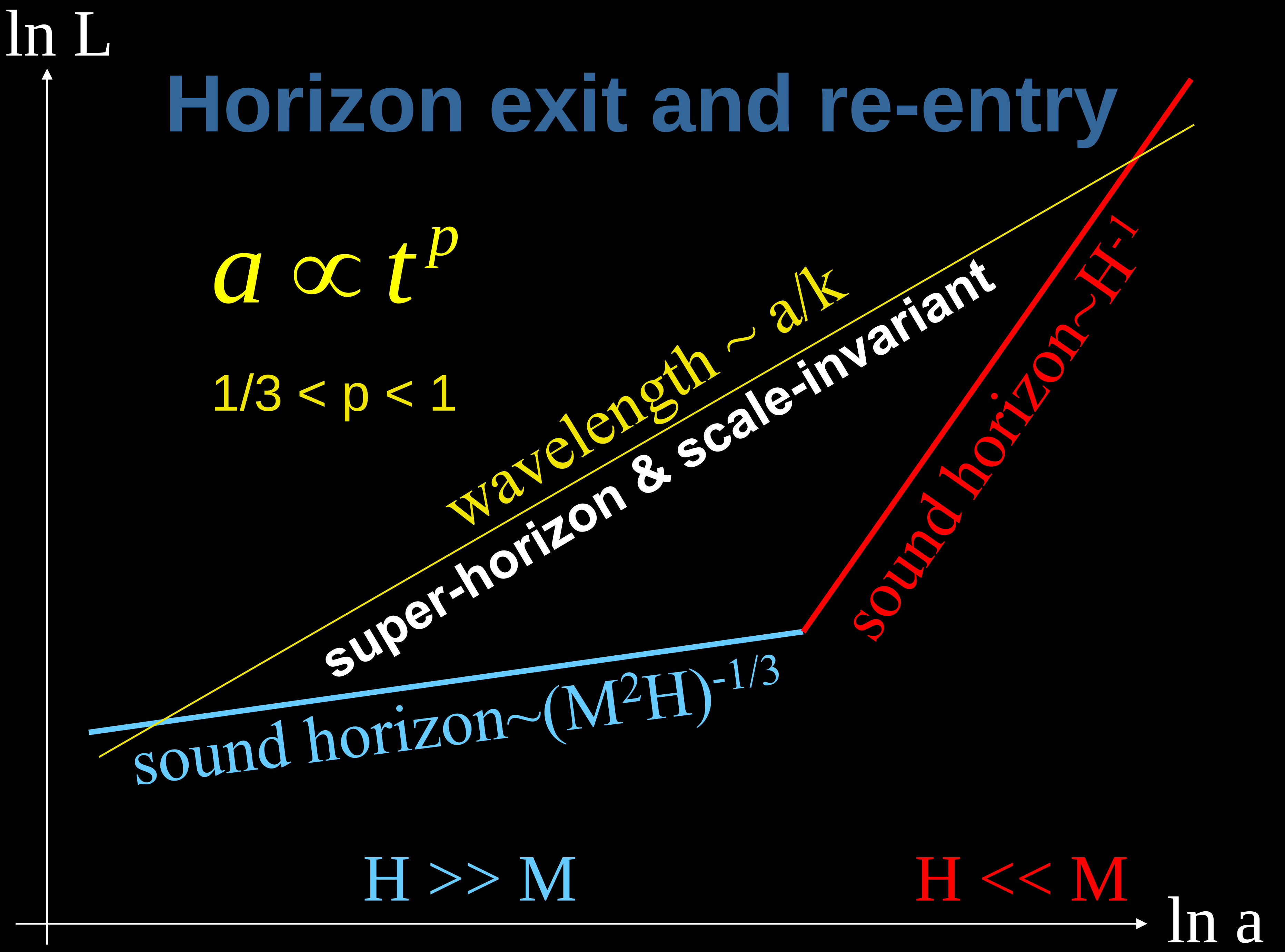
  $$\Rightarrow \quad \delta\phi \propto E \sim H$$

  Scale-invariance requires almost const. H, i.e. inflation.

