Gauge Field Localization in Models with Large Extra Dimensions

Norisuke Sakai (Tokyo Woman's Christian University)

In collaboration with **Kazutoshi Ohta** ,

Progress of Theoretical Physics **124**, 71 (2010) [arXiv:1004.4078] ,Talk at Kobe workshop 2011.1.6

Contents

1 Physics Beyond the Standard Model

LHC Era: **Physics beyond the Standard Model**

Gauge HierarchyProblem: Huge ratio of Electroweak to Fundamental scales

$$\frac{m_{\rm W}^2}{M_{\rm GUT}^2} \approx \left(\frac{10^2}{10^{16}}\right)^2 \approx 10^{-28}, \quad \frac{m_{\rm W}^2}{M_{\rm Gravity}^2} \approx \left(\frac{10^2}{10^{18}}\right)^2 \approx 10^{-32}$$

Solutions (for Explanations) of the Gauge Hierarchy

1. **Composite Higgs** (Technicolor) : Realistic calculable models needed

   L. Susskind, *Phys. Rev.* **D20** (1979) 2619; S. Weinberg, *Phys. Rev.* **D19** (1979) 1277; **D13** (1976) 974; S. Dimopoulos, and L. Susskind, *Nucl. Phys.* **B155** (1979) 237; $\cdots$

2. **Supersymmetry (SUSY)**

$$\begin{array}{lll} \text{Spin}\,\mathbf{0} : m_B, & & \\ \updownarrow & m_B = m_F & \leftarrow \text{Supersymmetry} \\ \text{Spin}\,\frac{1}{2} : m_F, & m_F = 0 & \leftarrow \text{Chiral Symmetry} \end{array}$$

   light Higgs is protected by SUSY and chiral symmetry of Higgsino

   S.Dimopoulos, H.Georgi, Nucl.Phys.**B193** (1981) 150; N.Sakai, Z.f.Phys.**C11** (1981) 153; E.Witten, Nucl.Phys.**B188** (1981) 513; $\cdots$

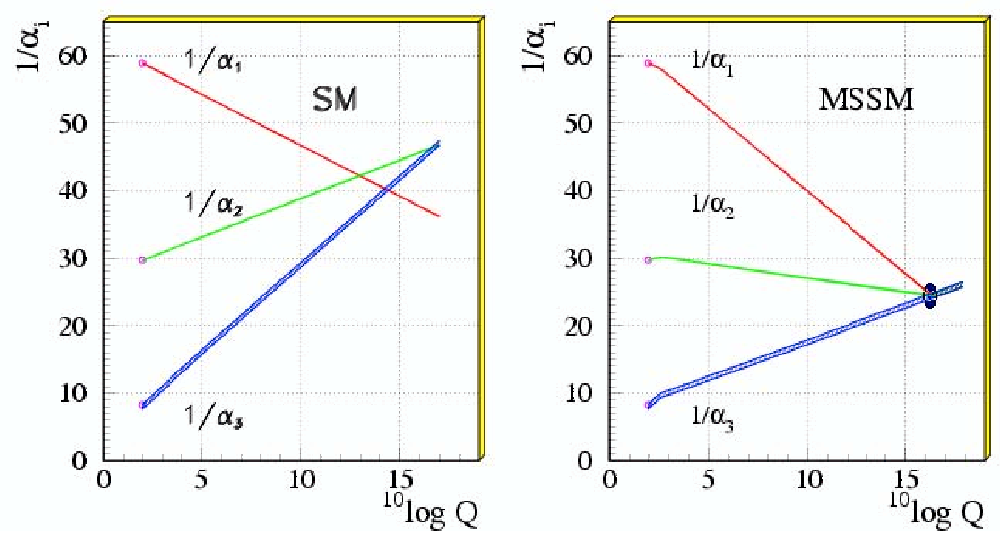


Figure 1: NonSUSY GUT (left) cannot unify gauge couplings. SUSY GUT (right) can unify gauge couplings. $\alpha_i = g_i^2/4\pi, (i = 1, 2, 3)$ are $U(1), SU(2), SU(3)$ gauge couplings.

