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1. Plenary talks

► SU GAO, *Descriptive Combinatorics of Countable Group Actions.*
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Descriptive combinatorics is an active direction of descriptive set theory which studies the combinatorial properties of topological or Borel graphs on Polish spaces. In this talk I will give a survey of selected results on Borel combinatorics of countable group actions, and discuss some recent results on Schreier graphs induced by actions of finitely generated countable abelian groups.

► FENRONG LIU, *Modelling Gameplay on Graphs in Modal Logics.*
Graphs can serve as a playground for various games, such as the Sabotage Game and the Game of Cops and Robbers. In these games, players move along graph edges with the goal of reaching a specific node or set of nodes, or maneuver to avoid being caught—or to catch the opponent. In this talk, I will show how to model these player interactions using modal dynamic logic and present recent developments in this area.

► TAKAKO NEMOTO, *Constructive reverse mathematics over formal systems.*
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Constructive reverse mathematics (cf. [3]) aims to characterize each mathematical theorem by a A combination of logical principles and a choice axioms which are necessary and sufficient to prove it over base theories with intuitionistic logic. To formalize it, we often use a system of analysis, which intends to treat natural numbers and functions over them (cf. [5]). In this talk, we will overview the study of constructive reverse mathematics using formal systems from the following three points of view:

1. Classification of theorems around WKL
   In constructive reverse mathematics, many theorems are characterized by variants of weak Kőnig's lemma (WKL), e.g., [1]. Over classical logic, some of them are equivalent or provable in the base theory, and so we can see the subtlety of classification over intuitionistic systems.
2. The role of Σ^0_1 induction
   Some recent results show that Σ^0_1 induction plays a crucial role in showing the equivalence (cf. [4]). We will see some of the examples.
3. Separation techniques
   In reverse mathematics, it is also important what theorems are NOT equivalent. Some recent studies have developed the technique to construct a model, which is useful to show "theorem A does not imply theorem B" (cf. [2]).

[1] B. Josef, H. Ishihara, T. Kihara and T. Nemoto, *The binary expansion and the intermediate value theorem in constructive reverse mathematics*, ***Archives for Mathematical Logic***, vol. 58 (2019), pp. 203–217.

[2] M. Fujiwara, H. Ishihara, T. Nemoto, N-Y. Suzuki and K. Yokoyama, *Extended frames and separations of logical principles*, ***The Bulletin of Symbolic Logic***, vol. 29 (2023), no. 3, pp. 311–353.

[3] H. Ishihara, *An introduction to constructive reverse mathematics*, ***Handbook of constructive mathematics*** (D. Bridges, H. Ishihara, M. Rathjen and H. Schwichtenberg, editors), Cambridge University Press, 2023, pp. 636–660.

[4] T. Nemoto, *Finite sets and infinite sets in weak intuitionistic arithmetic*, ***Archives for Mathematical Logic***, vol. 59 (2020), pp. 607–657.

[5] ——— *Systems for constructive reverse mathematics*, ***Handbook of constructive mathematics*** (D. Bridges, H. Ishihara, M. Rathjen and H. Schwichtenberg, editors), Cambridge University Press, 2023, pp. 661–699.

▶ KAZUSHIGE TERUI, *Algebraic proof theory: a tutorial.*
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This is a tutorial talk on an interplay between ordered algebras and (non-reductive) proof theory, which may well be called *algebraic proof theory*.

Giving algebraic semantics to a logic is often considered too trivial or circular: think of the notorious claim that Boolean algebras are meaningful as semantics of classical logic. Because of its triviality, however, it gives us a direct and useful means to derive various syntactic properties on logics and proof systems. The most prominent example is an algebraic proof of cut elimination for higher order impredicative logics. While the original idea due to (Schütte 1960) was to employ a three-valued interpretation of classical formulas, it later turns out that it can be more directly formalized as sort of *algebraic completion*, as demonstrated by (Maehara 1991). Although no ordinal information is gained by such an algebraic proof, it instead gives us a widely applicative technique for cut elimination in nonclassical logics, which further has an intimate connection with the reducibility candidates technique for polymorphic lambda calculi (Girard 1971).

Cut elimination is not the only example. For instance, Maehara's syntactic proof of the interpolation theorem can be algebraized, leading to a direct proof of the amalgamation property for various ordered algebras. Another striking example is the Ω-rule (Buchholz 1981), which was originally introduced for ordinal analysis of inductive definitions. It does algebraize too, leading to what we call the Ω-valuation technique, which makes sense in the Dedekind-MacNeille completions of various ordered algebras. Although ordinal information is lost again, one instead gains some interesting algebraic information from that practice.

The talk will consist of a gentle introduction to algebraic completion and its application to proof theory, with the only prerequisites being some basic understanding of lattices and Gentzen's sequent calculi.

▶ ARTEM CHERNIKOV, *Intersecting sets in probability spaces and model theoretic classification.*
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For any fixed n and $\varepsilon > 0$, given a sufficiently long sequence of events in a probability space all of measure at least ε, some n of them will have a common intersection. This follows from the inclusion-exclusion principle. A more subtle pattern: for any $0 < p < q < 1$, we cannot find events A_i and B_j so that the measure of A_i intersected B_j is less than p and of A_j intersected B_i is greater than q for all $1 < i < j < n$, assuming n is sufficiently large. This is closely connected to a fundamental model-theoretic property of probability algebras called stability. We will discuss these and more complicated patterns that arise when our events are indexed by multiple indices. In particular, how such results are connected to higher arity generalizations of de

Finetti's theorem in probability, structural Ramsey theory, hypergraph regularity in combinatorics, and of course model theory.

[1] H. Jerome Keisler, *Randomizing a model*, ***Advances in Mathematics***, vol. 143 (1999), no. 1, pp. 124–158.

[2] Itaï Ben Yaacov, *On theories of random variables*, ***Israel Journal of Mathematics***, vol. 194 (2013), no. 2, pp. 957–1012.

[3] Artem Chernikov and Henry Towsner, *Hypergraph regularity and higher arity VC-dimension*, ***Preprint, arXiv:1507.01482*** (2020).

[4] Artem Chernikov and Henry Towsner, *Perfect stable regularity lemma and slice-wise stable hypergraphs*, ***Preprint, arXiv:2402.07870*** (2024).

[5] Artem Chernikov and Henry Towsner, *Intersecting sets in probability spaces and Shelah's classification*, ***Proceedings of the 14th Panhellenic Logic Symposium, arXiv:2406.18772*** (2024).

[6] Artem Chernikov and Henry Towsner, *Averages of hypergraphs and higher arity stability*, ***Preprint, arXiv:2508.05839*** (2025).

▶ DANIEL TURETSKY, *True stages for computability and effective descriptive set theory.*

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Priority arguments are hard. The true stages machinery was conceived as a technique for organizing complex priority constructions in computability theory, much like Ash's metatheorem. With a little modification, however, it can prove remarkably useful in descriptive set theory. Using this machinery, we can obtain nice proofs of results of Wadge, Hausdorff and Kuratowski, and Louveau, sometimes strengthening the result in the process. I will give the ideas behind the machinery and some examples of how it applies to both computability theory and descriptive set theory.

2. Special sessions

Model theory

► HIROTAKA KIKYO, *Dividing and forking in some generic structures.*
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The main part of this talk is a joint work with Akito Tsuboi. We investigate the class of m-hypergraphs in which substructures with l elements have more than s subsets of size m that do not form a hyperedge. The class has a (unique) Fraïssé limit, if $0 \leq s < \binom{l-2}{m-2}$. We show that the theory of the Fraïssé limit has SU-rank one if $0 \leq s < \binom{l-3}{m-3}$, and dividing and forking will be different concepts in the theory if $\binom{l-3}{m-3} \leq s < \binom{l-2}{m-2}$. We also provide some structures produced by Hrushovski's predimension construction with a control function where dividing and forking will be different concepts in the theory.

[1] G. Conant, *An axiomatic approach to free amalgamation*, ***Journal of Symbolic Logic***, vol. 82 (2017), no. 2, pp. 648–671.

[2] G. Conant, *Forking and dividing in Henson graphs*, ***Notre Dame Journal of Formal Logic***, vol. 58 (2017), no. 4, pp. 555–566.

[3] D.M. Evans, M.W.H. Wong, *Some remarks on generic structures*, ***Journal of Symbolic Logic***, vol. 74 (2009), no. 4, pp. 1143–1154.

[4] H. Kikyo, A. Tsuboi, *Dividing and forking in random hypergraphs*, ***Annals of Pure and Applied Logic***, vol. 176 (2025), 103521.

► SCOTT MUTCHNIK, *A low-information unstable forking sequence.*
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The stable forking conjecture, one of the most important problems in model theory, asks whether every instance of forking in a simple theory is exhibited by a stable formula. A closely related question is whether the forking relation is stable in any simple theory: that is, whether there is no indiscernible sequence $\{a_i, b_i\}_{i<\omega}$ such that $a_i \not\perp b_j$ exactly when $i < j$. Peretz, in [1], shows that the forking relation is stable between types of SU-rank 2 in a supersimple theory, and that when $\{a_i, b_i\}_{i<\omega}$ exhibits the instability of the forking relation between types of SU-rank 3, the limit type of $\{b_i\}_{i<\omega}$ does not fork over the first two terms $b_0 b_1$. Informally, this means that *every* sequence exhibiting the instability of the forking relation between types of rank three is given by a small amount of information.

However, while Peretz suggests generalizations of some of his results to types of larger SU-rank, his rank-three result does not have a direct generalization to larger ranks, at least as long as the pseudolinearity conjecture is true. Nonetheless, we can show that if the forking relation is unstable between types of larger rank, then there *exists* a sequence exhibiting this instability given by a small amount of information. We discuss this existential version of Peretz's result and, if time allows, discuss our proof.

[1] Peretz, Assaf. "Geometry of Forking in Simple Theories." *Journal of Symbolic Logic* 71, no. 1 (2006): 347–59.

► JINHE YE, *Revisiting Zilber's trichotomy.*
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Zilber's Trichotomy Conjecture remains a central theme in model theory, offering a powerful lens through which to understand the geometry of definable sets in well-behaved contexts. Although numerous counterexamples exist, the conjecture continues to inspire fruitful investigation. In this talk, I will introduce a new axiomatic framework —Hausdorff Geometric Structures —that captures a broad range of contexts where the Trichotomy holds. This framework provides a unified approach to studying the trichotomy in such structures. Using this, we fully resolve the restricted trichotomy conjecture in the theory of algebraically closed fields. The talk is based on joint work with Ben Castle and Assaf Hasson.

▶ CHIEU-MINH TRAN, *Large implies henselian*.
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In the classic paper "Henselian implies large", Pop showed that a field is large if it is elementarily equivalent to the fraction field of a proper Henselian local domain. We show that it can, in fact, be improved to an "if and only if" statement. We will explain how this result arises from the effort to introduce canonical topologies over a field, namely, a result comparing the etale-open topology with a newly introduced finite-closed topology. (Joint with Will Johnson, Erik Walsberg, and Jinhe Ye)

Set theory

▶ SARKA STEJSKALOVA, *Weak Kurepa trees.*
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In the talk, we will survey recent results related to Kurepa trees and weak Kurepa trees at ω_1 . We will provide a work-in-progress solution to the question of whether it is possible to add a weak Kurepa tree (or even a Kurepa tree) with a ccc forcing of size ω_1 over a model with no weak Kurepa trees. This problem is a generalization of the question of Jin and Shelah, who asked the same question for Kurepa trees. We will also discuss these questions for other forcings besides small ccc forcings and compare known results for Kurepa and weak Kurepa trees.

▶ TERUYUKI YORIOKA, *Aspero-Mota iteration and the size of the continuum.*

Aspero-Mota iteration is a type of forcing iterations which enables us to obtain the consistency results, which are conclusions from Proper Forcing Axiom, together with the size of the continuum larger than $\aleph_2$. The most segnificant points of Aspero-Mota iteration is the use of symmetric systems of countable elementary submodels of the model of sets of hereditarily size less than the fixed uncountable regular cardinal κ equipped with markers. Markers stands for activations of countable elementary submodels to be generic. In the talk, we demonstrate that Aspero-Mota iteration forces that the size of the continuum is not greater than $\aleph_2$ if we require that markers are also symmetric.

▶ MONROE ESKEW, *Ideals and blobs.*

Last year the author and Hayut showed the consistency of dense ideals on every successor cardinal. A key component to this was a new collapse forcing. By relativizing its definition to various classes of posets, we get various simple forcings that we call "blobs" which amalgamate all posets from the class in a very random way. This works for many classes and allows us to avoid many iteration theorems. For example, we can force MM without having to know about RCS and without using Laver functions. This is joint work with Curial Rodriguez.

▶ TANMAY C. INAMDAR, *A Ramsey theorem for the reals.*
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In 1970 Galvin conjectured that for every colouring of pairs of reals with finitely many colours, there is a set homeomorphic to the rationals on which at most two colours are taken. By Sierpiński's 1933 colouring, the number of colours is optimal. I will discuss my proof of Galvin's conjecture.

▶ MIGUEL A. CARDONA, *Around the constant prediction number and constant evasion number.*
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Let $2 \leq n \leq \omega$ and call a function $\sigma\colon {}^{<\omega}n \to n$ a predictor. By Σ_n, we denote the set of all predictors. Say σ *constantly predicts a real* $f \in {}^{\omega}n$, written as $f \sqsubseteq^{\mathrm{cp}} \sigma$

$$f \sqsubseteq^{\mathrm{cp}} \sigma \text{ iff } \exists k \in \omega\ \forall i \in \omega\ \exists j \in [i, i+k)\ f(j) = \sigma(f{\restriction}j).$$

We define the constant evasion number

$$\mathfrak{e}_n^{\mathrm{const}} := \min\{|F| : F \subseteq {}^{\omega}n \text{ and} \neg\exists\sigma \in \Sigma_n\ \forall f \in F\colon f \sqsubseteq^{\mathrm{cp}} \sigma\}$$

and the constant prediction number

$$\mathfrak{v}_n^{\mathrm{const}} := \min\{|S| : S \subseteq \Sigma_n \text{ and } \forall f \in {}^{\omega}n \; \exists \sigma \in S \colon f \sqsubset^{\mathrm{cp}}_{=} \sigma\}.$$

Motivated by the previous cardinal invariants, this talk aims to introduce different versions of these cardinals. Mainly, we offer characterizations of cardinal invariants based on Cichoń's diagram and obtain several bounds and consistency results pertaining to them.

This work is joint with Miroslav Repický.

Proof theory and reverse mathematics

▶ TAISHI KURAHASHI, *Provability and consistency principles for the second incompleteness theorem.*
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The second incompleteness theorem (G2) states that every consistent and computably axiomatized theory containing sufficient arithmetic cannot prove its own consistency. However, this formulation of G2 is not accurate. The unprovability of consistency statements depends both on the choice of provability predicates and on how consistency statements are formalized. Hilbert and Bernays [1], Löb [5], Jeroslow [2], and others established sufficient conditions on provability predicates under which G2 holds. On the other hand, Kurahashi [3] pointed out that the unprovability of consistency statements guaranteed by these sufficient conditions depends on the formulation of consistency statements. Kurahashi also systematized the relationships between the properties of provability predicates and the variations of G2 based on different formulations of consistency statements.

