Gauge-Higgs unification in Randall Sundrum Spacetime at Finite Temperature

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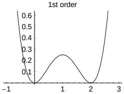
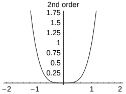
Electroweak phase transition and thermal effects

- Our universe : baryon asymmetric (no anti-proton in our daily life)
- Baryogenesis - Sakhalov's three conditions
  1. B violation process
  2. C and CP symmetry is broken
  3. out of thermal equiriblium
- Electroweak baryogenesis - the 3rd condition requires the first-order phase transition and the expanding bubbles (inside : broken phase)



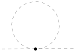


- Order of thermal EWPT
  1. Standard Model - 2nd
  2. SUSY - 1st for some models [Funakubo et.al.]

For other extension of the SM (little higgs, gauge-Higgs unification, etc.), we need to clarify the order of thermal EWPT.

Gauge-Higgs unification (GHU)

- Hierarchy Problem $\Leftarrow$ quadratic divergent δm_h^2

$$m_h^2 = m_{\text{bare}}^2 + (\quad$$

$$\sim g\Lambda^2), \quad \Lambda : \text{cutoff} \tag{1}$$

$$m_h = \mathcal{O}(100\text{GeV}), \quad m_{\text{bare}}, \Lambda \sim M_{GUT} \gg m_h \tag{2}$$

$\rightarrow$ fine-tuning between m_{bare} and Λ.

- Gauge-Higgs unification [N.S.Manton(1983), ...]:
  - extra-dimensional component of the gauge field = the Higgs field

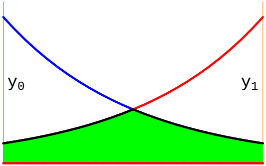
$$A_M = (A_\mu, A_y = h) \tag{3}$$

  - gauge symmetry is spontaneously broken by nonzero $\langle A_y \rangle$
  - Effective potential and the Higgs-mass is finite, thanks to the gauge symmetry in the higher-dimensional spacetime
    $\rightarrow$ solve the fine-tuning problem [Inami-Lim-HH (1998)]

GHU on Randall Sundrum space-time

- Fermions in S^1/Z_2 extra space:
  - zero-mode wave-function : domain-wall profile due to bulk mass term
  - Yukawa-coupling : overlap of wave-functions of fermions and gauge zero modes:

$$H(x)\bar{\psi}_R(x)\psi_L(x) = \int dy \bar{\psi}_R(x,y) A_y(x,y) \psi_L(x,y) \tag{4}$$



→ lightest-mode mass depends exponentially on the bulk mass parameter!

- Higgs effective potential (and Higgs mass) are enhanced [Hosotani et.al,2007, HH 2007].

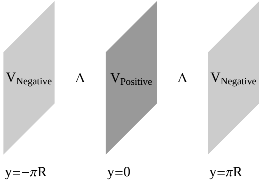
$$m_h \sim \mathcal{O}(100\mathrm{GeV}) \tag{5}$$

Randall-Sundrum space-time

- non-factorizable metric:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \quad k : AdS_5 \text{ curvature} \tag{6}$$

- circle with identification : $y \to -y$ fundamental region : $[0, \pi R]$ fixed points : $y_0 = 0, \quad y_1 = \pi R$



- Hierarchy
  1. Planck (hidden brane) scales : $\Lambda, M_5, k, R \sim M_{pl}$
  2. Kaluza-Klein (visible brane) scales : $m_{KK} = \pi k e^{-kR\pi} \frac{1}{1-e^{-\pi k R}}$
  3. $kR \simeq 12 \to e^{kR\pi} \simeq M_{pl}/\text{TeV}$

Finite-Temperature Effects with non-periodic KK tower

- 1-loop correction for effective potential (per d.o.f of the field):

$$V_{\rm eff} = \frac{(-1)^{2\eta}}{2}\frac{1}{\beta}\int\frac{d^3p}{(2\pi)^3}\sum_{n=-\infty}^{\infty}\sum_{\ell}\ln\left[\left(\frac{2\pi(n+\eta)}{\beta}\right)^2+\vec{p}^2+m_\ell^2\right],$$
$$\eta = 0(\text{boson}), \tfrac{1}{2}(\text{fermion}), \quad \beta\equiv 1/T. \tag{7}$$

- When the extra dimension is compactified on S^1 (radius R),

$$m_\ell^2 = \left(\frac{2\pi\ell+\theta}{2\pi R}\right)^2 + M^2, \quad M : \text{bulk mass} \tag{8}$$

$\rightarrow$ one may use many tricks (Poisson sum formula, etc...)