UV fixed point with z=3

- oscillation → freeze-out iff $d(H^2/\omega^2)/t > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\, x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

Horizon exit and re-entry



GOING BACK TO HORAVA‘S IDEA

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2dt^2 + g_{ij}\,(dx^i + N^idt)(dx^j + N^jdt)$
- Foliation-preserving deffeomorphism
  $t \to t'(t), \quad x^i \to x'^i(t,x^j)$
- Anisotropic scaling with z=3 in UV
  $t \to b^z\, t, \quad x^i \to b\, x^i$
- Ingredients in the action

$$Ndt \qquad \sqrt{g}\,d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R \right.$$

$$\left. R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad \right]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R_i^j R_j^i \quad R^2 \quad]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad R \quad]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x [\quad 1 \quad]$$

IR action with z=1

- **UV: z=3** , power-counting renormalizability

⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

kinetic term

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\, d^3x\left(K_{ij}K^{ij}-\lambda K^2+c_g^2 R-2\Lambda\right)$$

IR potential

note:

Renormalizability has not been proved.

RG flow has not yet been investigated.

Projectability condition

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

Dark matter as integration constant in Horava-Lifshitz gravity

arXiv:0905.3563 [hep-th]

See also arXiv:0906.5069 [hep-th]
Caustic avoidance in Horava-Lifshitz gravity

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No "local" Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  can be integrated to give
  Friedmann eq with
  "dark matter as
  integration constant"

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt\sqrt{g}\,d^3x\left(K_{ij}K^{ij}-\lambda K^2+R-2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x\sqrt{g}(G^{(4)}_{\mu\nu}+\Lambda g^{(4)}_{\mu\nu}-8\pi G_N T_{\mu\nu})n^\mu n^\nu=0$$

$$n_\mu dx^\mu=-Ndt,\quad n^\mu\partial_\mu=\frac{1}{N}(\partial_t-N^i\partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu}+\Lambda g^{(4)}_{i\mu}-8\pi G_N T_{i\mu})n^\mu=0$$

$$G^{(4)}_{ij}+\Lambda g^{(4)}_{ij}-8\pi G_N T_{ij}=0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu} \right)$$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$

Micro to Macro

- Overall behavior of smooth $T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu$ is like pressureless dust.
- Microscopic lumps (sequences of caustics & bounces) of ρ^{HL} can collide and bounce. (cf. early universe bounce [Calcagni 2009, Brandenberger 2009]) If asymptotically free, would-be caustics does not gravitate too much.
- Group of microscopic lumps with collisions and bounces → When coarse-grained, can it mimic a cluster of particles with velocity dispersion?
- Dispersion relation of matter fields defined in the rest frame of "dark matter"
  → Any astrophysical implications?

Summary so far

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.

Vainshtein effect in massive gravity

- Linearized analysis results in vDVZ discontinuity of the massless limit.
- However, perturbative expansion completely breaks down and cannot be trusted.
- Non-perturbative analysis shows continuity and GR is recovered in the massless limit.
- Continuity is not uniform as a function of distance. (e.g. 1/r expansion does not work.) However, Vainshtein radius can be pushed to infinity in the massless limit.

Analogue of Vainshtein effect

- Breakdown of perturbation in the limit $\lambda \rightarrow 1$

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = e^{2\zeta}\left[e^h\right]_{ij}$$

$$B = \frac{3\lambda - 1}{\lambda - 1}\frac{\dot\zeta}{\partial^2}, \quad n_i = 0$$

momentum constraint

$$\begin{aligned} I_{kin} \;=\; & M_{Pl}^2 \int dt d^3\vec{x} \left\{ (1+3\zeta)\left[\frac{3\lambda-1}{\lambda-1}\dot\zeta^2 + \frac{1}{8}\dot h^{ij}\dot h_{ij}\right] \right. \\ & +\frac{1}{2}\zeta\partial^i(\partial_i B\partial^2 B + 3\partial^j B\partial_i\partial_j B) + \frac{1}{2}(\partial^k h_{ij}\partial_k B - 3\dot h_{ij}\zeta)\partial^i\partial^j B \\ & \left. -\frac{1}{4}(\dot h^{ij}\partial_k h_{ij})\partial^k B \right\} + O(\epsilon^4), \end{aligned}$$

- No negative power of (λ-1) in potential part
- Non-perturbative analysis is needed for scalar graviton sector!