In this talk, we will present results from the author' s recent work [4], which is a continuation of [3]. We will show that several weak principles such as **E** and **C** inspired by non-normal modal logic are sufficient for deriving various refined forms of G2. We will introduce the consistency statements $\mathrm{Con}^{\mathrm{L}}_T$, $\mathrm{Con}^{\mathrm{S}}_T$, and $\mathrm{Con}^{\mathrm{H}}_T$ and the rule **Ros** as follows:

- $\mathrm{Con}^{\mathrm{L}}_T :\equiv \neg \mathrm{Pr}_T(\ulcorner 0 = 1 \urcorner)$,
- $\mathrm{Con}^{\mathrm{S}}_T := \{\mathrm{Pr}_T(\ulcorner \varphi \urcorner) \to \neg \mathrm{Pr}_T(\ulcorner \neg\varphi \urcorner) \mid \varphi \text{ is a sentence}\}$,
- $\mathrm{Con}^{\mathrm{H}}_T :\equiv \forall x(\mathrm{Pr}_T(x) \to \neg \mathrm{Pr}_T(\dot{\neg} x))$,
- **Ros** : If $T \vdash \neg \varphi$, then $T \vdash \neg \mathrm{Pr}_T(\ulcorner \varphi \urcorner)$.

It is shown that $T \vdash \mathrm{Con}^{\mathrm{H}}_T \Rightarrow T \vdash \mathrm{Con}^{\mathrm{S}}_T \Rightarrow \mathbf{Ros} \Rightarrow T \vdash \mathrm{Con}^{\mathrm{L}}_T$. The main results of this talk are as follows:

1. For every Σ_1 provability predicate $\mathrm{Pr}_T(x)$ satisfying the converse Barcan principle **CB** and the uniform version of formalized Δ_0-completeness ${}_{\mathbf{0}}\mathbf{C}^{\mathrm{U}}$, we have $T \nvdash \mathrm{Con}^{\mathrm{H}}_T$. We denote this statement simply as "$\{\mathbf{CB}, {}_{\mathbf{0}}\mathbf{C}^{\mathrm{U}}\} \Rightarrow T \nvdash \mathrm{Con}^{\mathrm{H}}_T$".
2. $\{\mathbf{E}, \mathbf{D3}\} \Rightarrow T \nvdash \mathrm{Con}^{\mathrm{S}}_T$.
3. $\{\mathbf{C}, \mathbf{D3}\} \Rightarrow$ non-**Ros**.
4. $\{\mathbf{E}, \mathbf{C}, \mathbf{D3}\} \Rightarrow T \nvdash \mathrm{Con}^{\mathrm{L}}_T$.
5. $\{\mathbf{E}^{\mathrm{U}}, \mathbf{CB}_{\exists}\} \Rightarrow {}_{\mathbf{1}}\mathbf{C}$.

These results provide a more fine-grained understanding of the interplay between provability conditions and consistency statements, clarifying the structural aspects of G2.

[1] David Hilbert and Paul Bernays, ***Grundlagen der Mathematik. Vol. II***, Springer, Berlin, 1939.

[2] Robert G. Jeroslow, *Redundancies in the Hilbert–Bernays derivability conditions for Gödel's second incompleteness theorem*, ***The Journal of Symbolic Logic***, vol. 38 (1973), no. 3, pp. 359–367.

[3] Taishi Kurahashi, *A note on derivability conditions*, ***The Journal of Symbolic Logic***, vol. 85 (2020), no. 3, pp. 1224–1253.

[4] Taishi Kurahashi, *Refinements of provability and consistency principles for the second incompleteness theorem*, arXiv:2507.00955.

[5] Martin Hugo Löb, *Solution of a problem of Leon Henkin*, ***The Journal of Symbolic Logic***, vol. 20 (1955), no. 2, pp. 115–118.

► SHUWEI WANG, *Realisability semantics and choice principles for Weaver's third-order conceptual mathematics.*
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In [1], Nik Weaver introduced a formal semi-intuitionistic third-order arithmetic as an alternative foundation for mainstream mathematics. This system is claimed to follow his philosophical vision of *mathematical conceptualism*, which is a variant of predicativitism and admits all and only countable procedurals built up from below as concrete mathematical objects. Supplemented by a layer of class-like third-order objects, Weaver maintains that his arithmetic can be utilised for much of modern mathematics that mainly concerns uncountable structure with a tame, countable description, such as separable metric spaces and second-countable topological spaces.

In section 2.3 of his paper, Weaver proposed a direction to further extend this theory to accommodate for uncountable choice principles that are commonly used in mainstream mathematics, by assuming a global well-ordering structure on the second-order objects. However, it remains suspectable how such an extension may stay loyal to his predicative philosophical ideals. In this talk, we shall present a realisability interpretation for Weaver's mathematics through hyperarithmetic functions in a classical second-order meta-theory. By doing this, we compare the proof-theoretic strength of Weaver's base theory and possible extensions to well-known fragments of second-order arithmetic — ranging in strength from Σ^1_1-choice to Bar induction — and use this as a benchmark to discuss how well they fit into the predicativity picture.

Detailed proofs of many results in this talk are available in the speaker's arXiv preprint [2], with some more recent improvements.

[1] Nik Weaver, *Axiomatizing mathematical conceptualism in third order arithmetic*, 2009. Preprint available at arXiv:0905.1675 [math.HO].

[2] Shuwei Wang, *An ordinal analysis of CM and its extensions*, 2025. Preprint available at arXiv:2501.12631 [math.LO].

► TIN LOK WONG, *Definable solutions to combinatorial problems.*
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It is popular endeavour in recursion theory to craft specially definable solutions to combinatorial problems. Such studies have led to various conservation results in reverse mathematics. We prove that, for certain combinatorial problems, the existence of definable solutions is equivalent to induction. In particular, one may be faced with the peculiar situation where familiar combinatorial problems have no definable solution at all in a model of arithmetic.

This work is joint with Chi Tat Chong (Singapore), and is still ongoing.

► YUDAI SUZUKI, *The pseudo-hierarchy method and transfinite leftmost path principle.*
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A sequence of sets of natural numbers indexed by a well-ordering is called a jump-hierarchy if each i-th segment is obtained as the Turing jump of the initial segment of the sequence bounded by i. That is, a jump-hierarchy represents a transfinite iteration of the Turing jump operator.

The system ATR_0, the second strongest of the big five systems in reverse mathematics, is characterized by the axiom 'for any well-ordering W and set X, there is a jump-hierarchy indexed by W starting with X'. However, this axiom implies the existence of pseudo-hierarchies. Here, a pseudo-hierarchy is a jump-hierarchy-like sequence indexed by an ill-founded linear ordering.

To study ATR_0, it is important to study pseudo-hierarchies. This is because a pseudo-hierarchy is more complicated than a jump-hierarchy in the sense of computability, and hence ATR_0 proves a lot of complicated theorems thanks to pseudo-hierarchies.

In recent studies of reverse mathematics, the system TLPP_0 standing for transfinite leftmost path principle is of interest. TLPP_0 is characterized by the axiom 'for any ill-founded tree T and well-ordering α, there is a $\Delta^0_{\alpha+1}$-leftmost path'. Here, a $\Delta^0_{\alpha+1}$-leftmost path is a path of T that has no $\Delta^0_{\alpha+1}$-definable path to its left. Since $\Delta^0_{\alpha+1}$-definability is defined by using the jump-hierarchy of length α, the system TLPP_0 is deeply related to jump-hierarchies. In this talk, I will study TLPP_0 from the point of view of pseudo-hierarchies.

Philosophical logic

▶ SUJATA GHOSH, *A modal logic study of substitutions.*
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Substitutions play a crucial role in a wide range of contexts, from analyzing the dynamics of social opinions and conducting mathematical computations to engaging in game-theoretical analysis. For many situations, considering one-step substitutions is often adequate, but, for more complex cases, iterative substitutions become indispensable.

In this talk, we will explore logical frameworks that model both single-step and iterative substitutions. We will study a number of properties of these logics, including their expressive power, Hilbert-style proof systems, and satisfiability problems. We will also try to delineate the relationship between single-step substitutions and the standard syntactic replacements commonly found in many classical logics. We will end with a comparison of the proposed framework for iterative substitution with existing ones involving such reasoning.

This is a joint work with Yaxin Tu, Dazhu Li and Fenrong Liu.

▶ SUZUKI, NOBU-YUKI, *Disentangling and Reconnecting Constructivity: The Role of the Quantifier Annihilation Axiom.*
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Constructivity, a core concept in intuitionistic logic, is often marked by two meta-properties: the Existence Property (EP), ensuring witnesses for existential proofs, and the Disjunction Property (DP), ensuring a definite disjunct. EP and DP often appear together; for example, many systems — such as recursive extensions of Heyting arithmetic — preserve both (cf. [1, 2]).

The Quantifier Annihilation Axiom

$$Z : \exists x A(x) \to \forall x A(x),$$

has appeared in [3] (as Z_1). By employing Z, the properties EP and DP can be disentangled — no longer necessarily holding together in intermediate predicate logics ([5]). Moreover, when the use of Z is restricted — formalized as Z-normality — the implication EP $\Rightarrow$ DP is restored.

Analyzing both the separation and the recovery of EP and DP opens new questions about the boundaries of constructive reasoning, and may lead to a sharper understanding of its underlying structure.

[1] H. Friedman, *The disjunction property implies the numerical existence property*, ***Proceedings of the National Academy of Sciences of the United States of America***, vol. 72 (1975), no. 8, pp. 2877–2878.

[2] T. Kurahashi, *On partial disjunction properties of theories containing Peano arithmetic*, ***Archives Mathematical Logic***, vol. 57 (2018), no. 7–8, pp. 953–980.

[3] H. Ono, *A study of intermediate predicate logics*, ***Publications of the Research Institute for Mathematical Sciences, Kyoto University***, vol. 7 (1987), pp. 85–93.

[4] ——— *Some problems in intermediate predicate logics*, ***Reports on Mathematical Logic***, no.21 (1987), pp. 55–67 ;Supplement no.22(1988), 117–-118.

[5] N.-Y. Suzuki, *A Negative Solution to Ono's Problem P52: Existence and Disjunction Properties in Intermediate Predicate Logics*, ***Hiroakira Ono on Substructural Logics, (Outstanding Contributions to Logic 23)***, (Nikolaos Galatos and Kazushige Terui), Springer, Cham, 2022, pp. 319–337.

► KOJI MINESHIMA, *Language, Logic, and LLMs.*
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Large Language Models (LLMs) have undoubtedly acquired linguistic abilities sufficient for meaningful communication. But can we also say that, as an extension of this, they have potentially acquired logical reasoning abilities? Beyond humans, we have never before encountered a creature capable of manipulating language as skillfully as LLMs. Understanding the linguistic and logical capacities of LLMs, and how we should theorize about them, is not only a philosophical concern but also a pressing issue for a technology deeply embedded in daily life with broad social consequences. If LLMs were to lack logical reasoning abilities in any fundamental sense, then alternative approaches to their design and use would be required. In such a case, what kinds of supplementary methods would be needed? Drawing on recent research from our group, I will consider what methodologies are appropriate for evaluating the logical reasoning capacities of LLMs, including their extension to mathematical reasoning, and I will provide an overview of the challenges that remain.

► HANTI LIN, *What is inductive logic?*.
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In inductive logic, the Bayesian approach is the most prominent, advocated by R. Carnap (objective Bayesian) and C. Howson (subjective Bayesian). Bayesians regard inductive logic as closely resembling deductive logic: it slightly weakens the ideal of truth preservation in deductive logic. That is, while truth preservation in deduction amounts to 100% of the worlds that make the premises true also making the conclusion true, Bayesian inductive logic aims instead at truth in a *high* proportion of worlds rather than the *maximal* proportion, with proportions measured by probabilities (logical probabilities for Carnap, subjective probabilities for Howson).

By contrast, Peirce, an earlier pioneer of inductive logic, appears to offer a different view. His approach is often presented as treating inductive logic as a matter of *convergence* to the truth, which sounds very different from truth preservation, let alone truth in a *high* proportion of worlds. I argue, however, that Peirce's view can be reformulated so that inductive logic is still a variant of deductive logic: Peircean convergence to the truth—in a sense I will make precise—is equivalent to truth in an ever-increasing proportion of worlds, a proportion bounded only by the *maximal* proportion.

Two upshots follow. First, whether inductive logic is understood in Bayesian or Peircean terms, it remains quite close to deductive logic—both approaches seek to weaken the deductive ideal of truth preservation as little as possible. Second, it is not surprising that there can be an inductive logic that is both Bayesian and Peircean, and I argue that it already exists in a somewhat unfamiliar form within a most active branch of Bayesian statistics: nonparametric Bayesian statistics.

In my earlier work, I argued that Peircean inductive logic systematizes frequentist statistics and machine learning theory. Now, with the points made in the present paper, I defend a stronger claim: Peircean inductive logic systematizes a broad range of data science—from frequentist statistics to machine learning theory to a significant part of Bayesian statistics—in a way that preserves Carnap's idea that inductive logic is a variant of deductive logic. All this is logic after all.

Computability theory

► WANG WEI, *Iterated jumps of perfect subtrees of positive trees.*
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A (binary) tree is *positive* iff its infinite paths form a set with positive Lebesgue measure. It is a well-known phenomenon in algorithmic randomness that positive trees contain infinite paths with weak computational power. By Cantor, positive trees always have perfect subtrees and even positive perfect subtrees. Chong et al. [1] studied computational power of perfect subtrees of positive trees, and to some extend showed that positive trees also contain perfect subtrees of weak computational power. This work was followed up by Barmpalias and Wang [2], by Barmpalias and Zhang [3], and also by Greenberg, J. Miller and Nies [4]. This talk will survey the main results in the above works, and also sketch a recent result that every Δ_n positive tree contains a low_n perfect subtree.

[1] Chong, Chitat and Li, Wei and Wang, Wei and Yang, Yue, *On the computability of perfect subsets of sets with positive measure*, ***Proceedings of the American Mathematical Society***, vol. 147 (2019), no. 9, pp. 4021–4028.

[2] Barmpalias, George and Wang, Wei, *Pathwise-randomness and models of second-order arithmetic*, ***Information and Computation***, vol. 299 (2024), 105181.

[3] Barmpalias, George and Zhang, Xiaoyan, *Growth and irreducibility in path-incompressible trees*, ***Information and Computation***, vol. 297 (2024), 105136.

[4] Greenberg, Noam and Miller, Joseph S. and Nies, André, *Highness properties close to PA completeness*, ***Israel Journal of Mathematics***, vol. 244 (2021), pp. 419–465.

► MATTHEW DE BRECHT, *Effective subcategories of quasi-Polish spaces.*
Kyoto University.