- after Poisson re-summation, we have

$$V_{\rm eff} = V_{\rm eff}^{T=0} + 2(-1)^{2\eta-1}\sum_{\ell}\sum_{\tilde{n}=1}^{\infty}(-)^{2\eta\tilde{n}}\frac{(n|m_\ell|\beta)^2K_2(\tilde{n}|m_\ell|\beta)}{(\sqrt{2\pi}\tilde{n}\beta)^4} \tag{9}$$

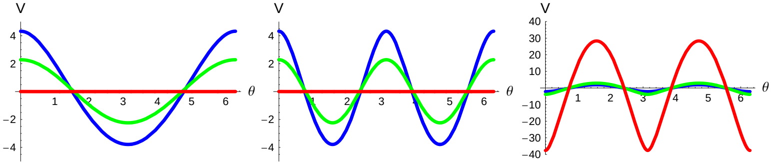
Example – $SU(2)$ on $M^3 \times S^1$

cf. Ho-Hosotani (1990)

- Wilson line phase

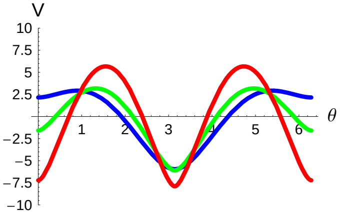
$$W = \exp(i\theta\sigma_3) = \begin{cases} 1_2 & \theta = 0, \\ -1_2 & \theta = \pi, \\ \mathrm{diag}(e^{i\theta}, e^{-i\theta}) & \text{otherwise} \end{cases} \tag{10}$$

- Contributions from fundamental fermion, adjoint fermion, and gauge+ghost fields (blue (cold) $\leftrightarrow$ red (hot))



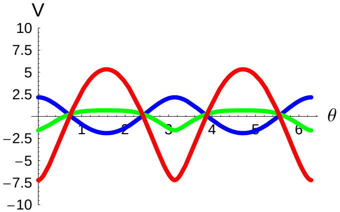
$SU(2)$ on $M^3 \times S^1$ (continued)

- gauge+ghost+ 1 fundamental fermion:



$$SU(2) : \text{unbroken} \qquad (11)$$

- gauge+ghost+1 adjoint fermion:



$$SU(2) \to \begin{cases} SU(2) & T > T_c \\ U(1) & T < T_c \end{cases} \qquad (12)$$

- GHU in RS
  - KK modes [Hosotani-Noda etal, 2005]:

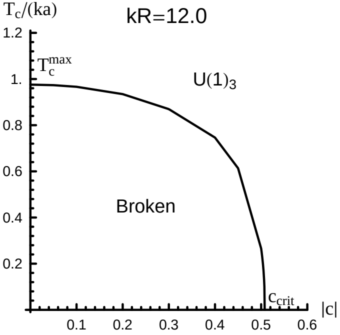
$$\begin{aligned} m_n &= k\lambda_n, \quad G(\alpha,\theta,\lambda_n)=0, && (13)\\ G(\alpha,\theta,\lambda_n) &\equiv \lambda_n^2 z_1 F_{\alpha-1,\alpha-1}(\lambda_n,z_1)F_{\alpha,\alpha}(\lambda_n,z_1) - \frac{4}{\pi^2}\sin^2\frac{\theta}{2}, \\ \alpha &= \begin{cases} \frac{1}{2}\pm(M/k) & \text{fermion} \\ 0,1 & \text{gauge-ghost-higgs fields} \end{cases} && (14)\\ \theta &: \text{Wilson-line phase}, \\ F_{\alpha,\beta}(\lambda,z) &\equiv Y_\beta(\lambda)J_\alpha(\lambda z) - J_\beta(\lambda)Y_\alpha(\lambda z), \quad z_1 = e^{\pi k R}, && (15)\end{aligned}$$

