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Appendix to a review article for The Tokyo Foundation for Policy Research

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1. Simulation results

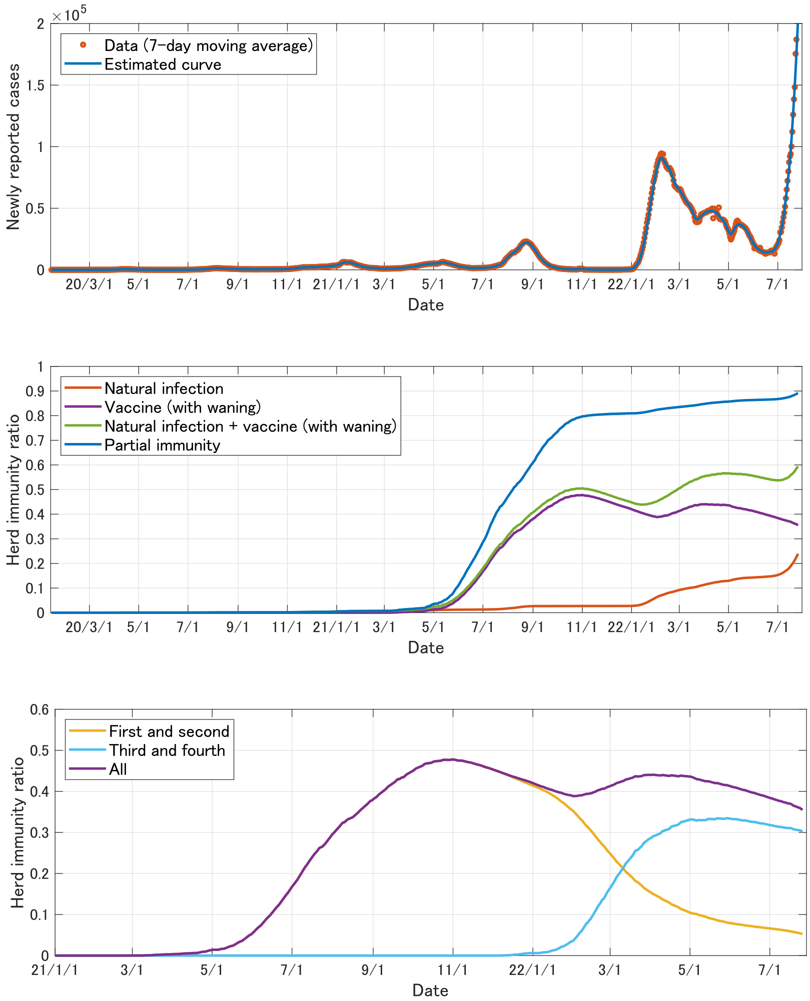
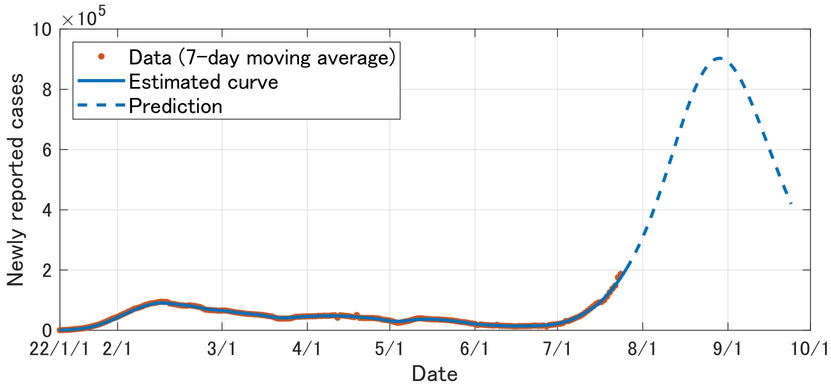


Figure 1: Time variation of newly reported cases (top), estimated herd immunity ratio (middle) and estimated vaccine-induced herd immunity ratio (bottom) for COVID-19 in Japan (2020/1/14 - 2022/7/25).



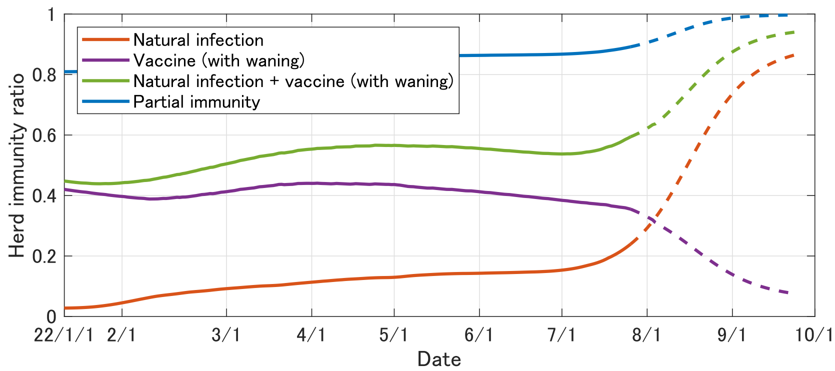


Figure 2: Prediction of newly reported cases (top) and herd immunity ratio (bottom) for COVID-19 in Japan (2022/1/1 - 2022/10/1).