In previous work, we constructed the category of quasi-Polish spaces as a represented space, and showed that limits, coproducts, and standard powerspace monads are computable in the sense of Type Two Theory of Effectivity (TTE).

In this talk, we show that some important sub-categories of quasi-Polish spaces (in particular the subcategories of overt discrete quasi-Polish spaces and compact Hausdorff quasi-Polish spaces) can be constructed as effective quasi-Polish spaces (i.e., effective internal categories of the category of quasi-Polish spaces).

To show that the constructions are natural, we show that Stone duality (for both objects and morphisms) is computable, in the sense that the dual contravariant functors and the natural transformations demonstrating their adjointness are computable.

► HEER TERN KOH, *Effective Polish Spaces.*
Nanyang Technological University.

We explore a recent program in comparing effective topological spaces up to (not necessarily computable) homeomorphism. We give a survey of some earlier results separating some of the common notions of presentations for Polish spaces, and discuss some new results regarding punctual Polish spaces. Additionally, we will also discuss some ‘pointless’ notions like computable topological spaces and computable topological groups. Finally, following in the pattern of computable structure theory, we also look at the degrees of categoricity for some ‘standard’ Polish spaces.

3. Contributed talks

Monday, September 8

▶ KENSHI MIYABE, *Variation of weakly computable reals in Solovay reducibility.*
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In computability theory, a real number α is termed *weakly computable* if there exists a computable sequence of rational numbers $(a_n)_n$ that converges to α and whose total variation, given by $|a_0| + \sum_{n\in\omega} |a_{n+1} - a_n|$, is finite. Miller (2017) defined a weakly computable real α as *variation nonrandom* if it possesses a computable approximation $(a_n)_n$ where the total variation is not Martin-Löf random. If no such approximation exists, α is called *variation random*.

This work investigates the relationship between a weakly computable real and its variation in the context of Solovay reducibility.

For any weakly computable real α, its variation β is always a left-c.e. real, and α is Solovay reducible to β ($\alpha \leq_S \beta$). Conversely, if β is a left-c.e. real such that $\alpha \leq_S \beta$, then we can find a computable approximation $(a_n)_n$ and a natural number q such that the variation of this sequence is $q\beta$. Consequently, determining the variation is equivalent to finding a left-c.e. real that is Solovay above α. This result further implies an algebraic characterization of the Solovay reducibility relation ($\alpha \leq_S \beta$) when α is weakly computable and β is left-c.e.

[1] Joseph S. Miller, *On work of Barmpalias and Lewis-Pye: A derivation on the d.c.e. reals*, ***Computability and Complexity –Essays Dedicated to Rodney G. Downey on the Occasion of His 60th Birthday***, (Adam R. Day, Michael R. Fellows, Noam Greenberg, Bakhadyr Khoussainov, Alexander G. Melnikov and Frances A. Rosamond, editors), vol. 10010, Springer, 2017, pp. 644–659.

▶ MANAT MUSTAFA, *On Learning Existentially Definable Subsets in a Computable Structure.*
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We investigate algorithmic learning of existentially definable subsets in computable structures from positive data. In this setting, a learner attempts to identify the Gödel number of an existential formula defining a target set within the structure. We analyze learnability under classical criteria—explanatory, vacillatory, behaviorally correct, and confident learning—for several classes of structures including equivalence structures and Boolean algebras. Our main results include: (i) for computable structures with decidable $\forall\exists$-theories, explanatory and behaviorally correct learnability coincide; (ii) for equivalence structures, vacillatory and behaviorally correct learnability are equivalent; (iii) for Boolean algebras, every computable algebra has an isomorphic copy where confident learning is achievable; and (iv) we construct examples separating learning criteria, such as an equivalence structure where vacillatory learnability does not imply explanatory learnability. These results illuminate the nuanced interactions between logic, structure theory, and algorithmic inference.

This talk base on join work with N.Bazhenov.

[1] Ash, C.J., Knight, J., *Computable Structures and the Hyperarithmetical Hierarchy*, ***Stud. Logic Found. Math., vol. 144. Elsevier, Amsterdam (2000)***

[2] Gold, E.M. *Language identication in the limit.* ***Inf. Control 10(5), 447-474 (1967).***

[3] Stephan, F., Ventsov, Y., *Learning algebraic structures from text.*, ***Theor. Comput.Sci. 268(2), 221-273 (2001).***

▶ KATSUHIKO SANO, *Uniform Interpolation of Basic Tense Logic.*
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The primary contribution of this talk is to establish the uniform interpolation theorem for basic tense logic, which is also known as two-way modal logic or modal logic with converse. Basic tense logic **Kt** was first proposed by Arthur Prior [2, 3, 4]. It is defined as a syntactic expansion of basic modal logic with a converse modality, whose corresponding accessibility relation is the converse relation R^{-1} defined in a given Kripke model $M = (W, R, V)$, where W is a non-empty set, $R \subseteq W \times W$, and V is a valuation. By regarding a Kripke frame (W, R) as representing a temporal order, we are able to talk not only about the future but also about the past using this additional converse modality. Define $\mathsf{P}(\varphi)$ as the set of all propositional variables occurring in a formula φ. We say that **Kt** has the *uniform interpolation property* if, for every pair (φ, p) of a formula φ and a propositional variable p, there exists a *post-uniform interpolant* $\exists p.\varphi$ such that $\mathsf{P}(\exists p.\varphi) \subseteq \mathsf{P}(\varphi) \setminus p$ and the following equivalence holds: $\mathbf{Kt} \vdash \varphi \to \psi$ iff $\mathbf{Kt} \vdash \exists p.\varphi \to \psi$ for all formulas ψ not containing p. As far as the author is aware, the uniform interpolation property of **Kt** has not yet been established. We extend Albert Visser's semantic argument [1] (cf. [5]) for the uniform interpolation property of basic modal logic **K** to establish the uniform interpolation property of **Kt**.

[1] Albert Visser, *Uniform Interpolation and Layered Bisimulation*, ***Gödel'96: Logical foundations of mathematics, computer science and physics–Kurt Gödel's legacy***, Spriinger-Verlag, 1996, pp.139–164.

[2] Arthur Prior, ***Time and Modality***, Oxford: Clarendon Press, 1957.

[3] Arthur Prior, ***Past, Present and Future***, Oxford: Clarendon Press, 1967.

[4] Arthur Prior, ***Papers on Time and Tense***, Oxford: Clarendon Press, 1968.

[5] Taishi Kurahashi, *Uniform Lyndon interpolation property in propositional modal logics*, ***Archive for Mathematical Logic***, vol. 59 (2020), pp. 659–678.

▶ YUITO MURASE AND AKINORI MANIWA, *Modality with explicit states of knowledge.*
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Kripke semantics offers an intuitive framework based on relational structures for intuitionistic and modal logics. In this paper, we propose an intuitionistic modal logic whose language can describe interactions between intuitionistic and modal transitions. This contrasts with standard modal languages, which refer only to modal transitions.

Building upon the informal idea that intuitionistic transitions capture the growth of knowledge, our language extends modal languages to account explicitly for knowledge states. Such states are represented by atomic *classifiers* γ, and the relation $\gamma_1 \preceq \gamma_2$ models the growth of knowledge from γ_1 to γ_2. We introduce two new constructs using classifiers:

1. A **bounded modality** $\Box^{\gamma} A$ relativizes the usual necessity operator to a lower bound of knowledge γ, asserting that A holds under standard modal accessibility

and above the knowledge level γ.

2. A **polymorphic classifier quantifier** $\forall\gamma_1 \succeq \gamma_2. A$ states that A holds for every knowledge state γ_1 that has grown from γ_2.

Thus, bounded modalities capture local interactions between modal and intuitionistic transitions, while polymorphic classifier quantifiers impose global constraints on the growth of knowledge.

This language is motivated by multi-stage programming, where modalities correspond to code types and classifiers to variable scopes under the Curry–Howard correspondence. We give a constructive formulation of our logic via a natural-deduction proof system and Kripke semantics, and report our current results.

► DIEGO A. MEJÍA, *Cardinal characteristics of the null-additive and meager-additive ideals.*

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Although many characterizations and properties of the null-additive and meager-additive ideals (e.g. due to Bartozsyński and Shelah) are known, there is very little reference to the study of their cardinal characteristics. In this work, we begin an investigation into these cardinal characteristics and examine their connections with classical cardinal characteristics (such as those in Cichoń's diagram), as well as related consistency results.

For instance, we prove that the additivity and uniformity numbers of the null-additive ideal coincide. However, whether the same holds for the meager-additive ideal remains an open question. While we may not resolve this in the present talk, this quest has led us to discover new characterizations of the meager ideal.

This is a joint work with Miguel Cardona and Ismael Rivera-Madrid [1, 2].

[1] Miguel A. Cardona, Diego A. Mejía, and Ismael E. Rivera-Madrid, *Uniformity numbers of the null-additive and meager-additive ideals*, preprint (2024), arXiv:2401.15364.

[2] Miguel A. Cardona, Diego A. Mejía, and Ismael E. Rivera-Madrid, *Directed schemes of ideals and cardinal characteristics, I: the meager additive ideal*, preprint (2025).

► FRANCESCO PARENTE, *On the Rudin-Frolík ordering of ultrafilters on complete Boolean algebras.*

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The Rudin-Frolík ordering of ultrafilters over ω was introduced by Frolík [2], who used it to show that the topological space $\beta\omega \setminus \omega$ is not homogeneous. Two generalizations of the Rudin-Frolík ordering to ultrafilters on complete Boolean algebras have been proposed independently by Balcar and Dow [1] and Murakami [3]. In this talk, we compare the two notions and investigate to what extent they coincide on the class of c.c.c. Boolean algebras.

This talk is based on joint work in progress with Jörg Brendle.

[1] Bohuslav Balcar and Alan Dow, *Dynamical systems on compact extremally disconnected spaces*, ***Topology and its Applications***, vol. 41 (1991), no. 1, pp. 41–56.

[2] Zdeněk Frolík, *Sums of ultrafilters*, ***Bulletin of the American Mathematical Society***, vol. 73 (1967), no. 1, pp. 87–91.

[3] Masahiko Murakami, *Standardization principle of nonstandard universes*, ***The Journal of Symbolic Logic***, vol. 64 (1999), no. 4, pp. 1645–1655.

▶ RYOMA SIN'YA, *Measure-Theoretic Aspects of First-Order Definable Languages and Group Languages.*
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A language L is said to be $\mathcal{C}$*-measurable*, where $\mathcal{C}$ is a class of languages, if there is an infinite sequence of languages in $\mathcal{C}$ that "converges" to L in the following sense: L is $\mathcal{C}$-measurable if and only if there exists an infinite sequence of pairs of languages $(K_n, M_n)_{n\in\mathbb{N}}$ in $\mathcal{C}$ such that $K_n \subseteq L \subseteq M_n$ holds for all n and the *density*[1] of the difference $M_n \setminus K_n$ tends to zero as n tends to infinity. The notion of $\mathcal{C}$-measurability was introduced in [3] and used for classifying non-regular languages by using regular languages. The notion of $\mathcal{C}$-measurability can be considered as a non-commutative and high-dimensional extension of Buck's measure theoretic notion for subsets of natural numbers so-called *measure density* [1], and the notion of $\mathcal{C}$-measurability can be defined by using a purely measure theoretic notion called *Carathéodory extension*. A language without $\mathcal{C}$-measurability has a complex shape in a measure-theoretic sense so that it can not be (asymptotically) approximated by languages belonging to $\mathcal{C}$. For example, while several context-free languages are shown to be REG-measurable where REG is the class of all regular languages, the set of all *primitive words* [2] is REG-*im*measurable [3].

Recently, we have investigated the properties of $\mathcal{C}$-measurability in the case that $\mathcal{C}$ is a subclass of regular languages; $\mathcal{C}$ = the class of all *star-free languages* (= first-order definable languages) and $\mathcal{C} = \mathsf{G}$ the class of all *group languages* (= languages definable by finite groups). In this talk, we describe why this notion was introduced, and give a brief overview of decidability results relating to the measurability on subclasses (local subvarieties) of regular languages, eg., star-free, generalised definite, and group languages.

We also demonstrate three recent results obtained with Takao Yuyama (Zen University) [7].

1. It is shown that a language L is SF-measurable if and only if L is GD-measurable, where GD is the class of all generalised definite languages (a more restricted subclass of star-free languages). This means that GD and SF have the same "measuring power", whereas GD is a very restricted proper subclass of SF. Moreover, we give a purely algebraic characterisation of SF-measurable regular languages, which is a natural extension of Schützenberger's theorem stating the correspondence between star-free languages and aperiodic monoids.
2. We also show the *probabilistic independence* of star-free and group languages as follows: the density of $L \cap K$ is the product of the densities of L and K for any star-free language L and group language K.
3. Finally, while the measuring power of star-free and generalised definite languages are equal, we show that the situation is rather opposite for subclasses of group languages as follows. For any two local subvarieties $\mathcal{C} \subsetneq \mathcal{D}$ of group languages, we have $\{L \mid L \text{ is } \mathcal{C}\text{-measurable}\} \subsetneq \{L \mid L \text{ is } \mathcal{D}\text{-measurable}\}$.

[1] The density $\delta_A(L)$ of a language L over an alphabet A is defined as $\delta_A(L) = \lim_{n\to\infty} \frac{1}{n}\sum_{i=0}^{n-1} \frac{|L\cap A^n|}{|A^n|}$.

[2] A non-empty word is said to be primitive if it can not be represented as a repetition of any shorter word.

[1] Robert C. Buck. The measure theoretic approach to density. *American Journal of Mathematics*, 68(4):560–580, 1946.

[2] Kazuhiro Inaba, Ryoma Sin'ya, Yoshiki Nakamura, and Yutaro Yamaguchi. Computational complexity of AT-, PT- and GD-measurability for regular languages. In *The 26-th Workshop on Programming and Programming Langugaes*. 2024. (written in Japanese, English version will be prepared).

[3] Ryoma Sin'ya. Asymptotic approximation by regular languages. In *Current Trends in Theory and Practice of Computer Science*, pages 74–88, 2021.

[4] Ryoma Sin'ya. Carathéodory extensions of subclasses of regular languages. In *Developments in Language Theory*, pages 355–367, 2021.

[5] Ryoma Sin'ya. Measuring power of locally testable languages. In Volker Diekert and Mikhail Volkov, editors, *Developments in Language Theory*, pages 274–285, Cham, 2022. Springer International Publishing.

[6] Ryoma Sin'ya. Measuring power of generalised definite languages. In *Implementation and Application of Automata*, pages 278–289. Springer International Publishing, 2023.

[7] Ryoma Sin'ya and Takao Yuyama. Measure-theoretic aspects of star-free and group languages. 2025.

[8] Takao Yuyama and Ryoma Sin'ya. Measuring power of commutative group languages. In *Implementation and Application of Automata*, pages 347–362. Springer International Publishing, 2024.