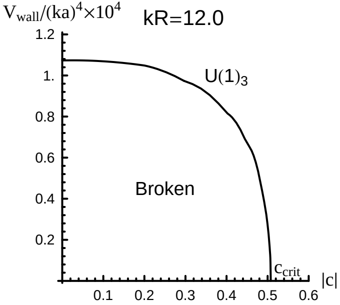
  - $V_{\rm eff}(T=0)$ [Falkowski (2006), HH(2007), Hosotani et.al(2008), Yamashita et.al(2008)]
    $a \equiv e^{-k\pi R}$

$$\begin{aligned} V^{T=0}(\alpha,\theta) &= \frac{k^4a^4}{16\pi^4}\int_0^\infty dt\, t^3\, \mathrm{Re}\ln\left[\frac{G(\alpha,\theta,it)}{G_{\rm asymp}(\alpha,it)}\right], && (16)\\ & G(\alpha,\theta,it) \stackrel{t\to\infty}{\to} G_{\rm asymp}(\alpha,it) && (17)\end{aligned}$$

$SU(2)$ model on RS(preliminary)

- gauge+ghost + 1 fermion (bulk mass $M = ck$)
  - ($SU(2)$ is broken to $U(1)_3$ by orbifold b.c.)
  - gauge field + fundamental fermion : $U(1)_3$ unbroken
  - gauge field + adjoint fermion :





$SO(5) \times U(1)$ GHU model

As application to the particle physics, we study the finite-temperature effect on the model proposed by Hosotani et.al (2008-),

(cf. Hosotani-san's Talk)

- $SO(5) \times U(1) \to SU(2)_L \times \cancel{SU(2)_R} \times U(1) \to U(1)_{\rm em}$
- $m_{KK}(\simeq \pi k a) \sim 1.5\mathrm{TeV}$ for $kR = 12$
- $m_h \sim 70\mathrm{GeV}$
- The model have "H-parity"
  - $P_H = -1$ is assigned for h and $+1$ for other SM fields
  - All P_H-odd interactions $(hWW, hZZ, h\bar{f}f, hhh)$ vanish.
    $\to$ the model can avoid the LEP constraint $(m_h \leq 114\mathrm{GeV})$
  - h is stable and can be the candidate of dark matter (higgs dark-matter).

- Effective potential - we adopt the following approximation:

$$V_{\text{eff}} \simeq \underbrace{V_W + V_Z}_{W,Z\text{boson}} + \underbrace{V_{\text{top}}}_{\text{top quark}}, \tag{18}$$

$$V_W + V_Z \sim 3V_W = 3 \cdot 3V(1, 2\theta), \tag{19}$$

$$V_{\text{top}} = -4V(0.063, 2\theta) \tag{20}$$

  other fermions' contribution ($b, c, s, d, u,\ e, \mu, \tau$, and non-SM heavy particles) are negligible.

- Result for $kR = 12$ (preliminary)
  - critical temperature : $\beta_c \sim 2.2/ka \to T_c \sim 200\text{GeV}$
  - height of potential barrier : $7 \times 10^{-5}(ka)^4 \sim (50\text{GeV})^4$
  - $V_c \equiv V_{\text{eff}}(T = T_c) \simeq V_{\text{eff}}(T = 0) \simeq 250\text{GeV}$
    $\to$ satisfies Shaposhnikov's criteria: $V_c/T_c > 1$

Summary

- Numerically studied Hosotani mechanism at finite-temperature
  - correction from the zero-temperature is obtained by summing up hundreds of Kaluza-Klein masses
  - obtain critical temperature and the height of the potential wall
- Apply to the gauge-Higgs unification ($SO(5) \times U(1)$ model) [preliminary]
  - Critical temperature $T_c \simeq 200\mathrm{GeV}$
  - Height of the potential wall $\sim \mathcal{O}(10^6)\mathrm{GeV}^4$
  - $V_c/T_c \geq 1$

$\therefore$ sufficiently strong 1st order p.t.

Perspective

- Spharelon process in higher-dimensional space-time
- Flavor mixing, CP violation phase in GHU $\rightarrow$ Maru san's talk