2. Parameters

Parameter	Description	Value
S	Susceptible population (unvaccinated)	-
E	Exposed population (unvaccinated)	-
I	Infectious population (unvaccinated)	-
R	Removed population (unvaccinated)	-
S_1	Susceptible population (vaccinated once)	-
E_1	Exposed population (vaccinated once)	-
I_1	Infectious population (vaccinated once)	-
R_1	Removed population (vaccinated once)	-
S_2	Susceptible population (vaccinated twice)	-
E_2	Exposed population (vaccinated twice)	-
I_2	Infectious population (vaccinated twice)	-
R_2	Removed population (vaccinated twice)	-
S_3	Susceptible population (vaccinated more than 3 times)	-
E_3	Exposed population (vaccinated more than 3 times)	-
I_3	Infectious population (vaccinated more than 3 times)	-
R_3	Removed population (vaccinated more than 3 times)	-
t	Time	-
a	Class age (time elapsed since the vaccination)	-
β	Infection rate	Estimated using data in [8]
ε	Onset rate	0.2 (incubation period $1/\varepsilon = 5$ days) [3]
γ	Removal rate	0.1 (infection period $1/\gamma = 10$ days) [1]
λ	Force of infection	Equation (1)
$1 - \sigma$	Efficacy of one time vaccination	0.46 [5]
v_n	Vaccination rate (for n-th)	Estimated using data in [6]
T	Duration between the vaccination	180 days
$1 - p(a)$	Efficacy of full vaccination at class age a	$0.8e^{-0.003a}$ (estimated using data in [5])
δ	Detection rate	0.5 (estimated using data in [4])
N	Total population in Japan	1.26×10^8 [7]

See [2] for the details of how to estimate each parameter.

3. Model

Before vaccination policy (January 14, 2020 - February 16, 2021).

$$
\begin{aligned}
S'(t) &= -\beta S(t)I(t),\\
E'(t) &= \beta S(t)I(t) - \varepsilon E(t),\\
I'(t) &= \varepsilon E(t) - \gamma I(t),\\
R'(t) &= \gamma I(t).
\end{aligned}
$$

Under vaccination policy (February 17, 2021 - July 25, 2022).

- Unvaccinated population:

$$
\begin{aligned}
S'(t) &= -\lambda(t)S(t) - v_1 S(t),\\
E'(t) &= \lambda(t)S(t) - (\varepsilon + v_1)E(t),\\
I'(t) &= \varepsilon E(t) - (\gamma + v_1)I(t),\\
R'(t) &= \gamma I(t) - v_1 R(t).
\end{aligned}
$$

- Vaccinated once:

$$
\begin{aligned}
S_1'(t) &= v_1 S(t) - \sigma\lambda(t)S_1(t) - v_2 S_1(t),\\
E_1'(t) &= v_1 E(t) + \sigma\lambda(t)S_1(t) - (\varepsilon + v_2)E_1(t),\\
I_1'(t) &= v_1 I(t) + \varepsilon E_1(t) - (\gamma + v_2)I_1(t),\\
R_1'(t) &= v_1 R(t) + \gamma I_1(t) - v_2 R_1(t).
\end{aligned}
$$

- Vaccinated more than twice ($n = 2, 3$):

$$
\begin{aligned}
S_n(t,0) &= \begin{cases} v_2 S_1(t), & n = 2,\\ v_3 \int_T^\infty S_2(t,a)da + v_4 \int_T^\infty S_3(t,a)da, & n = 3,\end{cases}\\
E_n(t,0) &= \begin{cases} v_2 E_1(t), & n = 2,\\ v_3 \int_T^\infty E_2(t,a)da + v_4 \int_T^\infty E_3(t,a)da, & n = 3,\end{cases}\\
I_n(t,0) &= \begin{cases} v_2 I_1(t), & n = 2,\\ v_3 \int_T^\infty I_2(t,a)da + v_4 \int_T^\infty I_3(t,a)da, & n = 3,\end{cases}\\
R_n(t,0) &= \begin{cases} v_2 R_1(t), & n = 2,\\ v_3 \int_T^\infty R_2(t,a)da + v_4 \int_T^\infty R_3(t,a)da, & n = 3,\end{cases}\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) S_n(t,a) = -p(a)\lambda(t)S_n(t,a) - q_n(a)S_n(t,a),\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) E_n(t,a) = p(a)\lambda(t)S_n(t,a) - [\varepsilon + q_n(a)]E_n(t,a),\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) I_n(t,a) = \varepsilon E_n(t,a) - [\gamma + q_n(a)]I_n(t,a),\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) R_n(t,a) = \gamma I_n(t,a) - q_n(a)R_n(t,a),
\end{aligned}
$$

where

$$q_n(a) = \begin{cases} 0, & a < T, \\ v_{n+1}, & \text{otherwise.} \end{cases}$$

- Force of infection:

$$\lambda(t) = \beta \left[I(t) + I_1(t) + \sum_{n=2}^{3} \int_0^\infty I_n(t,a)da \right]. \tag{1}$$

- Efficacy of full vaccination at class age a: $1 - p(a) = 0.8e^{-0.003a}$, which is fitted to the data in [5] as shown in Figure 3.