▶ YUTO TAKEDA, *Modal logic in second-order arithmetic.*
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This presentation is a survey and analysis of several theorems for modal propositional logic from the viewpoint of reverse mathematics. There have been several attempts to analyze logic in second-order arithmetic. In [1], Simpson studied the relations of several well-known theorems of classical logic in second-order arithmetic and showed that, over RCA_0, the Gödel' s completeness theorem is equivalent to WKL_0. Similar studies have been conducted on intuitionistic logic, and in [2], Yamazaki showed, over RCA_0, the strong completeness theorem for intuitionistic predicate logic is equivalent to ACA_0. In [3], we extend these studies to modal propositional logic. We show that the weak completeness theorem for modal propositional logic is provable in RCA_0. Moreover, we see that the strong completeness theorem by means of canonical models requires ACA_0, while the strong completeness theorem itself is equivalent to WKL_0 over RCA_0. In this talk, we will discuss the results of our analysis of various modal logic theorems, including these theorems, in second-order arithmetic.

[1] Stephen G. Simpson, *Subsystems of second order arithmetic*, Perspectives in Logic, Cambridge University Press, Cambridge; Association for Symbolic Logic, Poughkeepsie, NY, 2009.

[2] Takeshi Yamazaki, *Reverse mathematics and completeness theorems for intuitionistic logic*, ***Adv. Math***, 42(3):143–148, 2001.

[3] Sho Shimomichi, Yuto Takeda, Keita Yokoyama., *Completeness theorems for modal logic in second-order arithmetic.*, ***Crossroads of Computability and Logic: Insights, Inspirations, and Innovations***, Cham, Beckmann, Arnold and Oitavem, Isabel and Manea, Florin, Springer Nature Switzerland, 2025, pp. 452–466.

▶ YUSHIRO AOKI, *A property of forcing posets stronger than preclaiber $\aleph_1$ and its forcing axiom.*

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A property of forcing notions we call stat-pc was investigated by Kruger [1]. A forcing notion has stat-pc if any sequence of conditions indexed by a stationary set in ω_1 has a centered subsequence whose index set is also stationary. We investigate the separation of the forcing axiom for stat-pc forcings (MA(stat-pc)) from other forcing axioms.

Our first result shows that MA(stat-pc) does not imply MA(precaliber $\aleph_1$). To establish this, we analyze the uniformization property of ladder system colorings.

Second, we show that the consistency of the existence of an Aronszajn tree satisfying a specific condition implies the consistency of MA(σ-centered) + $\neg$MA(stat-pc). Hanazawa [2] introduced the notion of SS trees—Aronszajn trees in which every stationary subset contains a stationary antichain—and posed the question of whether every SS Aronszajn tree is special. We refer to an Aronszajn tree that serves as a counterexample to this question, that is, an SS Aronszajn tree that is not special, as a Hanazawa tree. We show that if a Hanazawa tree exists in the ground model, then an $\aleph_1$-σ-centered poset forces MA(σ-centered) + $\neg$MA(stat-pc). Here, a poset is $\aleph_1$-σ-centered if every subset of size $\aleph_1$ of it can be decomposed into countably many centered sets. The consistency of the existence of a Hanazawa tree remains an open question.

[1] Krueger, John, *A forcing axiom for a non-special Aronszajn tree*, ***Annals of Pure and Applied Logic***, vol. 171 (2020), no. 8, pp. 102820, 23.

[2] Hanazawa, Masazumi, *On a refinement of anti-Souslin tree property*, ***Tsukuba Journal of Mathematics***,

► HIDETAKA NORO, *cardinal invariants and generalized prediction numbers.*
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A cardinal invariant is a cardinal number lying between the least uncountable cardinal $\aleph_1$ and the cardinality of the continuum $\mathfrak{c}$. The study of cardinal invariants investigates how these cardinals can differ. In particular, some prediction principles admit various types (e.g., local type, constant types [1]) and can produce cardinal characteristics such as $\mathbf{add}(\mathcal{N})$ and $\mathfrak{b}$, which serve as important examples of this framework. In this talk, we propose a local version of constant prediction, which we call the local constant type, and examine its connections with other cardinal invariants.

[1] Shizuo Kamo, *Cardinal invariants associated with predictors*, ***Logic Colloquium '98 (Prague)***, vol. 13 of *Lecture Notes in Logic*, Assoc. Symbol. Logic, Urbana, IL, 2000, pp. 280–295.

► KOSHIRO ICHIKAWA, *A Modular Ordinal Analysis for Fragments of Induction Principles.*

Modular ordinal analysis, presented by Dieter Probst in his habilitationsschrift [5], aims to compute proof-theoretic invariants of a theory by decomposing the theory into some modules. It enables us to compute the invariants of a theory from that of a weaker one. Recently, Fedor Pakhomov and James Walsh [3, 4] proposed a new approach to compute invariants by iterated reflection principles, and Aguilera and Pakhomov [1] defined new higher invariants of theories in terms of Girard's dilators and ptykes. We provide an idea that makes it possible to understand relations between mathematical theories and to find a ray of hope for an ordinal analysis for full second order arithmetic Z_2. As a pilot study, we calculate usual and higher proof-theoretic invariants of syntactic reflection principles Π^1_{n+1}-$\mathsf{RFN}(T)$ and, by combing Frittation's result [2], fragments of second-order induction principles $T + \mathsf{I}\Pi^1_{n+1}$ for a reasonable theory T.

[1] JUAN PABLO AGUILERA AND FEDOR PAKHOMOV, *Reducing The* Π^1_2 *consequences of a theory*, ***Journal of the London Mathematical Society-Second Series***, vol. 107 (2023), no. 3. pp. 1045–1073.

[2] EMANNUELE FRITTAION, *A note on fragments of uniform reflection in second order arithmetic*, ***The Bulletin of Symbolic Logic***, vol. 28 (2022), no. 3. pp. 451–465.

[3] FEDOR PAKHOMOV AND JAMES WALSH, *Reflection ranks and ordinal analysis*, ***The Journal of Symbolic Logic***, vol. 86 (2021), no. 4, pp. 1350–1384.

[4] FEDOR PAKHOMOV AND JAMES WALSH, *Reducing* ω*-model reflection to iterated syntactic reflection*, ***The Journal of Symbolic Logic***, vol. 23 (2021), no. 02.

[5] DIETER PROBST, *A modular ordinal analysis of metapredicative subsystems of second order arithmetic*, ***Habilitationsschrift, Institut für Informatik Universität Bern, 2017.***

► OTO ARAKI, *Singular integrals in weak subsystems of second-order arithmetic.*
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By studying theorems of ordinary mathematics within weak subsystems of second-order arithmetic, it becomes clear which set existence axioms are not only sufficient but also necessary for proving those theorems. Studies on measure theory and L^p space theory from the perspective of Reverse Mathematics have shown that general results in measure theory, such as countable additivity of measures and the Vitali covering lemma, require WWKL_0[1][2], while an even stronger axiom ACA_0 is necessary when dealing with differentiation and limits[3]. In this talk, however, we provide a framework to deal with singular integrals within a weaker subsystem RCA_0. In particular, we formalize a RCA_0 version of the following well-known theorem stating the L^p boundedness of convolution-type operators, whose scope of application includes important operators such as the Riesz transform:

Let $K \in L^2(\mathbb{R}^d)$ satisfy $\sup_{\xi\in\mathbb{R}^d} |\widehat{K}(\xi)| \leq B$ and $\sup_{|x|>0} |\nabla f(x)| \leq \frac{B}{|x|^{d+1}}$ for some $B > 0$. Then, the convolution-type operator $K * (\cdot)$ is a bounded operator on the L^p space.

The idea is to define a dense subclass of the L^p space and restrict applications of integrals and measure-theoretic arguments to elements of this specific class in order to remain within RCA_0. Properties established within RCA_0 for this subspace is then extended to the entire class L^p by the linearity of the operator.

[1] XIAOKANG YU, STEPHEN G. SIMPSON, *Measure theory and weak König's lemma*, ***Archive for Mathematical Logic***, vol. 30 (1990), pp. 171–180.

[2] DOUGLAS K. BROWN, MARIAGNESE GIUSTO, STEPHEN G. SIMPSON, *Vitali's Theorem and WWKL*$_0$, ***Archive for Mathematical Logic***, vol. 41 (2002), pp. 191–206.

[3] XIAOKANG YU, *Riesz representation theorem, Borel measures and subsystems of second-order arithmetic*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 65–78.

Tuesday, September 9

► NIKOLAY BAZHENOV, *On computable and punctual Stone spaces.*
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The talk discusses computability-theoretic properties of separable Stone spaces. A *computably compact presentation* of a Polish space M is a computable metrization of M equipped with an effective enumeration $\{(B^i_0, B^i_1, \ldots, B^i_{n_i})\}_{i\in\omega}$ of all finite open covers of M that consist of basic open balls with rational radii. For background on computable compactness, we refer to the survey [1].

We say that a computably compact Polish space M is *effectively categorical* if for any pair of computably compact X and Y homeomorphic to M, there exists an effectively continuous, surjective homeomorphism from X onto Y. We show that effectively categorical Stone spaces are precisely the duals of computably categorical Boolean algebras.

If one replaces 'computable' by 'primitive recursive' throughout the standard definition of a computable Polish space, then one gets the notion of a punctual Polish space. We prove that every computably compact, punctual Stone space is primitively recursively homeomorphically embeddable into Cantor space.

The talk is based on joint works [2, 3] and an ongoing work with Badaev, Goncharov, Kalmurzayev, and Melnikov.

[1] R. G. Downey and A. G. Melnikov, *Computably compact metric spaces*, ***Bulletin of Symbolic Logic***, vol. 29 (2023), no. 2, pp. 170–263.

[2] N. Bazhenov, M. Harrison-Trainor, and A. Melnikov, *Computable Stone spaces*, ***Annals of Pure and Applied Logic***, vol. 174 (2023), no. 9, article 103304.

[3] R. Bagaviev, I. I. Batyrshin, N. Bazhenov, D. Bushtets, M. Dorzhieva, H. T. Koh, R. Kornev, A. G. Melnikov, and K. M. Ng, *Computably and punctually universal spaces*, ***Annals of Pure and Applied Logic***, vol. 176 (2025), no. 1, article 103491.

► TOSHIO SUZUKI, *The field generated by left c.e. reals below a weakly computable real.*
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This talk is based on joint research with Masahiro Kumabe and Kenshi Miyabe, and is a continuation of Miyabe's talk. A real number α is weakly computable if it is a limit of a computable sequence of rational numbers whose variation (the sum of the absolute values of adjacent terms) is finite. If it is increasing, α is left-c.e. Solovay reducibility is a preorder that compares algorithmic randomness of real numbers. Extending Miller's work [1], we study a subfield of the real numbers generated by left c.e. real numbers below a weakly computable real number with respect to Solovay reducibility. This field is a real closed field, closed under Solovay reducibility, and each element has its variation in this field. We discuss, from algorithmic randomness perspective, the relationship between the following two operations on a weakly computable real number; (i) operation of taking algebraic extension of the computable real numbers, and (ii) operation of taking the lower Solovay cone.

[1] J. Miller, *On work of Barmpalias and Lewis-Pye: a derivation on the d.c.e. reals*, ***Computability and Complexity*** (A. Day, M. Fellows, N. Greenberg, B. Khoussainov, A. Melnikov, F. Rosamond), LNCS 10010, Springer, 2017, pp. 644–659.

▶ MICHAEL HRUSAK, *The bounded topology.*
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We shall discuss the topology of *bounded convergence* on ideals on countable sets. We shall concentrate on question when this topology is metrizable, regular or forms a topological group, giving both ZFC and consistency results. The talk is based on the upcoming article [1] and a joint work in progress with O. Guzmán, E. Landeros and L. Reyes Saenz.

[1] F. Hernández, M. Hrušák, N. Rivas, *The bounded topology*, ***Topology and Its Applications***,(2025) to appear.

▶ GULNAZ AKIMBEKOVA, BEIBUT KULPESHOV, *On connection of the countable spectrum with algebras of binary formulas.*
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The present lecture concerns the notion of *weak o-minimality* originally studied by H.D. Macpherson, D. Marker and C. Steinhorn in [1]. A *weakly o-minimal structure* is a linearly ordered structure $M = \langle M, <, \dots \rangle$ such that any parametrically definable subset of M is a finite union of convex sets in M.

Algebras of binary formulas are a tool for describing relationships between elements of the sets of realizations of an one-type at the binary level with respect to the superposition of binary definable sets. A *binary isolating formula* is a formula of the form $\varphi(x, y)$ such that for some parameter a the formula $\varphi(a, y)$ isolates a complete type in $S_1(\{a\})$. In recent years, algebras of binary formulas for weakly o-minimal theories have been studied in [2, 3].

Theorem 1. *Let T be a small binary weakly o-minimal theory of convexity rank n for some $1 \leq n < \omega$, Γ_1 and Γ_2 are maximal pairwise weakly orthogonal families of irrational and quasirational 1-types over $\emptyset$ respectively. Then T has at most ω countable models iff both Γ_1 and Γ_2 are finite, and the algebra $\mathfrak{P}_p$ of binary isolated formulas is finite for any non-algebraic $p \in S_1(\emptyset)$.*

This research has been funded by Science Committee of Ministry of Science and Higher Education of the Republic of Kazakhstan (AP19674850, BR20281002).

[1] H.D. Macpherson, D. Marker, and C. Steinhorn, *Weakly o-minimal structures and real closed fields*, ***Transactions of the American Mathematical Society***, Vol. 352, No. 12 (2000), pp. 5435–5483.

[2] D.Yu. Emelyanov, B.Sh. Kulpeshov, S.V. Sudoplatov, *Algebras of distributions for binary formulas in countably categorical weakly o-minimal structures*, ***Algebra and Logic***, Vol. 56, No. 1 (2017), pp. 13–36.

[3] A.B. Altayeva, B.Sh. Kulpeshov, S.V. Sudoplatov, *Algebras of distributions of binary isolating formulas for almost ω-categorical weakly o-minimal theories*, ***Algebra and Logic***, Vol. 60, No. 4 (2021), pp. 241–262.

▶ AKITO TSUBOI, *Rethinking Ehrenfeucht Theories.*
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Let L be a countable language, and let T be a complete theory formulated in L.

$I(\omega, T)$ denotes the number of countable models of T, up to isomorphism. A non-ω-categorical theory T is called an Ehrenfeucht theory if $1 < I(\omega, T) < \omega$. There is a conjecture that Ehrenfeucht theories must possess the strict order property. We demonstrate that, by mimicking the constructions of the most well-known examples of Ehrenfeucht theories, any theory constructed in this manner has a definable dense ordering (in eq-sense).

► AIBAT YESHKEYEV AND INDIRA TUNGUSHBAYEVA, *On the Kaiser class of an inductive theory.*
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We study Jonsson theories [1] in a countable first-order language L. In this research, we describe a particular subclass of models of an arbitrary inductive L-theory. This subclass is shown to be a Jonsson invariant of the inductive theory, which enables the investigation of inductive theories by means of the tools developed for the study of Jonsson theories.