Analogue of Vainshtein effect

- Spherically symmetric, static ansatz

$$N = 1, \quad N_i dx^i = \beta(x) dx, \quad g_{ij} dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$\Downarrow$ $R \equiv \beta^{(\lambda-1)/(2\lambda)} r$ without HD terms

$$R'' + \frac{\lambda - 1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2 R}{4\lambda^2\beta^2} + \frac{(\lambda-1)\beta' R'}{\lambda\beta} - \frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta} - \frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2 + \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1 + 4AB}}{2A},$$

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)}$$

- “-” branch recovers GR in the $\lambda \to 1$ limit

Analogue of Vainshtein effect

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

➡ choose the "-" branch

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'} \frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)}$$

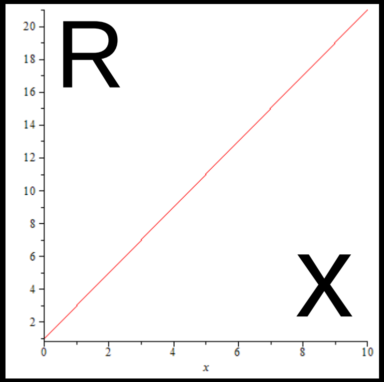
- $(3\lambda-1)\beta^2 \ll (\lambda-1)$
  perturbative regime, $1/r$ expansion
- $(3\lambda-1)\beta^2 \gg (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius???

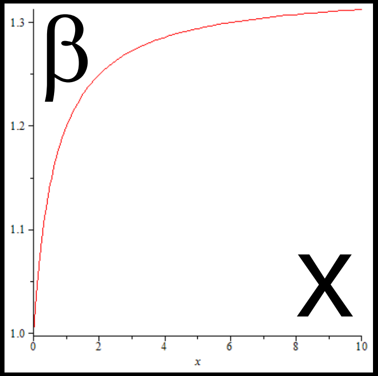
Izumi & Mukohyama 2009
"Steller center is dynamical"

Analogue of Vainshtein effect

- Numerical integration in the "-" branch with $\beta(x=0)=1$, $r(x=0)=1$, $r'(x=0)$ given

for
$\lambda-1=10^{-6}$
$r'(x=0)=2$



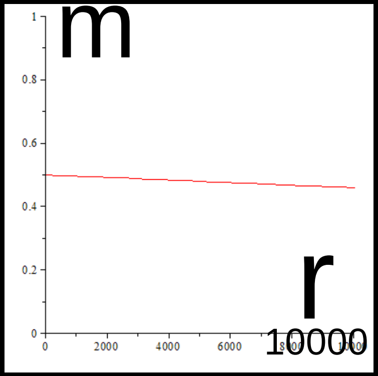


- Misner-Sharp energy

$$m \equiv \frac{r}{2}\left[1-\left(1-\beta^2\right)\left(r'\right)^2\right]$$

almost constant

GR is recovered!