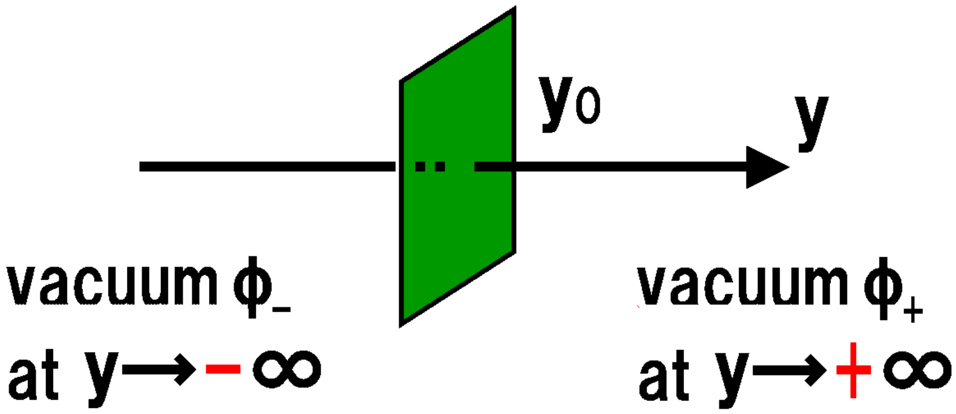
Gauge coupling unification: Indirect Evidence for SUSY

3. Models with **Large Extra Dimensions** (Brane-World scenario)

Standard model particles should be **localized on a brane**

Gravity propagates in higher dimensional bulk: $M^2_{\mathrm{Gravity}} = M^{n+2}_{\mathrm{TeV}} R^n$

P.Horava and E.Witten, Nucl.Phys.**B475**, 94 (1996); N.Arkani-Hamed, S.Dimopoulos, G.Dvali, Phys.Lett.**B429** (1998) 263 ; I.Antoniadis, N.Arkani-Hamed, S.Dimopoulos, G.Dvali, Phys.Lett.**B436** (1998) 257; Randall, Sundrum, Phys.Rev.Lett.**83** (1999) 3370; 4690; $\cdots$



y: **extra dimension**

Models beyond the Standard Model can be tested at LHC and other facilities

2 Gauge Field Localization on Domain Wall

Let us take the simplest case of a brane : **Domain Wall**

Gauge symmetry broken in the bulk (Higgs phase) and restored on a wall

Flux is absorbed by the superconducting bulk

$\longrightarrow$ gauge boson acquires a mass of order **1/(wall width)**

Gauge fields should be **confined in the bulk** (outside of wall)

G.Dvali, M.Shifman, Phys.Lett.**B396** (1997) 64; N.Arkani-Hamed, S.Dimopoulos, G.Dvali, Phys.Lett.**B429** (1998) 263; N.Maru, N.Sakai, Prog.Theor.Phys.**111** (2004) 907; $\cdots$

Warped (Randall-Sundrum) model does not help to localize gauge fields

Our Pourpose: Propose a model to

Localize Non-Abelian Gauge Fields on a Wall

Dielectric vacua as a classical representation of confiniment

$$D = \epsilon_0 E + P \equiv \epsilon E$$

Confinement $\leftrightarrow \epsilon = 0$ (**perfect dia-electric medium**)

Relativisitic version of dielectric vacuum

$$\mathcal{L} = -\frac{1}{4}\epsilon(x) F_{\mu\nu} F^{\mu\nu}, \quad \epsilon(x) = \frac{1}{g^2(x)}$$

J.Kogut, L.Susskind, Phys.Rev.**D9** (1974) 3501; R.Fukuda, Phys.Lett.**B73** (1978) 305; Mod.Phys.Lett.**A24** (2009) 251; $\cdots$

Electric **permiability** $\epsilon(x)$: **Position-dependent gauge coupling**

g^2 finite on the wall, and $g^2 \to \infty$ for the bulk asymptotically

3 Position-Dependent Gauge Coupling

Explore **properties** of our model first, and construct explicit models later

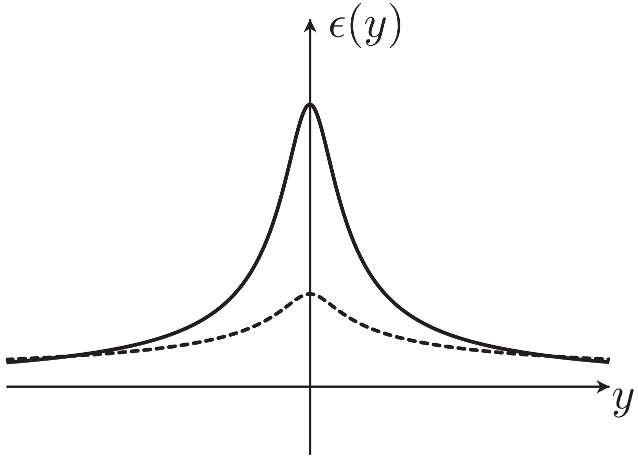


Figure 2: Example of position-dependent gauge coupling in $\boldsymbol{y}$.