Let T be an inductive L-theory. The Kaiser class K_T of T is the following class of L-structures:

$$K_T = \{M \mid M \in Mod(T) \text{ and } Th_{\forall\exists}(M) \text{ is a Jonsson theory}\}.$$

The following results were obtained.

THEOREM 1. *Let T be an inductive theory, and let M be a model of T. $M \in K_T$ iff M is $\forall$-elementary equivalent to an existentially closed model of some Jonsson theory T' such that $T \subseteq T'$ and M is a model of T'.*

Theorem 1 gives the following corollary:

COROLLARY 2. *Given an inductive theory T, $K_T \neq \varnothing$ iff there exists a Jonsson theory T' such that $T \subseteq T'$.*

[1] J. BARWISE, ***Handbook of mathematical logic***, Part 1: Model theory, Izdatel'stvo Nauka (Moscow), 1982.

► YUAN CHAO, *On the Deep Structure of Xunzi's Linguistic Logic System.*
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This paper explores the deep structure of Xunzi's linguistic logic system by analyzing the logical chain of "name (名) → proposition (辞) → distinction (辨) → argumentation (说) → cause (故)" that he constructed. Xunzi begins with the principle of name-reality congruence as the logical foundation and emphasizes the formal role of "proposition (辞)" in expressing judgments. The stages of "distinction (辨)" and "argumentation (说)" provide semantic differentiation and argumentative validation, ultimately leading to "cause (故)" as a truth-justified conclusion. This logical sequence is grounded in the metaphysical structure of "Heaven (天) → resonance (感) → mind (心) → knowing (知) → wisdom (智)," with the purpose of demonstrating the ethical legitimacy of "humanity (人) → benevolence (仁) → ritual (礼) → righteousness (义) → Dao (道)." We argue that Xunzi thereby constructs a systematic linguistic logic that bridges cosmology and ethics, language and reality, logic and morality. This unique and integrated logical configuration in the East Asian Confucian tradition offers an important non-Western paradigm for comparative logic.

▶ MASAYA TANIGUCHI, *Proof-Theoretic Analysis and Normalization in Categorial Grammars for Context-Free Languages.*
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This paper presents a proof-theoretic analysis of *CG*s, a propositional logic system constructed using only two types of implication operators. The motivation for studying this system lies in its potential to formalize aspects of formal language theory. Specifically, research in this area has advanced by establishing a correspondence between the provability of propositions within the logic and the recognizability of strings by formal languages.

Our investigation centers on a subclass of *CG*s that directly corresponds to context-free grammars (CFGs). Within this scope, we analyze the structure of formal proofs. A key result of our study is the demonstration that, although this system retains the expressive power of traditional categorial grammars, its proofs can always be normalized into a specific, well-defined form.

What is particularly striking about this normal form is its structure. Unlike the general, arbitrarily branching trees commonly encountered in syntactic analysis, we prove that the normal forms in this system are consistently "extreme" —consisting exclusively of either right-branching or left-branching structures. That is, the resulting proof trees follow a strictly unidirectional branching pattern.

As a consequence, our findings establish that categorial grammars for context-free languages are inherently compatible with an incremental, unidirectional parsing strategy —one that proceeds consistently either left-to-right or right-to-left. This theoretical insight suggests the possibility of developing streamlined parsing algorithms for context-free languages within this specific type of categorial grammar, potentially offering new efficiencies in computational processing.

▶ JILIN WANG, ALESSANDRO DUARTE, AND BRUNO BENTZEN, *The logic of Frege's Begriffsschrift is actually non-Fregean.*
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It is commonly assumed that Frege' s *Begriffsschrift* [1] represents a sound and complete system for classical logic with propositional quantifiers. In this talk, we argue that this assumption is mistaken. We show that *Begriffsschrift* is better understood as a non-Fregean logic. In non-Fregean logic, there is an identity connective $\equiv$ for sentences. This connective does not reduce to truth-functional equivalence [2]. In other words, the scheme $(\phi \to \psi) \to (\psi \to \phi) \to \phi \equiv \psi$ does not generally hold [2, 3, 4]. This means that the identity between sentences depends on more than just having the same truth value. Instead, it requires that the sentences share the same content in a technical sense.

Our results are based on previous work by Suzko and Bloom [2, 5] as well as significant results by Duarte [7] and Bertran San Millán [6] concerning the propositional fragment of *Begriffsschrift.* Suszko' s theories of kind W was originally to capture the logic of Wittgenstein' s *Tractatus*. However, as we shall demonstrate in this talk, it is an appropriate semantics for *Begriffsschrift.* To be precise, we prove that the first-order fragment of *Begriffsschrift* with propositional quantification is deductively equivalent to the theory of kind W, extending Bertran-Millan's [6] result that the propositional fragment corresponds to the SCI logic of Bloom and Suszko. After that, we showed that Frege' s logic is complete with respect to the theories of kind W.

In sum, our main contribution is twofold. First, our results establish the completeness of *Begriffsschrift*' s first-order fragment with respect to non-Fregean semantics. Second, our results correct a common mistake in the existing literature on Frege and thus give a clearer view of what *Begriffsschrift* really captures: a meaning-based idea of identity, which accommodates well his early semantic notions of judgeable contents and identity of content. This helps us better understand Frege' s early logical views and shows how they connect to other theories of non-Fregean logic.

[1] Gottlob Frege, ***Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens***, L. Nebert 1879

[2] Suszko, Roman, *Ontology in the Tractatus of L. Wittgenstein*, ***Notre Dame Journal of Formal Logic***, vol. 9, no. 1, pp. 7–33.

[3] ——— *Quasi-completeness in non-Fregean logic*, ***Studia Logica***, vol. 29 (1971), pp. 7–16.

[4] ——— *Identity connective and modality*, ***Studia Logica***, vol. 27 (1971), pp. 7–41.

[5] Stephen L. Bloom, *A completeness theorem for "Theories of Kind* W*"*, ***Studia Logica: An International Journal for Symbolic Logic***, vol. 27 (1971), pp. 43–56.

[6] Joan Bertran San Millán, *La Lógica de Gottlob Frege: 1879–1903*, PhD thesis, Universitat de Barcelona, 2016.

[7] Alessandro Bandeira Duarte, *Lógica e Aritmética na Filosofia da Matemática de Frege*, PhD thesis, Pontifical Catholic University of Rio de Janeiro, 2009.

Wednesday, September 10

▶ MASATO FUJITA, *On definable compactness in locally o-minimal structures.*
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Definable compactness was first proposed by Peterzil and Starchenko in o-minimal structures [6]. Johnson generalized this concept to more general setting [5]. Andújar Guerrero et al. found equivalence conditions for definable topological spaces to be definably compact in o-minimal structures [2]. Recently, Andújar Guerrero extended his research in [1].

A generalization of o-minimality called local o-minimality has been studied (e.g. [7, 4]). In this talk, we introduce a generalization of the study on definably compact definable topological spaces to locally o-minimal cases [3]. As an application, we investigate definably compact definable topological groups in locally o-minimal structures.

[1] P. Andújar Guerrero, *Definable compactness in o-minimal structures*, Model theory, **4** (2025), 101–130.

[2] P. Andújar Guerrero, M. E. M. Thomas and E. Walsberg, *Directed sets and topological space definable in o-minimal structures*, J. London Math. Soc. (2), **104** (2021), 989–1010.

[3] M. Fujita, *Definable compactness in definably complete locally o-minimal structures*, Fund. Math., **267** (2024), 129–156.

[4] M. Fujita, *Locally o-minimal structures with tame topological properties*, J. Symbolic Logic, **88** (2023), 219–241.

[5] W. Johnson, *Interpretable sets in dense o-minimal structures*, J. Symbolic Logic, **83** (2018), 1477–1500.

[6] Y. Peterzil and S. Starchenko, *Definable compactness and definable subgroup of o-minimal groups*, J. London Math. Soc. (2), **59** (1999), 769–786.

[7] C. Toffalori and K. Vozoris, *Notes on local o-minimality*, Math. Logic Quart., **55** (2009), 617-632.

▶ NIGEL PYNN-COATES, *Tame pairs of transseries fields.*
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Interest in transseries and Hardy fields comes from several fields, including asymptotic analysis, dynamical systems, and model theory of the real numbers. The first-order theory of the differential field of (logarithmic-exponential) transseries and maximal Hardy fields is completely axiomatized by the theory of closed H-fields, which is model complete, as M. Aschenbrenner, L. van den Dries, and J. van der Hoeven have shown in a long series of works culminating in [1, 2]. I will describe my extension of this model completeness to tame pairs of closed H-fields and related results from [3], in order to better understand large closed H-fields, such as maximal Hardy fields, hyperseries, or surreal numbers.

[1] Matthias Aschenbrenner, Lou van den Dries, and Joris van der Hoeven, ***Asymptotic differential algebra and model theory of transseries***, Annals of Mathematics Studies, Princeton University Press, Princeton, NJ, 2017.

[2] ——— *The theory of maximal Hardy fields*, ***arXiv.org e-Print archive***, `arXiv:2408.05232`.

[3] Nigel Pynn-Coates, *Tame pairs of transseries fields*, ***arXiv.org e-Print archive***, arXiv:2408.07033.

► YETAO HU, *Is the Recombination Paradox an Inclosure Paradox?*.
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David Lewis's modal realism, with its Principle of Recombination (PR) for constructing possible worlds, faces the Recombination Paradox, identified by Armstrong and Forrest. Lewis proposed restricting PR via 'size and shape permitting,' while Daniel Nolan suggested that the aggregate of all possible worlds forms a proper class rather than a set. This paper argues that these are 'consistency-repairing' attempts that may be ad hoc or fail to address the core issue.

The central thesis is that the Recombination Paradox is an instance of Graham Priest's Inclosure Paradox. The structure of the Inclosure Paradox, which involves a totality (Ω) and a function (δ), is such that three conditions (Existence, Closure, and Transcendence) jointly entail the contradiction that $\delta(\Omega)$ is both in Ω and outside Ω ($\delta(\Omega) \in \Omega \wedge \delta(\Omega) \notin \Omega$).Through an analysis of this structure, the author evaluates Lewis's and Nolan's solutions, critiquing them as attempts to repair consistency and avoid the contradictory outcome by either restricting Ω or the application of δ.

Subsequently, the paper highlights Priest's dialetheic resolution, which employs paraconsistent logic. This approach accepts the paradoxical outcome as a true contradiction, reflecting genuine features at the limits of thought or reality (such as with 'all possible worlds' or unlimited recombination). These situations are often characterized by his Inclosure Schema, where an operation upon a totality generates an object that is paradoxically both within and beyond that totality.

The paper argues that this dialetheic perspective yields a more fundamental understanding of the Recombination Paradox, with significant implications. For modal logic, the dialetheic approach suggests 'impossible worlds' should be incorporated not merely as theoretical tools but as worlds where the very contradictions identified by the paradox hold true. Consequently, this approach transforms the semantic analysis of counterfactual conditionals, particularly those with impossible antecedents. It challenges the classical Lewisian account of vacuous truth for such conditionals, proposing instead a paraconsistent treatment where antecedents can be true and false in relevant impossible worlds, allowing for their non-vacuous evaluation.

However, Priest's solution is not metaphysically neutral. For a modal realist, accepting impossible worlds as concrete entities would entail the existence of concrete contradictions, a consequence that undermines the very foundations of Lewis's realism. Thus, Priest's solution, in resolving the paradox, appears to compel a move away from concrete modal realism towards an abstract, ersatzist conception of worlds, making it a replacement, not a repair, of the original theory.

[1] F. Berto and M. Jago. ***Impossible Worlds***, Oxford University Press, 2019.

[2] F. Berto and M. Jago, *Impossible Worlds*, in E. N. Zalta and U. Nodelman (eds.), ***The Stanford Encyclopedia of Philosophy***, Metaphysics Research Lab, Stanford University, 2023.

[3] D. Edgington, *On conditionals*, ***Mind***, 1995, 104(414): pp. 235–329.

[4] D. Lewis, *Counterfactual Dependence and Time's Arrow*, ***Noûs***, 1979, 13(4): pp. 455–476.

[5] D. Nolan, *Recombination unbound*, ***Philosophical Studies***, 1996, 84(2-3): pp. 239–262.

[6] B. Russell, *On some difficulties in the theory of transfinite numbers and order types*, ***Proceedings of the London Mathematical Society***, 1905, 4(14): pp. 29–53.

[7] D. Lewis, ***On the Plurality of Worlds***, Wiley-Blackwell, 1986.

[8] G. Priest, ***Beyond the limits of thought***, Oxford University Press, 2002.

[9] G. Priest, ***In Contradiction: A Study of the Transconsistent***, New York: Oxford University Press, 2006.

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[11] G. Priest, *Mission impossible*, in ***Saul Kripke on Modal Logic***, Springer, 2024, pp. 347–364.

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► OSVALDO GUZMAN, *Ramsey clubs and square bracket operations.*
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A celebrated theorem of Stevo Todorcevic is that there is a coloring $c : [\omega_1]^2 \longrightarrow \omega_1$ such that every color appears in every uncountable subset of ω_1. Given a set $X \subseteq \omega_1$, we can define the graph $G_X = (\omega_1, \sim_X)$ where $\alpha \sim_X \beta$ if $c(\alpha, \beta) \in X$. Todorcevic proved that this families of graphs are very interesting in case c one of the square bracket partitions defined in his book on *Walks on Ordinals.* In this talk, we will study the following question: *When does G_X contains an uncountable complete graph?*

This is joint work with Stevo Todorcevic.

► PURBITA JANA, PRATEEK KWATRA, *Continuous Algebra: algebraic semantics for Continuous Propositional Logic.*
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Continuous Propositional Logic (CPL), as defined by Itaï Ben Yaacov et al. [3], introduces a new unary connective $\frac{1}{2}$, which semantically halves the degree of falsity of a sentence, to the Łukasiewicz Logic. This gives CPL a continuous analogue of the functional completeness property (called fullness) to $\{\rightarrow, \neg, \frac{1}{2}\}$. In [2], we have introduced continuous algebra as the algebraic semantics for CPL. A Continuous algebra is an MV-algebra together with an unary operator κ, analogous to the unary connective $\frac{1}{2}$ in CPL. We establish structural results, including the subdirect representation theorem. We also introduce ℓu^*-groups, which are lattice ordered groups with strong unit u, denoted by ℓu-groups[1], with a partial operator * that mimics the behavior of κ over the interval $[id, u]$. This addition enables a natural correspondence between ℓu^*-groups and the continuous algebras, allowing us to prove the Chang's completeness theorem for the continuous algebras. Furthermore, we are working on proving the strong approximated completeness theorem and the strong approximated Craig interpolation for CPL using continuous algebras.

[1] Roberto Cignoli, and Daniele Mundici, *An Elementary Proof of Chang' s Completeness Theorem for the Infinite-Valued Calculus of Łukasiewicz.*, ***Studia Logica***, vol. 58 (1997), no. 1, pp. 79–97.