Caustic avoidance

JCAP 0909:005,2009

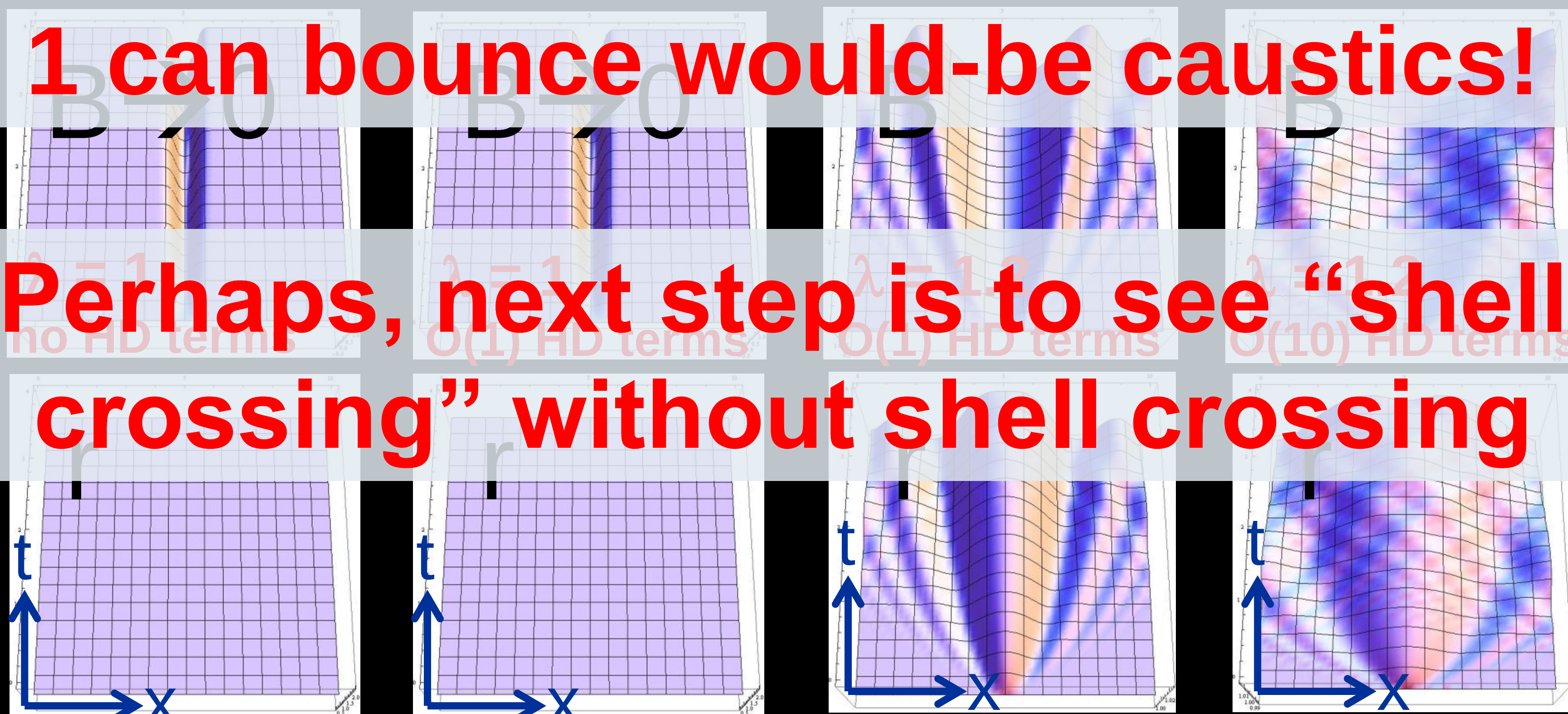
- In GR, congruence of geodesics forms caustics because gravity is attractive.
- HL gravity is repulsive at short distances, due to higher curvature terms.
(c.f. bouncing FRW universe)
- With codimension 2 and 3, higher curvature terms can bounce would-be caustics.
- With codimension 1, deviation of λ from 1 is also needed to bounce would-be caustics.

Caustic avoidance

$$N = 1 \qquad N_i = 0$$

$$ds^2 = B(t,x)dx^2 + r^2(t,x)(dy^2 + dz^2)$$

HD terms and deviation of λ from 1 can bounce would-be caustics!



Perhaps, next step is to see “shell crossing” without shell crossing

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- While there are many fundamental issues to be addressed, it is interesting to investigate cosmological implications.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity does NOT recover GR at low-E but can instead mimic GR+CDM: "dark matter as an integral constant". Constraint algebra is smaller than GR since the time slicing and the "dark matter" rest frame are synchronized.
- For spherically-symmetric, static, vacuum configurations, GR is recovered in the limit $\lambda \rightarrow 1$ non-perturbatively.
- Caustics avoidance requires higher curvature terms and deviation of λ from 1 in the UV. Next step is to see if bounce of shells can mimic shell crossing.

Future works

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ? Does it satisfy the stability condition for the scalar graviton?
  ($|c_s| < \mathrm{Max}[|\Phi|^{1/2}, HL]$ for $\mathrm{Max}[M^{-1}, 0.01\mathrm{mm}] < L < H^{-1}$)
- Is Vainshtein effect generic?
  e.g. superhorizon nonlinear cosmological perturbations (to appear soon, with K.Izumi)
- Can we get a common "limit of speed" ?
  (i) $M_{z=3} << M_{pl}$, (ii) supersymmetry, (iii) other ideas?
- Micro & macro behavior of "CDM"
- Adiabatic initial condition for "CDM" from the z=3 scaling
- Spectral tilt from anomalous dimension
- Extensions of the original theory: Blas, et.al; Horava & Melby-Thompson …