Domain wall in $\mathbf{4+1}$ dimensions : $\boldsymbol{x^M, M = 0, 1, \cdots, 4}$,

Wall profile depends on $\boldsymbol{y = x^4}$, World-volume $\boldsymbol{x^\mu, \mu = 0, 1, 2, 3}$,

Our model $\quad (\boldsymbol{F^a_{MN} \equiv \partial_M W^a_N - \partial_N W^a_M - f^{abc} W^b_M W^c_N})$

$$\boldsymbol{\mathcal{L} = -\frac{1}{4}\epsilon(y) F^{MNa} F^a_{MN}}, \qquad \boldsymbol{\epsilon(y) \geq 0}$$

$\boldsymbol{\epsilon(y) \to 0, \quad y \to \pm\infty}$: (profile **localized on the wall**)

Mode Analysis **spectrum** of effective fields

Linearized field equation

$$\boldsymbol{0 = \epsilon(y)(\partial^N \partial_N W^a_M - \partial^N \partial_M W^a_N) + (\partial^y \epsilon(y))(\partial_y W^a_M - \partial_M W^a_y)}$$

Axial gauge $\boldsymbol{W_y^a = 0}$

Decomposition to **transverse** and **longitudinal** components

$$\boldsymbol{W_\mu^a = W_\mu^{aT} + W_\mu^{aL}, \qquad W_\mu^{aL} \equiv \frac{1}{\partial^2}\partial_\mu \partial^\nu W_\nu^a}$$

Field eq. for **longitudinal** component (applying $\boldsymbol{\partial^\mu}$ to $\boldsymbol{M = \mu}$)

$$\boldsymbol{0 = -\partial_y \left(\epsilon(y)\partial_y(\partial^\mu W_\mu^a)\right)}$$

Solution has no propagating modes ($\boldsymbol{F^a(x)}$ arbitrary function of $\boldsymbol{x}$)

$$\boldsymbol{\partial^\mu W_\mu^a = Y(y) F^a(x)}$$

$$\boldsymbol{Y(y) = \int^y dy' \frac{1}{\epsilon(y')}} \tag{3.1}$$

Field eq. for **transverse** component

$$\boldsymbol{0 = \epsilon(y)\partial^\nu \partial_\nu W_\mu^{aT} - \partial_y(\epsilon(y)\partial_y W_\mu^{aT})}$$

Eigenvalue eq. for the mode functions

$$\boldsymbol{\left[-\frac{1}{\epsilon(y)}\frac{d}{dy}\epsilon(y)\frac{d}{dy} - m_n^2\right] u_n(y) = 0}$$

$u_n(y)$ assumed to be a **complete set** of wave functions in y

Mode expansion (Kaluza-Klein decomposition)

$$W_\mu^{aT} = \sum_n w_\mu^{a(n)}(x) u_n(y), \quad (\partial^\nu \partial_\nu + m_n^2) w_\mu^{a(n)}(x) = 0$$

Mapping to a (threshold) bound state using Y in Eq.(3.1) instead of y

$$H u_n(Y) = 0$$

$$H \equiv -\frac{1}{2}\frac{d^2}{dY^2} + U(Y), \quad U(Y) = -\frac{1}{2} m_n^2 \epsilon^2(y(Y)),$$