[2] Purbita Jana and Prateek, *Continuous Algebra: Algebraic Semantics for Continuous Propositional Logic*, (2025), eprint=2501.13114, url=https://arxiv.org/abs/2501.13114.

[3] Itaï Ben Yaacov and Arthur Paul Pedersen, *A proof of completeness for continuous first-order logic*, ***Journal of Symbolic Logic***, vol. 75 (2010), no. 1, pp. 168–190.

▶ YUNSONG WANG, *Completeness of intuitionistic logic based on problem semantics.*
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Finite problem semantics was proposed by Yu. T. Medvedev [1] aiming to give an intuitive semantics for intuitionistic logic. However, the weakest logic defined by finite problem semantics is called *Medvedev logic* **ML**, which is an intermediate logic with some interesting properties, such as the finite model property and the disjunction property. It has been proved that there is no finite axiomatization for **ML** [2] and whether there is recursive axiomatization for **ML** is a long-standing open problem.

Nowadays, instead of understanding **ML** in finite problem semantics, **ML** is usually defined by a class of Kripke frames (called *Medvedev frames*) in intuitionistic Kripke semantics [3]. Due to the author's limited knowledge, a detailed proof of the equivalence was not presented in the literature. Hence, we introduce the equivalence in detail in this paper.

However, we still want to formalize **IPL** and some other important nonclassial logics in a intuitive way. In 2000 [4], Friedman showed that minimal propositional logic **MPC** is complete w.r.t. a variant of problem semantics and announced that it can be extended to **IPL**. In this paper, we rewrite his proof and fill in some gaps. Moreover, in this paper, we discussed how to analyze some other subintuitionistic logics and supintuitionistic logics, such as Jankov's logic **KC**, by variants of problem semantics. From this point of view, we can compare them in an intuitive way and we find some interesting new result about **IPL** and relevant logics.

[1] Yu. T. Medvedev, *Finitive problems*, ***Doklady Akademii nauk SSSR***, vol. 142 (1962), no. 5, pp. 1015–1018.

[2] L. Maksimova, V. Shehtman and D. Skvortsov, *The Impossibility of a Finite Axiomatization of Medvedev' s Logic of Finitary Problems*, ***Soviet Mathematics Doklady***, vol. 20 (1979), pp. 394–398.

[3] Alexander V. Chagrov and Michael Zakharyaschev. ***Modal Logic***, Oxford University Press, 1997.

[4] Harvey Friedman, *Completeness of intuitionistic propositional calculus*, ***https://philpapers.org/rec/FRICOI***, 2000.

▶ WENFANG WANG AND MINGDE CAO, *Decoding Yijing with Algebra: A Novel Exploration of Hexagram Transformation from Mathematical Structures.*
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As an ancient Chinese divinatory and philosophical text, Zhouyi (the Book of Changes, also known as Yijing or I Ching) contains a sophisticated hexagram system with profound logical and mathematical structures. This study presents a formal analysis of Zhouyi hexagrams using modern mathematical logic and algebraic frameworks to uncover their intrinsic relationships and symbolic meanings.

The research begins by examining the historical origins of Zhouyi, demonstrating its connection to early numeral systems and tracing the evolution of hexagrams from concrete counting tools to abstract divinatory symbols. We then employ Boolean algebra to formalize key Yijing concepts, including guabian (hexagram transformation), zhengwei (correct positioning), and ying (correspondence), providing an algebraic representation of hexagram transformations and their properties.

Furthermore, Boolean lattice theory is introduced to explain hexagram generation and classification, revealing previously unexplored structural relationships among the 64 hexagrams. Our findings indicate that Zhouyi's symbolic system can be effectively modeled using Boolean algebraic structures, offering a rigorous mathematical foundation for analyzing its logical patterns.

The study highlights how classical philosophical systems like Zhouyi can provide novel insights for contemporary scientific paradigms, particularly in low-power, reversible computing. This interdisciplinary approach opens new research directions connecting cultural heritage with cutting-edge logic design.

Keywords: Yijing, Boolean algebra, modular arithmetic, computational philosophy, symbolic logic

► SATOSHI NAKATA, *A kripke frame semantics defined along local operators in a topos.*
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Recent studies on local operators (a.k.a. Lawvere-Tierney topologies) have found practical applications in computability theory and intuitionistic proof theory. In particular, Kihara introduced a new degree structure isomorphic to the poset of local operators in the effective topos [1]. This degree structure provides a manageable realizability interpretation through the corresponding *j-translation*, and is expected to have future applications to intuitionistic arithmetic and intuitionistic set theory.

The purpose of this talk is to propose a new translation of formulas that combines the structure of Kripke frame and j-translation, with a view toward further applications in intuitionistic proof theory. Given a formula φ in the internal language of a topos and an internal poset $\mathbb{P}$ of local operators, we define a new formula $j \Vdash_{\mathbb{P}} \varphi$ to express, roughly, that the k-translation of φ holds for all local operators $k \geq j$ in $\mathbb{P}$. Our translation is based on van den Berg's Kuroda-style j-translation [2] and Lawvere's construction of the associated sheaf functor, as referenced in [3]. We prove that it satisfies the axioms and preserves the inference rules of first-order intuitionistic predicate logic. Furthermore, for formulas concerning the natural number object, we show that the axioms of Heyting arithmetic are also satisfied.

As a byproduct, we naturally obtain a Kripke frame semantics in which the possible worlds are interpreted as Kleene realizabilities relativized to a Turing degree.

[1] Takayuki Kihara. Lawvere-Tierney topologies for computability theorists. ***Transactions of the American Mathematical Society***, Series B, 10, pp. 48-85, 2023.

[2] Benno van den Berg. A Kuroda-style j-translation. ***Archive for Mathematical Logic***, Vol. 58, pp. 627–634, 2019.

[3] Peter Johnstone. ***Topos Theory***, London Mathematical Society Monographs, Vol. 10, 1977.

► RUSSELL MILLER, *Questions on computability and Galois groups.*
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The *absolute Galois group* Gal(F) of a field F is the Galois group of its algebraic closure $\overline{F}$ relative to F, containing precisely those automorphisms of $\overline{F}$ that fix F itself pointwise. Even for a field as simple as the rational numbers $\mathbb{Q}$, $\mathrm{Gal}(\mathbb{Q})$ is a complicated object. Indeed (perhaps counterintuitively), $\mathrm{Gal}(\mathbb{Q})$ is among the thorniest of all absolute Galois groups normally studied.

When F is countable, $\mathrm{Gal}(F)$ usually has the cardinality of the continuum. However, it can be presented as the set of all paths through an F-computable finite-branching tree, built by a procedure uniform in F. We will first consider the basic properties of this tree, which depend in some part on F. Then we will address questions about the subgroup consisting of the computable paths through this tree, along with other subgroups similarly defined by Turing ideals. One naturally asks to what extent these are elementary subgroups of $\mathrm{Gal}(F)$ (or at least elementarily equivalent to $\mathrm{Gal}(F)$). This question is connected to the computability of Skolem functions for $\mathrm{Gal}(F)$, and also to the arithmetic complexity of definable subsets of $\mathrm{Gal}(F)$. When $F = \mathbb{Q}$, we have a few answers and many more questions. In the simpler situations of the absolute Galois group of a finite field, or of the Galois group of the cyclotomic field over $\mathbb{Q}$, much more is known, thanks in part to joint work by Jason Block and the speaker.

Thursday, September 11

► YUKI NISHIMURA, LEONARDO PACHECO, *A hybrid logic for the present King of France*.
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Hybrid logic extends modal logic by introducing *nominals*, special propositional symbols referring to exactly one state in a model. With nominals, we can express concepts such as a moment in a temporal flow or an agent in a community.

But what happens when a name refers to nothing? Russell [1] provided a well-known example: "The present King of France is bald." Naturally, there is no current King of France (nor was there in 1905). Russell concludes that this sentence is false.

We consider Kripke models which allow nominals not referring to any possible world. For these models, we propose a Hilbert-style axiomatization **Ke**(@) and prove its soundness and completeness. Furthermore, we show that this system has a constructive flavor: by adding the formula $@_i i$ to our **Ke**(@), it becomes equivalent to the standard hybrid logic **K**(@) introduced in [2].

[1] Bertrand Russell, *On denoting*, ***Mind***, vol. 14 (1905), no. 56, pp. 479–493.
[2] Patrick Blackburn and Balder ten Cate, *Pure extensions, proof rules, and hybrid axiomatics*, ***Studia Logica***, vol. 84 (2006), no. 2, pp. 277–322.

► YOSUKE FUKUDA, *A modal linear logic: its proof theory and semantics*.
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The proof-theoretic investigation of modal logics has a rich history, both in the style of sequent calculus and natural deduction (cf. [1, 2]). A significant challenge in the proof-theoretic investigation of modal logics lies in designing "well-behaved" formal system of modal logic that enjoys the cut-elimination theorem or the normalization theorem. This difficulty is often emphasized in relation to the presence of modalities (for example, the side condition of the necessitation rule, the cut-elimination step on modalities, etc.).

In order to provide a new perspective on formal system of modal logic, we consider what we call *modal linear logic* to analyze modal logic in terms of linear logic, and to develop a basis of formal system of modal logic. The new (and old) idea of designing modal linear logic is to revisit Schellinx's "modal linear decoration" [3], a method for embedding from modal logic into the so-called *subexponential linear logic*.

In this talk, we first introduce modal linear logic as an integration of modal logic and linear logic. The key idea of this system is to introduce a new modality that combines the powers of a modality of modal logic and an exponential of linear logic; and the system is technically characterized as a subsystem of subexponential linear logic (or equivalently, the so-called *adjoint logic*). We then discuss its proof-theoretical properties w.r.t. modal logic and its semantic characterization.

This talk is partially based on the previous study of the linear-logical reconstruction of intuitionistic modal logic S4 [4].

[1] Francesca Poggiolesi. ***Gentzen Calculi for Modal Propositional Logic***,

Springer, Dordrecht, 2011.

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[3] Harold Schellinx, *A Linear Approach to Modal Proof Theory*, ***Proof Theory of Modal Logic***, Springer, Dordrecht, 1994, pp. 33–43.

[4] Yosuke Fukuda and Akira Yoshimizu, *A Linear-Logical Reconstruction of Intuitionistic Modal Logic S4*, ***Formal Structures for Computation and Deduction*** Dortmund, Germany, vol. 131, Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2019, pp. 20:1–20:24.

▶ KOKI OKURA, *A construction of NIP expansions of the p-adic fields without the rationality of Poincaré series.*
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Fix a prime p. To any subset S of the nth power of the p-adic integers, we can associate a formal power series called the Poincaré series of S. Although we can take a subset with a highly intricate Poincaré series, Denef [1] showed a remarkable result: If a subset S is definable in the structure of the p-adic field, then the Poincaré series of S is always a rational function.

The main interest of our work is the relationship between the rationality of Poincaré series of definable sets and stability-theoretic properties. It easily follows from previous studies [3][4] that any dp-minimal expansion of the p-adic field admits the rationality of Poincaré series. On the other hand, Denef himself [2] provided an example with the rationality of Poincaré series which is not dp-minimal, but admits a weaker property, i.e., NIP. Due to these observations, it is natural to ask whether any NIP expansion of the p-adic field has the rationality of Poincaré series. In this talk, we give a counterexample to this question.

[1] J. Denef, *The rationality of the Poincaré series associated to the p-adic points on a variety*, ***Inventiones mathematicae***, vol. 77 (1984), no. 1, pp. 1–23.

[2] ——— *p-adic semi-algebraic sets and cell decomposition*, ***Journal für die reine und angewandte Mathematik***, vol. 369 (1986), pp. 154–166.

[3] P. C. Kovacsics, and E. Leenknegt, *Integration and cell decomposition in P-minimal structures*, ***The Journal of Symbolic Logic***, vol. 81 (2016), no. 3, pp. 1124–1141.

[4] P. Simon, and E. Walsberg, *Dp and other minimalities*, arXiv:1909.05399 (2019).

▶ YOUAN SU, *A proof-theoretic approach to uniform interpolation property of multi-agent modal logic K_n.*
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In Pitts[3], the uniform interpolation property (UIP) was established as a strengthening of Craig interpolation. The original proof is based on a G3-style sequent calculus. In Bílková [1], this method was developed to established UIP in modal logic.

UIP in multi-agent modal logic has been established by model-theoretic methods as in Fang et al.[2]. However, the proof-theoretic approaches are not studied enough. This talk extends the methods originated from Bílková[1]. In a revised sequent calculus, an explicit algorithm to construct an uniform interpolant formula is introduced for a multi-agent modal logic K_n.

[1] Bílková, Marta, *Uniform interpolation and propositional quantifiers in modal logics*, ***Studia Logica***, vol.85. pp. 1–31, 2007.

[2] Fang, Liangda and Liu, Yongmei and van Ditmarsch, Hans, *Forgetting in multi-agent modal logics*, ***Artificial Intelligence***, vol.266. pp. 51–80, 2019.

[3] Pitts, Andrew M, *On an interpretation of second order quantification in first order intuitionistic propositional logic*, ***The Journal of Symbolic Logic***, vol. 57, no. 1, pp. 33–52, 1992.

► KENTA TSUKUURA, *The dissection of* (†).
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The principle (†), asserting that every ω_1-stationary preserving poset is semiproper, was introduced by Shelah and has been extensively studied. It is known that (†) has many equivalent formulations, such as the semiproperness of all Namba forcings $\mathrm{Nm}(\lambda)$, semistationary reflection principles, and the strong Chang's conjecture.

A concrete model of (†) can be obtained by collapsing a strongly compact cardinal κ to ω_2 using $\mathrm{Coll}(\omega_1, {<}\kappa)$. On the other hand, in extensions by $\mathrm{Coll}(\omega_2, {<}\kappa)$, $\mathrm{Coll}(\omega_3, {<}\kappa)$, and so on, the semiproperness of $\mathrm{Nm}(\lambda)$ holds for all $\lambda \geq \kappa$. Even when κ is merely measurable, $\mathrm{Nm}(\kappa)$ is semiproper in the extension. Thus, (†) partially holds in these models.

Motivated by this, we are interested in evaluating to what extent (†) holds. To this end, we introduced the notion of idealized two-cardinal Namba forcings $\mathrm{Nm}(\kappa, \lambda, I)$ and proposed a hierarchy of fragments of (†), represented diagrammatically. We refer to this diagram as the dissection of (†).

In this talk, we focus on the separation of the statements appearing in the dissection of (†) and present several results in this direction.

► IDO FELDMAN, *All Hajnal-Máté graphs on* $\aleph_2$ *may have small chromatic number*.
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In [1], Hajnal and Máté introduced a class of ladder-system graphs on $\aleph_1$, and gave an example from $\diamondsuit$ of such a graph of uncountable chromatic number. To compare, Martin's Axiom (and the failure of the Continuum Hypothesis) implies that all Hajnal-Máté graphs have countable chromatic number.