Normalization of the position-dependent coupling function ϵ

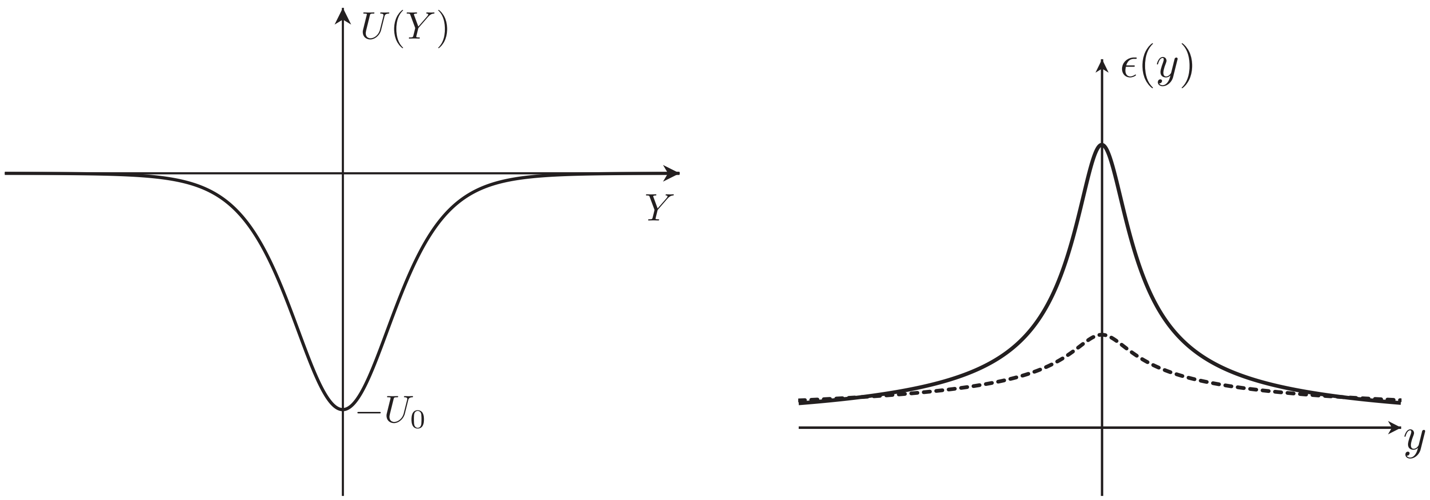
$$\frac{1}{g_4^2} = \int dy\, \epsilon(y) = \int dY\, \epsilon^2(y(Y))$$

$$\mathcal{L}_{\rm eff} \equiv \int dy\, \mathcal{L}_5$$

Canonical kinetic terms $\longrightarrow$ Normalization of mode functions

$$\int_{-\infty}^{\infty} dy\, [g_4^2\, \epsilon(y)] u_n(y)^* u_l(y) = \delta_{nl}$$

There is always a **zero energy solution**: u_0 =constant, $H u_0 = 0$



(a) Potential in $\boldsymbol{Y}$ (b) Position-dependent gauge coupling in $\boldsymbol{y}$

Figure 3: A dashed line represents the wave function of the localized massless vector field.

4-dimensional gauge invariance is maintained intact.

Solvable Example

$$U(Y) = -\frac{U_0}{\cosh^2 \alpha Y}$$

$$\epsilon(y) = \frac{1}{g_4}\sqrt{\frac{\alpha}{4g_4^2 + 2g_4\alpha^3 y^2}}, \quad y = (g_4\sqrt{2}/\alpha^{3/2}) \sinh \alpha Y$$

Mass spectra: a **massless mode** $n = 0$ and a **mass gap** $m_1^2 = 4g_4^2\alpha$

$$m_n^2 = 2g_4^2\alpha n(n+1), \qquad n = 0, 1, 2, \ldots$$

All modes are **normalizable**.

Square well potential is also solvable : A massless mode and a mass gap

4 Four-Dimensional Coulomb law

We wish to demonstrate **4 dimensional Coulomb law**

for the **static source** on the world volume of wall

(to clarify the role of non-transverse component)

$$\mathcal{L} = -\frac{1}{4}\epsilon(y)F^a_{MN}F^{aMN} + \mathcal{J}^a_M W^{aM}, \quad \mathcal{J}^a_M(x,y) = \delta(y)\delta_{M\mu}J^a_\mu(x)$$

Field eq. for W_y has **no source** term

$$0 = \epsilon(y)\partial^M\partial_M W^a_y \quad \longrightarrow \quad 0 = \partial^M\partial_M W^a_y$$

No external source at infinities $\rightarrow$ no nontrivial solution : $W^a_y = 0$

Field equation for W^a_μ in the Landau gauge $\partial^M W^a_M = 0$

$$0 = \epsilon(y)\partial^\nu\partial_\nu W^a_\mu + \partial^y\left(\epsilon(y)\partial_y W^a_\mu\right) - \delta(y)J^a_\mu(x)$$

Thin wall limit with a regularization in the bulk by $1/g_5^2$ ($\epsilon \neq 0$)

$$\epsilon_{\text{thin}}(y) = \frac{\delta(y)}{g_4^2} + \frac{1}{g_5^2}$$

Going to **Euclidean** space, and momentum space only in $\mathbf{4}$-dimensions

$$\begin{aligned} 0 &= \frac{(p^2 - \partial_y^2)}{g_5^2} \tilde{W}^a_\mu(p,y) \\ &+ \frac{\delta(y) p^2 \tilde{W}^a_\mu(p,y)}{g_4^2} - \frac{\partial_y \left(\delta(y) \partial_y \tilde{W}^a_\mu(p,y) \right)}{g_4^2} - \delta(y) \tilde{J}^a_\mu(p) \end{aligned}$$

Solution for $y \neq 0$

$$\tilde{W}^a_\mu(p,y) = C^{a+}_\mu(p) e^{-py} \theta(y) + C^{a-}_\mu(p) e^{py} \theta(-y)$$

Souce terms at $y = 0$ determines $C^{a+}_\mu(p)$, $C^{a-}_\mu(p)$: Result is

$$\tilde{W}^a_\mu(p, y=0) = \frac{g_4^2}{p^2 + \frac{2g_4^2}{g_5^2} p} \tilde{J}^a_\mu(p)$$

After regularization is removed ($g_5 \longrightarrow \infty$: strong coupling in the bulk)

$$\lim_{g_5^2 \to \infty} \tilde{W}^a_\mu(p, y=0) = \frac{g_4^2}{p^2} \tilde{J}^a_\mu(p) \longrightarrow W^a_\mu(x, y=0) = \frac{g_4^2 Q^a}{4\pi r} \delta_{\mu 0}$$

for a static charge $\tilde{J}^a_\mu(p) = Q^a \delta_{\mu 0}$: **4-dimensional Coulomb law**

Finite width for the wall

$$\boldsymbol{\epsilon_{\text{step}}(y) \equiv \begin{cases} 1/(2\varepsilon) & |y| < \varepsilon \\ 1/g_5^2 & |y| > \varepsilon \end{cases}}$$

Conclusion is unchanged

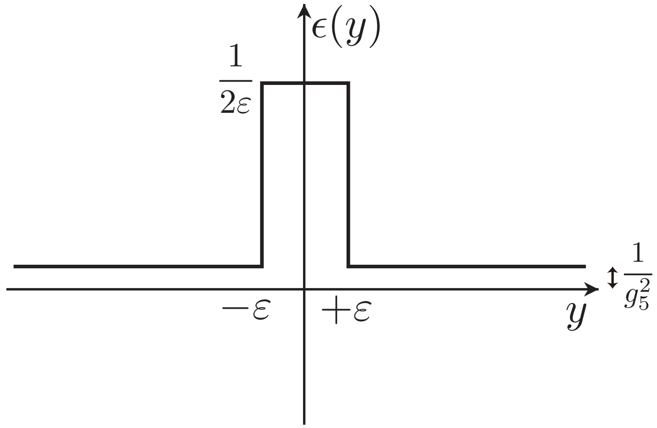


Figure 4: Step function for the position-dependent coupling.

5 Supersymmetric Models for BPS Walls

$\mathbf{5}$-dimensions $\longrightarrow$ $\mathbf{8}$ SUSY

vector multiplet (gauge field) and hypermultiplet (matter)

We display bosonic part only (relevant for wall solutions and gauge fields)

Wall sector

$U(1) \times U(1)$ gauge fields ($A^I_\mu, I = 1, 2$), neutral scalars Σ^I

charged Scalars $H_A, A = 1, \cdots, N_{\mathrm{F}}$, with charge q^A and mass m_A

$$\mathcal{L}_{\mathrm{wall}} = -\frac{1}{4e_I^2}(F^I_{MN})^2 + \frac{1}{2e_I^2}(\partial_M \Sigma^I)^2 + |\mathcal{D}_M H_A|^2 - V$$

$$V = |(q_I^A \Sigma^I - m_A) H_A|^2 + \frac{1}{2e_I^2}(Y^I)^2, \quad Y^I = e_I^2 \left(c_I - q_I^A |H_A|^2\right)$$

$\mathcal{D}_M H_A = (\partial_M + i W^I_M q^A_I) H_A$, gauge coupling e_I, FI parameter c_I