Here, we consider a natural higher analogue of Hajnal-Máté graph on $\aleph_2$. While there are no higher analogs of Martin's Axiom that are known to imply that all of such graphs have chromatic number $\aleph_1$, we show some direction of a possible way to solve this problem and discuss partial results using a forcing with two types of models as side conditions introduced by Neeman [3], further developed by Mohammadpour and Veličković [2].

[1] A. Hajnal and A. Máté, *Set mappings, partitions and chromatic numbers*, ***Proccedings Logic Colloquium '73, Stud. Logic Found. Math.***, vol. 80 (1975), pp. 347–380.

[2] Rahman Mohammadpour and Boban Veličković, *Guessing models and the approchability ideal*, ***Journal of Mathematical Logic***, vol. 21 (2021), no. 2, Paper No. 2150003, 35 pp.

[3] Itay Neeman, *Forcing with sequences of models of two types*, ***Notre Dame Journal Formal Logic,*** vol. 55 (2014), no. 2, pp. 265–298.

► JOACHIM MUELLER-THEYS, *On Elementhood and Comprehension*.

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The art of formalisation is a central part of logic. One of the laws that it is subject to is the preservation of logical categories.

In the theory of binary relations $R \subseteq M \times M$, there is a logical theorem indeed, stating that there exists *no* $a \in M$ such that for all $b \in M$: $b\ R\ a$ if and only if $b\ R\ b$ is not the case. May this theorem be applied to *elementhood* $\in\ \subseteq M \times \wp(M)$?

To some extent, properties can be considered as sets; logical incidence may be defined by elementhood then. We have observed a *Primitive Comprehension Theorem*: For all properties P on M, there is a set $A \subseteq M$ such that $a \in A$ if and only if a has P.

This should particularly be true for *definable* properties. Let therefore $\mathcal{L}$ be some first-order logic with the language L. In the following, we sketch our construction of an evident (cautious, tentative) extension $\mathcal{L}'$ with set-building expressions $\{x \mid \phi(x)\}$ and restricted second-order quantification $\exists X$, which will *regain comprehension*:
1.1. Atomic formulæ $t \doteq s$, $Pt_1 \ldots t_n$ of L are (atomic) formulæ of L';
1.2. For any formula $\phi(x) \in L$, $t \mathrel{\dot\in} \{x \mid \phi(x)\}$ is a (atomic) formula of L' (where $\dot\in \notin L$);
1.3. If α, β are formulæ of L', then so are $\neg\alpha$, $\alpha \wedge \beta$, $\ldots$, $\forall x\, \alpha$, $\exists x\, \alpha$;
1.4. If $\alpha(\{x \mid \phi(x)\})$ is a formula of L', then so is $\exists X\, \alpha(X)$.
2. We add definitional schemata like:
$\forall y\ \big(y \mathrel{\dot\in} \{x \mid \phi(x)\} :\leftrightarrow \phi(y)\big)$,
as kind of logical axioms, and the inference rule:
$\alpha(\{x \mid \phi(x)\}) \vdash \exists X\, \alpha(X)$.

We thus obtain a schema of *Comprehension Theorems*: Let $\phi(x) \in L$. Then $\exists X\, \forall x\, \big(x \mathrel{\dot\in} X \leftrightarrow \phi(x)\big)$ is a theorem of $\mathcal{L}'$.
Sketch of proof. Let $\alpha(\{x \mid \phi(x)\}) := \forall y\ \big(y \mathrel{\dot\in} \{x \mid \phi(x)\} :\leftrightarrow \phi(y)\big)$, whence $\alpha(X) = \forall y\ \big(y \mathrel{\dot\in} X \leftrightarrow \phi(y)\big)$. Since $\vdash \alpha(\{x \mid \phi(x)\})$, $\vdash \exists X\, \alpha(X)$.

Semantically, $\{x \mid \phi(x)\}^{\mathcal{M}} := \{a \in |\mathcal{M}| \colon \mathcal{M} \models \phi\,[a]\}$, and, specifically, $\mathcal{I} \models t \mathrel{\dot\in} \{x \mid \phi(x)\}$ $:\Leftrightarrow t^{\mathcal{I}} \in \{x \mid \phi(x)\}^{\mathcal{M}}$, where $\mathcal{M}$ be some suitable model and $\mathcal{I} := (\mathcal{M}, V)$.

Likewise, new property symbols $P_{\phi(x)}$ may be added to L, which are defined by $\forall y\ \big(P_{\phi(x)}\, y :\leftrightarrow \phi(y)\big)$. So to speak, $P_{\phi(x)}$ is a *simple* name for $\phi(x) \in L$. In a combined system, we obtain the *correspondence*: $\forall y\ \big(P_{\phi(x)}\, y \leftrightarrow y \mathrel{\dot\in} \{x \mid \phi(x)\}\big)$.

Eventually, we have to thank all who have contributed more or less, intentionally or unintentionally, directly or indirectly.

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The no-no paradox and its generalization (called no-no type) are self-referential statements whose paradoxical natural arises from the conflict between classical logic assignments of these statements and thier syntactic symmetry. Their syntactic symmetry can be characterized by permutation groups—a permutation group is said to be the symmetry group of a given paradox if all permutations within this group (and only these permutations) leave this paradox invariant.

A natural question arises: can any group characterize a paradox of no-no type? (See. [6, p. 1930]). In [1], it is demonstrated that the alternating group A_4 cannot be the symmetry group of any Boolean system, which is finite set of self-referential statements. And since the no-no type paradoxes we concern are precisely classified to

Boolean systems, A_4 cannot be the symmetry group of no-no type paradox. In our current work extends this negative result: we now generalize it to all A_ns with $n \geq 4$.

By introducing the notion of invariance preorder for binary sequences, we refine the algebraic techniques from research on the representability of the permutation groups by Boolean functions. It can be proved that for $n \geq 4$, the alternating group A_n and the symmetric group S_n have the same orbit partition and invariance preorder. Based on our analysis, it implies realizing the symmetry of A_n always introduces additional symmetry with respect to S_n, and thereby precluding the possibility that symmetry is just A_n. Thus, we obtain the main result that there is no Boolean system (or no-no paradox) with A_n as its symmetry group.

A corollary of the main result proof is the preorder relation among sequences induced by the action of S_n. This enables us to delineate all possible constructions of Boolean systems with maximal symmetry. We provide a constructive analysis for two classes of paradoxes with S_n symmetry: one is for the Boolean paradoxes and the other for the no-no type paradoxes. Our results reveal the intrinsic connection between broader symmetry and semantic paradoxes.

[1] Hangjie Cao, & Ming Hsiung, *Nonrepresentability of the alternating group A_4 on boolean systems (in Chinese)*, ***Studies in Logic***, vol. 17 (2024) , no. 2, pp. 56–72.

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Friday, September 12

► KAKERU YOKOYAMA, *Ramdomness and agnostic c-online learning.*
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In this talk, we present a new perspective on the connection between Machine Learning and Algorithmic Randomness. An important field in Machine Learning is online learning, which operates under the following setting: a learner receives a sequence of data instances one by one and, based on the past data, aims to make accurate predictions for the upcoming instances.

This framework bears a notable resemblance to the martingale-based characterisation of Martin-Löf Randomness, where a binary sequence is deemed ML-random if no lower semicomputable martingale succeeds on it. Both frameworks involve a learner processing finite prefixes of a sequence, yet they differ in what it means for the learner to perform "well." In agnostic online learning, the criterion is whether the learner minimizes regret with respect to a given pool of experts. In algorithmic randomness, success is measured by whether a martingale can grow unbounded capital—reflecting the inherent complexity of the sequence.

We propose interpreting the class of sequences that agnostic c-online learning fails to predict accurately as a kind of randomness class. We then examine how this class relates to existing notions of randomness. Our comparison considers various approaches, including analyzing properties of martingales associated with online learning algorithms and constructing online learners derived from randomness tests.

This is joint work with Ming Ng.

► TAKAYUKI KIHARA, *Idealized effective toposes and the Katětov order.*
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Hyland [1] proposed the effective topos **Eff** as the world of computable mathematics. Examples of subtoposes of **Eff** include **Set** (the world of set-theoretic mathematics) and **1** (the world of inconsistent mathematics). Note that the stronger a theory is, the smaller its Lindenbaum algebra is; thus the smaller the corresponding subtopos is. The question is: What kind of intermediate toposes exist between **Eff** and **Set**?

Lee-van Oosten [3] provided a method for presenting all subtoposes of the effective topos. The speaker [2] explained that their presentation consists of two layers: *oracle* and *majority*. The notion of majority (dually, minority) is given by a filter (dually, an ideal). Given an ideal $\mathcal{I}$ on ω, the notion of $\mathcal{I}$-computability is given as follows: Run ω-many computations $(\Phi_n)_{n\in\omega}$ in parallel, and if the majority of these computations are correct, i.e., $\{n \in \omega : \Phi_n \text{ is incorrect}\} \in \mathcal{I}$, then the computation is considered successful.

Then, for two different ideals $\mathcal{I}$ and $\mathcal{J}$ on ω, what is the relationship between $\mathcal{I}$-computable mathematics and $\mathcal{J}$-computable mathematics? (Ultra)filters or ideals are compared by the Rudin-Keisler order or the Katětov order. On the other hand, toposes are compared by geometric morphisms. Our discovery is that a game-theoretic variant of the Katětov order between ideals is connected to the geometric inclusion relation between idealized subtoposes of **Eff**.

This is joint work with Ming Ng.

[1] J.M.E. Hyland, *The Effective Topos*, ***Studies in Logic and the Foundations of Mathematics***, vol. 110 (1982), pp. 165–216.

[2] Takayuki Kihara, *Lawvere-Tierney topologies for computability theorists*, ***Transactions of the American Mathematical Society, Series B***, vol. 10 (2023), pp. 48–85.

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▶ IKUO YONEDA, *Local modularity and one-basedness for type definable subsets in rosy theories.*

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Let X be a type definable subset in a rosy theory. By a modified proof of Remark 5.13 on pp.106 in [3] we show that if X is locally modular with the existence of weak canonical base for any strong type in $X^{\mathrm{eq}} := \mathrm{dcl}^{\mathrm{eq}}(X)$, then X^{eq} is one-based [7]. The converse does not hold by Vassiliev's $\mathrm{SU} = 1$ unary predicate set [5]. It is known that there exists a strong type without its weak canonical base in a non locally modular *o*-minimal theory with CF-property [4]. We want to find a locally modular type-definable set Y without weak canonical base for some strong type in Y^{eq}. And then Y^{eq} will not be one-based since one-basedness implies the existence of weak canonical base for any strong type in Y^{eq} [6]. We have the definition of weak one-basedness for geometric theories [2]. For rosy theories, weak one-basedness is equivalent to very weak local modularity [1]. For now we do not find the definition of weak one-basedness for type definable subsets in rosy theories.

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[7] ———, *Some remarks on weak one-basedness*, 2025, 1st revision for publication in the ***Annals of Pure and Applied Logic*** resubmitted.

▶ ALEKSANDER CIEŚLAK, TAKEHIKO GAPPO, ARTURO MARTÍNEZ-CELIS, AND TAKASHI YAMAZOE, *Cardinal invariants of products of ideals.*

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For an ideal $\mathscr{I}$ on ω, let $\mathcal{K}_\mathscr{I}$ denote the σ-ideal generated by sets of the form $\prod_{n<\omega} I_n$ where $I_n \in \mathscr{I}$ for each $n < \omega$. For example, when $\mathscr{I} = \mathrm{Fin}$ is the finite ideal, $\mathcal{K}_\mathscr{I} = \mathcal{K}$ is the σ-ideal generated by compact sets in ω^ω. Pawlikowski [1] particularly dealt with the σ-ideal $\mathcal{K}_\mathscr{Z}$ where $\mathscr{Z}$ denotes the asymptotic density zero ideal and studied its topological properties. However, we study σ-ideals $\mathcal{K}_\mathscr{I}$ for ideals $\mathscr{I}$ in general and focus on their cardinal invariants rather than their topological properties. As a ZFC result, we prove that in many cases the additivites $\mathrm{add}(\mathcal{K}_\mathscr{I})$ can be calculated and in fact they are equal to some classical cardinal invariants, such as ω_1, $\mathrm{add}(\mathcal{N})$ and $\mathfrak{b}$. As a consistency result, we show that for $\mathscr{I}$ in a specific class of ideals including tall analytic P-ideals, consistently $\mathfrak{b} < \mathrm{non}(\mathcal{K}_\mathscr{I})$ holds, and consequently $\mathrm{non}(\mathcal{K}_\mathscr{Z})$ and $\mathrm{cov}(\mathcal{K}_\mathscr{Z})$ can be added to a model of Cichoń's maximum with distinct values.

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► LEONARDO PACHECO, *Epistemic possibility in Artemov and Protopopescu's intuitionistic epistemic logic*.
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Epistemic logics formalize knowledge and related concepts. Artemov and Protopopescu [1] defined an epistemic logic IEL to formalize intuitionistic knowledge. The central idea of this logic is that intuitionistic truth implies intuitionistic knowledge. This heavily contrasts with the classical case, where classical knowledge implies classical truth.

The modality K is interpreted in IEL as

$$K\varphi \text{ holds iff it is intuitionistically known that } \varphi,$$

for all formula φ. IEL satisfies the principles of co-reflection $\varphi \to K\varphi$ and weak reflection $K\varphi \to \neg\neg\varphi$. Note that, in a classical setting, these imply that truth and knowledge coincide.

We extend IEL with a modality $\hat{K}$ for epistemic possibility. To do so, we use the semantics of diamonds in constructive modal logics [2]. We will show that, for all formula φ, $\hat{K}P$ is equivalent to $\neg\neg P$. This implies that φ is epistemically possible iff it one can show that it is impossible to prove the negation of φ.

Note that there are other intuitionistic approaches to epistemic logic in the literature. For a brief survey, see Section 6 of [1]. As far as the author is aware, epistemic possibility has not been studied in other intuitionistic approaches to epistemic logic. For more details, see [3].

[1] Sergei Artemov and Tudor Protopopescu, *Intuitionistic epistemic logic*, ***The Review of Symbolic Logic***, vol. 9, no. 2 (2016), pp. 266–298.

[2] Michael Mendler and Valeria de Paiva, *Constructive* CK *for contexts*, ***Context Representation and Reasoning***, vol. 13 (2005).

[3] Leonardo Pacheco, *Epistemic possibility in Artemov and Protopopescu's intuitionistic epistemic logic*, ***RIMS Kôkyûroku***, no. 2293, pp. 66–71 (2024).

► ZHE LIN, AND ZHIGUANG ZHAO, *Non-labelled sequent calculus for intuitionistic public announcement logic and classical public announcement and preference logic*.

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Public announcement logic [3] and the dynamic logic of preference upgrade [1] are about how knowledge and preference are updated after receiving new information. In this paper, we adapt methods from [4] to introduce a novel sequent calculus for Intuitionistic Public Announcement Logic (IPAL) [2] and prove that the cut-elimination theorem holds for this system. We then generalize these results to Classical Public Announcement and Preference Logic (CPPAL), establishing a terminating and cut-free sequent calculus for this logic.