SUSY vacua: $A = 1, \cdots, N_{\mathrm{F}}, \ \ I = 1, 2$

$$(q_I^A \Sigma^I - m_A) H_A = 0, \quad c_I - q_I^A |H_A|^2 = 0,$$

Energy density for a wall in the gauge $W_y^I = 0$

$$\mathcal{E} = \frac{1}{2e_I^2}(\partial_y \Sigma^I - e_I^2(c_I - q_I^A|H_A|^2))^2 + |\partial_y H_A + (q_I^A \Sigma^I - m_A) H_A|^2$$
$$+\partial_y \left(c_I \Sigma^I - q_I^A \Sigma^I |H_A|^2 + m_A |H_A|^2\right)$$

BPS equations

$$\left(\partial_y + q_I^A \Sigma^I\right) H_A = H_A m_A, \quad \partial_y \Sigma^I = e_I^2(c_I - q_I^A |H_A|^2)$$

Exact wall solution at the strong coupling limit $e_I^2 \to \infty$

$$H_A = H_{0A} \Omega_1^{-q_1^A/2} \Omega_2^{-q_2^A/2} e^{m_A y}, \qquad \Sigma^I = \frac{1}{2}\partial_y \log \Omega_I$$

$$c_I = |H_{0A}|^2 \Omega_1^{-q_1^A} \Omega_2^{-q_2^A} e^{2 m_A y}, \quad I = 1, 2$$

Two copies of two charged Higgs fields (four flavor model)

	$H_{A=1}$	$H_{A=2}$	$H_{A=3}$	$H_{A=4}$
q_1^A for $U(1)_1$	1	1	0	0
q_2^A for $U(1)_2$	0	0	1	1
m_A	$\frac{m}{2}$	$-\frac{m}{2}$	$\frac{m}{2}$	$-\frac{m}{2}$

Table 1: Charge and mass of Higgs fields (hypermultiplets) of the four-flavor model

$$c_1 = c_2 = c > 0$$

A pair of SUSY vacua

$$H_1 = 0, \quad H_2 = \sqrt{c}, \quad \Sigma^1 = \frac{m}{2}$$

$$H_1 = \sqrt{c}, \quad H_2 = 0, \quad \Sigma^1 = -\frac{m}{2}$$

Similarly by interchaging $H_1, H_2 \longrightarrow H_3, H_4$ and $\Sigma^1 \longrightarrow \Sigma^2$

Wall solution

$$H_1 = \sqrt{c}\frac{e^{\frac{m}{2}(y-y_1)}}{\sqrt{2\cosh m(y-y_1)}}, \quad H_2 = \sqrt{c}\frac{e^{\frac{-m}{2}(y-y_1)}}{\sqrt{2\cosh m(y-y_1)}}$$

$$\boldsymbol{\Sigma^1 = \frac{m}{2}\tanh m(y-y_1), \quad \Sigma^2 = \frac{m}{2}\tanh m(y-y_2)}$$

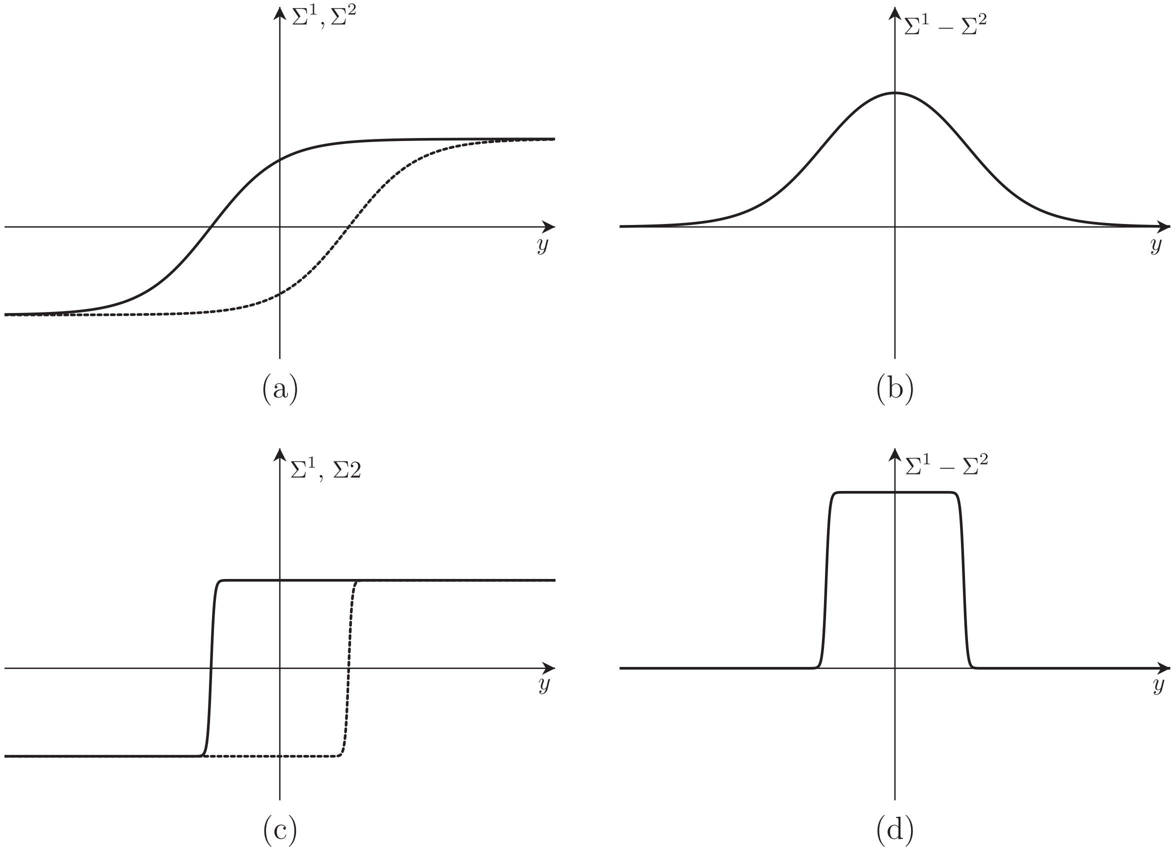


Figure 5: Profile of the domain wall with two copies of two Higgs flavors. (a), (b): $\boldsymbol{|y_1 - y_2| \sim 1/m}$. (c), (d): $\boldsymbol{|y_1 - y_2| \gg 1/m}$, the shape tends to a step-function.

$U(1) \times U(1)$ model with three Higgs flavors

	$H_{A=1}$	$H_{A=2}$	$H_{A=3}$
$U(1)_1$	1	1	0
$U(1)_2$	0	-1	1
m_A	m	0	0

Table 2: Charges and masses of the three Higgs fields

Two vacua: $c_1 > 0$, $c_2 > 0$

$$H_1 = 0, \quad H_2 = \sqrt{c_1}, \quad H_3 = \sqrt{c_1 + c_2}, \quad \Sigma^1 = \Sigma^2 = 0$$

$$H_1 = \sqrt{c_1}, \quad H_2 = 0, \quad H_3 = \sqrt{c_2}, \quad \Sigma^1 = m, \quad \Sigma^2 = 0$$

Wall solution

$$\Sigma^I = \frac{1}{2}\partial_y \log \Omega_I, \quad \Omega_1 = e^{2my} + e^{2my_0}\Omega_2$$

$$\Omega_2 = \frac{1 - e^{2m(y-y_0)} + \sqrt{(1 - e^{2m(y-y_0)})^2 + 4(1 + \frac{c_1}{c_2})e^{2m(y-y_0)}}}{2(1 + \frac{c_1}{c_2})}$$