[1] J. van Benthem and F. Liu. Dynamic logic of preference upgrade. *Journal of Applied Non-Classical Logics*, 17(2):157–182, 2007.

[2] M. Ma, A. Palmigiano, and M. Sadrzadeh. Algebraic semantics and model completeness for intuitionistic public announcement logic. *Annals of Pure and Applied Logic*, 165(4):963–995, 2014.

[3] Plaza, J. (2007). Logics of public communications. *Synthese*, 158(2):165–179.

[4] Z. Lin and M. Ma. Gentzen sequent calculi for some intuitionistic modal logics. *Logic Journal of the IGPL*, 27 (4):596-623, 2019

► AKIHIKO ARAI, *The isomorphism problem for ultraproducts of operator algebras in continuous model theory.*

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An ultraproduct-like construction appeared in the context of operator algebras as early as the 1950s. Around the same time, the model-theoretic notion of ultraproducts was introduced by Łoś, and Robinson made use of it to develop nonstandard analysis. However, no essential connection between these two perspectives seems to have been recognized at the time.

In recent years, continuous model theory – a generalization of classical model theory to the setting of metric structures – has gained increasing attention as a powerful tool for analyzing ultraproducts of operator algebras. In this talk, we will present the foundational aspects of continuous model theory in the context of operator algebras and, as an application, explore the isomorphism problem for their ultraproducts.

[1] Hart, Bradd, *An introduction to continuous model theory*, ***Model Theory of Operator Algebras*** (Isaac Goldbring, editor), De Gruyter, Berlin, Boston, 2023, pp. 83–131.

[2] Farah, Ilijas and Hart, Bradd and Sherman, David, *Model theory of operator algebras II: model theory*, ***Israel Journal of Mathematics***, vol. 201 (2014), no. 1, pp. 477–505.

► KAITO ICHIKURA, NOBU-YUKI SUZUKI, *Hyperdoctrine Semantics for Subminimal First-Order Logics.*

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Minimal logic, introduced by Johansson, is the logic obtained from intuitionistic logic by excluding *ex contradictione quodlibet*. It is the "minimal" logic that retains the intuitionistic negation ($\neg A := A \to \bot$) while preserving the positive fragment of intuitionistic logic. Vakarelov [4] introduced a weaker logic with the same positive fragment as intuitionistic logic by using a negation as a primitive logical connective. Such logics are called subminimal logics.

Previous studies by Colacito et al. [1], Ichikura [2], and Niki [3] have explored the hierarchy of subminimal logics using neighbourhood and algebraic semantics. However, these studies have been confined to propositional logics, and it seems difficult to extend neighbourhood semantics into semantics for the corresponding first-order logics.

In this talk, we give a hyperdoctrine semantics, which is an extension of algebraic semantics in [2], for subminimal first-order logics. We show the soundness and completeness of these logics with respect to our semantics, and we provide a characterization of Halldén-completeness within this framework.

[1] Almudena Colacito, Dick de Jongh and Ana Lucia Vargas, *Subminimal negation*, ***Soft Computing***, vol.21 (2017), pp.165–174.

[2] Kaito Ichikura, *Continua of logics related to intuitionistic and minimal logics, to appear in* ***Bulletin of the Section of Logic***.

[3] Satoru Niki, *Subminimal logics in light of Vakarelov's logic*, ***Studia Logica***, vol.108 (2020), no. 5, pp.967–987.

[4] Dimiter Vakarelov, *Nelson's negation on the base of weaker versions of intuitionistic negation*, ***Studia Logica***, vol.80 (2005), pp.393–430.

► MIRAI IKEBUCHI, *Homotopical methods in logic.*
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In his dissertation, William Lawvere observed that there is a correspondence between equational theories and categories with finite products. For any equational theory T, one can define a small category with finite products $\mathrm{Syn}(T)$, called the *syntactic category* of T.

Recent studies [3, 2, 1] show that one can apply homotopical or homological methods to syntactic categories, hence to equational theories, and obtain interesting results. One of the results is that, given an equational theory, its homology gives an information that how many equations are needed to present the theory. For example, we can show that the theory of groups needs at least two equations to present it, which was earlier proved by arguments specific to groups [4].

The authors' general motivation is to study broader classes of theories using homology/homotopy. For example, given two sets of axioms, if the homology groups of them are not isomorphic, one should be able to conclude that the two sets of axioms give different theories, though it is highly non-trivial how to compute homology groups, and it is an important future work.

[1] M. Ikebuchi, *Homological Invariants of Higher-Order Equational Theories*, ***on Logic in Computer Science (LICS '25)*** (Singapore), 2025 (to appear).

[2] ——— *A lower bound of the number of rewrite rules obtained by homo- logical methods*, ***Logical Methods in Computer Science***, vol. 18 (2022), no. 3, pp. 36:1–36:25.

[3] P. Malbos and S. Mimram, *Homological computations for term rewriting systems* ***1st International Conference on Formal Structures for Computation and Deduction (FSCD 2016)*** (Porto, Portugal), (Delia Kesner and Brigitte Pientka), vol. 52, Schloss Dagstuhl –- Leibniz-Zentrum für Informatik, 2016, pp. 27:1–27:17.

[4] B. H. Neumann, *Another single law for groups*, ***Bulletin of the Australian Mathematical Society***, vol. 23 (1981), no. 1, pp. 81–102.

▶ HARUKA KOGURE, *Arithmetical completeness for some extensions of the pure logic of necessitation*.
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Let T denote a primitive recursively axiomatized consistent extension of Peano Arithmetic PA. We say that a formula $\mathrm{Pr}_T(x)$ is a provability predicate for T if for any formula φ, $T \vdash \varphi \iff \mathsf{PA} \vdash \mathrm{Pr}_T(\ulcorner \varphi \urcorner)$ holds. Provability predicates are considered to be modalities. An arithmetical interpretation f based on $\mathrm{Pr}_T(x)$ is a mapping from modal formulas to sentences of arithmetic satisfying the following conditions.

- f commutes with propositional connectives and
- $f(\Box A)$ is $\mathrm{Pr}_T(\ulcorner f(A) \urcorner)$.

For each provability predicate $\mathrm{Pr}_T(x)$ of T, let $\mathsf{PL}(\mathrm{Pr}_T)$ denote the set of all modal formulas A satisfying $T \vdash f(A)$ for all arithmetical interpretations f based on $\mathrm{Pr}_T(x)$. The set $\mathsf{PL}(\mathrm{Pr}_T)$ is called the *provability logic* of $\mathrm{Pr}_T(x)$. A well-known result in the study of provability logics is Solovay's arithmetical completeness theorems saying that if T is Σ_1-sound, then $\mathsf{PL}(\mathrm{Prov}_T)$ is exactly the modal logic **GL**, where $\mathrm{Prov}_T(x)$ is a standard provability predicate of T.

We investigate the arithmetical completeness theorems of some extensions of the pure logic of necessitation **N** introduced by Fitting, Marek, and Truszczyński [1]. For $m, n \in \omega$, Kurahashi and Sato [2] introduced the logic $\mathbf{NA}_{m,n}$ obtained from **N** by adding the axiom scheme $\Box^n A \to \Box^m A$. In this talk, among other things, we show that for each $m, n \geq 1$, the logic $\mathbf{NA}_{m,n}$ becomes a provability logic. This talk is based on the results presented in [3].

[1] Melvin C. Fitting, V. Wiktor Marek, and Miroslaw Truszczyński, *The pure logic of necessitation*, ***Journal of Logic and Computation***, vol.2, no.3, pp.349-373, 1992.

[2] Taishi Kurahashi and Yuta Sato, *The finite frame property of some extensions of the pure logic of necessitation*, ***Studia Logica***, accepted.

[3] Haruka Kogure, *Arithmetical completeness for some extensions of the pure logic of necessitation*, arXiv:2409.00938.

▶ YUTA SATO, *Uniform Lyndon interpolation for* $\mathbf{N}^+\mathbf{A}_{m,n}$.
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N, the *pure logic of necessitation*[1], is a nonnormal modal logic obtained from **K** by removing the K axiom. For any $m, n \in \mathbb{N}$, a logic $\mathbf{N}^+\mathbf{A}_{m,n}$ is obtained by adding to **N** a modal reduction principle $\Box^n \varphi \to \Box^m \varphi$ and a rule $\frac{\neg \Box \varphi}{\neg \Box \Box \varphi}$. Here, the said rule is admissible in every normal modal logic, thus $\mathbf{N}^+\mathbf{A}_{m,n} \subseteq \mathbf{K} + \Box^n \varphi \to \Box^m \varphi$. It is proven in [2] that $\mathbf{N}^+\mathbf{A}_{m,n}$ has the finite frame property (FFP) for every $m, n \in \mathbb{N}$, while FFP of $\mathbf{K} + \Box^n \varphi \to \Box^m \varphi$ is, in general, still unknown to this day.

In this talk, we shall present a new result on $\mathbf{N}^+\mathbf{A}_{m,n}$, that is, it enjoys uniform Lyndon interpolation property (ULIP) for every $m, n \in \mathbb{N}$. This result is in stark contrast to the situation in normal modal logics, where it is known that ULIP of $\mathbf{K} + \Box^n \varphi \to \Box^m \varphi$ fails for some $m, n \in \mathbb{N}$. We shall introduce a general method, *propositionalization*, that enables one to prove ULIP of a logic if there is a special embedding of it into some weaker logic with ULIP. A cut-admissible sequent calculus for $\mathbf{N}^+\mathbf{A}_{m,n}$ is also introduced to construct such an embedding of $\mathbf{N}^+\mathbf{A}_{m,n}$ into classical propositional logic **Cl**. This talk is based on [3].

[1] Melvin C. Fitting, V. Wiktor Marek, and Miroslaw Truszczyński, *The pure logic of necessitation*, ***Journal of Logic and Computation***, 2(3):349-373, 1992.

[2] Taishi Kurahashi and Yuta Sato, *The finite frame property of some extensions of the pure logic of necessitation*, ***Studia Logica***, to appear, doi:10.1007/s11225-024-10154-w.

[3] Yuta Sato, *Uniform Lyndon interpolation for the pure logic of necessitation with a modal reduction principle*, submitted, arXiv:2503.10176.

▶ TAISHI KURHASHI, KOHEI TOMINAGA, *Smullyan's truth and provability.*
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In Smullyan' s paper "Truth and Provability" [1] published in *The Mathematical Intelligencer* (2013), Smullyan introduced a simple framework on finite strings, and showed that the ideas and the structures of the proofs of Gödel's First Incompleteness Theorem and Tarski's Undefinability Theorem are understandable through this framework. The framework is specified by determining a set of predicates, which are finite strings, and determining which set of finite strings each predicate names. In addition, Smullyan's framework employs two prefixes `n` and `r` for predicates and special rules regarding the naming relation for the predicates prefixed by these symbols. For such a simple system, Smullyan proved three theorems corresponding to Fixed-point Theorem, Gödel's First Incompleteness Theorem and Tarski's Undecidability Theorem, respectively.

We revisit [1] for three purposes. First, we introduce the notion of *Smullyan models* to give a precise definition for Smullyan' s framework. Second, we clarify the relationship between these three theorems and the symbols `n` and `r` in terms of both implications and non-implications. Third, we construct two Smullyan models based on arithmetical ideas and discuss the correspondence between the properties of these Smullyan models and those concerning truth and provability in arithmetic. This talk is based on [2].

[1] Raymond Merrill Smullyan, *Truth and Provability*, ***The Mathematical Intelligencer***, vol. 35 (2013), pp. 21–24.

[2] Taishi Kurahashi and Kohei Tominaga, *Smullyan's truth and provability*, ***Journal of Logic and Computation***, doi:10.1093/logcom/exaf001.

▶ RYOICHI SATO, *Generalizations of the measure zero ideal modulo ideals on the natural numbers.*
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Gavalova and Mejia introduced the generalization of the measure zero ideal with an ideal on omega. This is one of the study of the generalizations expanding the sets of all finite sets to ideals on the natural numbers. We can investigate the properties between the measure zero ideal and the closed measure zero ideal with this expansion. Indeed, these expanded ideals are known to lie between the measure zero ideal and the closed measure zero ideal. Some topological properties such as the Baire properties have a significant effect on the measure zero ideal, the closed measure zero ideal, and the generalized ideals. We studied the topoligical properties which has an effect on the notions of the measure zero ideals.

[1] GAVALOVÁ, VIERA AND MEJÍA, DIEGO A., *LEBESGUE MEASURE ZERO MODULO IDEALS ON THE NATURAL NUMBERS*, ***The Journal of Symbolic Logic***, Published online (2023), pp. 1–31.

▶ TRISTAN VAN DER VLUGT, *Higher Baire spaces for singular cardinals.*
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The higher Baire space ${}^{\kappa}\kappa$ is the set of functions from κ to κ, where κ is an uncountable cardinal. Recently the study of combinatorial properties of ${}^{\kappa}\kappa$ for regular uncountable κ has been a very active field of research. In this talk we will consider the case where κ is singular, as well as related spaces such as ${}^{\mathrm{cf}(\kappa)}\kappa$. We will compare several reasonable (and unreasonable) topologies on these spaces and discuss the $\mathrm{cf}(\kappa)$-meagre ideal (i.e. sets that are unions of $\mathrm{cf}(\kappa)$-many nowhere dense sets) and their associated cardinal invariants.

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Piotr Borodulin-Nadzieja and Barnabás Farkas introduced, in [1], the cardinal invariant $\mathfrak{p}_K(J)$ associated with an ideal J on ω, the set of natural numbers. This invariant is defined as the minimal cardinality of a family $\mathcal{A} \subseteq \mathcal{P}(\omega)$ with the finite union property such that $\mathcal{A} \not\leq_K J$, where $\leq_K$ denotes the well-known Katětov order. In their work, the authors also established several topological and analytic characterizations of $\mathfrak{p}_K(J)$ and proved some consistency results.

In this talk, we study the supremum of all cardinals $\mathfrak{p}_K(J)$ as J ranges over all ideals on ω, which we denote by $\mathfrak{p}_K$. To this end, we present several combinatorial and topological characterizations of $\mathfrak{p}_K$ and show that it can be defined in terms of logarithms of cardinals. In particular, this implies that $\mathfrak{p}_K$ is a regular cardinal. Finally, we study its behavior in various well-known models of set theory and discuss its relationship with classical cardinal invariants.

This talk is based on [2], a work in progress in collaboration with Miguel A. Cardona, Diego A. Mejía, Miroslav Repický, and Jaroslav Šupina.

[1] Piotr Borodulin-Nadzieja and Barnabás Farkas, *Cardinal coefficients associated to certain orders on ideals*, Archive for Mathematical Logic **51** (2012), no. 1-2, 187–202. *URL Address*: `https://doi.org/10.1007/s00153-011-0260-9`.DOI: 10.1007/s00153-011-0260-9.

[2] Miguel A. Cardona, Diego A. Mejía, Miroslav Repický, Jaroslav Šupina, and Andrés F. Uribe-Zapata, *Pseudo-intersection numbers.* In preparation.