$$\Sigma^2(-y) = \Sigma^2(y) > 0$$

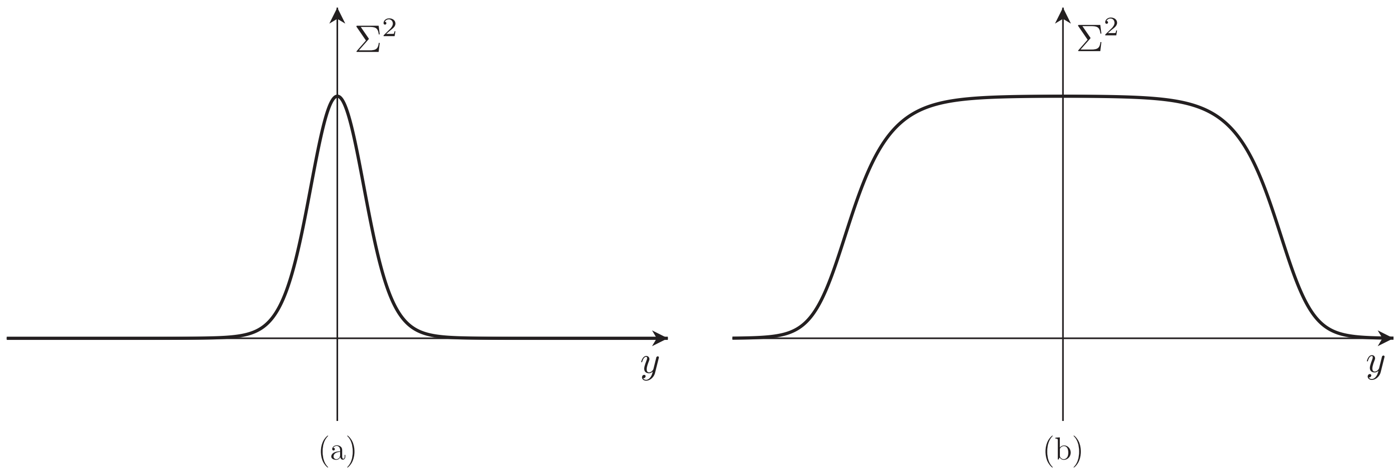


Figure 6: Profile Σ^2 of the domain wall with three Higgs flavors. (a): $c_1/c_2 \sim 1$. (b): $c_1/c_2 \gg 1$, step-function like profile.

Position-dependent coupling function from the cubic prepotential

8 SUSY Lagrangian determined by a **Prepotential** $a(\Sigma)$

$$a_I \equiv \frac{\partial a(\Sigma)}{\partial \Sigma^I}, \quad a_{IJ} \equiv \frac{\partial^2 a(\Sigma)}{\partial \Sigma^I \partial \Sigma^J}, \quad a_{IJK} \equiv \frac{\partial^3 a(\Sigma)}{\partial \Sigma^I \partial \Sigma^J \partial \Sigma^K}$$

complex scalars $H^{irA}, i = 1, 2$: $SU(2)_R$ doublet

$Y^{\alpha I}, c^{\alpha I}, \alpha = 1, 2, 3$: $SU(2)_R$ triplet

$$\mathcal{L} = a_{IJ}\left(-\frac{1}{4}F^I_{MN}F^{JMN} + \frac{1}{2}D_M\Sigma^I D^M\Sigma^J + \frac{1}{2}Y^{\alpha I}Y^{\alpha J}\right) - c^{\alpha I}Y^{\alpha I}$$

$$+a_{IJK}\Big\{-\frac{1}{24}\epsilon^{LMNPQ}W_L^I\Big(F_{MN}^J F_{PQ}^K+\frac{1}{2}[W_M,W_N]^J F_{PQ}^K$$
$$+\frac{1}{16}[W_M,W_N]^J[W_P,W_Q]^K\Big)\Big\}$$
$$+(\mathcal{D}_M H^{irA})^*\mathcal{D}^M H^{irA}-(H^{irA})^*[(q^I\Sigma^I-m_A)^2]^r{}_s H^{isA}$$
$$+(H^{irA})^*(\sigma^\alpha)^i{}_j q^I(Y^{\alpha I})^r{}_s H^{jsA},$$

Prepotential $a(\Sigma)$ should be at most a **cubic** polynomial

N.Seiberg, Phys.Lett.**B388** (1996) 753; $\cdots$

We can assume

$$a\,(\Sigma)=\frac{1}{2e_1^2}(\Sigma^1)^2+\frac{1}{2e_2^2}(\Sigma^2)^2+\frac{1}{2}(a_1\Sigma^1+a_2\Sigma^2)\Sigma^a\Sigma^a$$

Σ^1,Σ^2 for $U(1)\times U(1)$, Σ^a for non-Abelian gauge group

For 2 copies of 2 Higgs model, we choose : $a_1=-a_2\equiv a>0$

For 3 Higgs model, we choose : $a_1=0,\qquad a_2>0$

We obtain gauge coupling function $\epsilon(y)=1/g^2(y)$ which is

Positive definite, **Asymptotically vanishing** (confining)

6 Conclusion

1. A mechanism using the **position-dependent gauge coupling** is proposed to **localize non-Abelian gauge fields** on domain walls in five-dimensional space-time.

2. Low-energy effective theory possesses a **massless vector field**, and a **mass gap**.

3. The four-dimensional **gauge invariance** is maintained intact.

4. We obtain perturbatively the **four-dimensional Coulomb law** for static sources on the domain wall.

5. BPS domain wall solutions with the localization mechanism are explicitly constructed in the $\boldsymbol{U(1) \times U(1)}$ **supersymmetric gauge theory** coupling to the non-Abelian gauge fields only through the **cubic prepotential**, which is consistent with the general principle of supersymmetry in five-dimensional space-time.