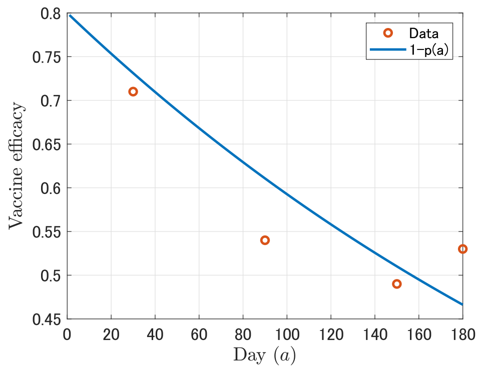


Figure 3:

- Let

$$M_0(t) := E(t) + I(t) + R(t), \quad M_1(t) := E_1(t) + I_1(t) + R_1(t),$$
$$M_n(t) := \int_0^\infty [E_n(t,a) + I_n(t,a) + R_n(t,a)]da, \quad n \geq 2.$$

Description for each curve in Figure 1:

- Natural infection: $\sum_{n=0}^{3} M_n(t)$.
- Vaccine (with waning): $(1-\sigma)S_1(t) + \sum_{n=2}^{3} \int_0^\infty [1-p(a)]S_n(t,a)da$.
- Natural infection + vaccine (with waning): $\sum_{n=0}^{3} M_n(t) + (1-\sigma)S_1(t) + \sum_{n=2}^{3} \int_0^\infty [1-p(a)]S_n(t,a)da$.
- Partial immunity: $1 - S(t)$.
- First and second: $(1-\sigma)S_1(t) + \int_0^\infty [1-p(a)]S_2(t,a)da$
- Third and fourth: $\int_0^\infty [1-p(a)]S_3(t,a)da$
- All: $(1-\sigma)S_1(t) + \sum_{n=2}^{3} \int_0^\infty [1-p(a)]S_n(t,a)da$.

How to estimate $\beta = \beta(t)$ and δ

See [2].

How to estimate the vaccination rates

Note that $v_1 \times [S(t) + E(t) + I(t) + R(t)] \times N$ is the number of the first vaccination at time t. Hence, we estimate $v_1 = v_1(t)$ as

$$v_1(t) = \frac{(number\ of\ the\ first\ vaccination\ at\ time\ t)}{[S(t) + E(t) + I(t) + R(t)] \times N}.$$

In a similar manner, we estimate $v_n = v_n(t)$ $(n \geq 2)$ as

$$v_n(t) = \begin{cases} \dfrac{(number\ of\ the\ second\ vaccination\ at\ time\ t)}{[S_1(t) + E_1(t) + I_1(t) + R_1(t)] \times N}, & n = 2, \\ \dfrac{(number\ of\ the\ n{-}th\ vaccination\ at\ time\ t)}{\int_T^\infty [S_{n-1}(t,a) + E_{n-1}(t,a) + I_{n-1}(t,a) + R_{n-1}(t,a)] da \times N}, & n \geq 3. \end{cases}$$

How to predict

We fixed the infection rate and vaccination rates using the latest 1 week data.

References

[1] R.M. Anderson, H. Heesterbeek, D. Klinkenberg, T.D. Hollingsworth, How will country-based mitigation measures influence the course of the COVID-19 epidemic?, The Lancet 395 (2020) 21–27.

[2] T. Kuniya, Appendix to a review article for The Tokyo Foundation for Policy Research, `http://www2.kobe-u.ac.jp/~tkuniya/appendix`, accessed on July 11, 2022.

[3] N.M. Linton, T. Kobayashi, Y. Yang, et al., Incubation period and other epidemiological characteristics of 2019 novel coronavirus infections with right truncation: a statistical analysis of publicly available case data, J. Clin. Med. 9 (2020) 538.

[4] m3.com, 自然感染による抗体保有率、東京 2.80%・大阪 3.78%に, `https://www.m3.com/news/open/iryoishin/1017268`, accessed on March 8, 2022.

[5] NIID 国立感染症研究所, 新型コロナワクチンの有効性を検討した症例対照研究の暫定報告（第三報）, `https://www.niid.go.jp/niid/ja/2019-ncov/2484-idsc/10966-covid19-71.html`, accessed on March 7, 2022.

[6] Prime Minister of Japan and His Cabinet, `https://www.kantei.go.jp/jp/headline/kansensho/vaccine.html`, accessed on July 27, 2022.

[7] Statistics Bureau Japan, Population estimates monthly report, `https://www.stat.go.jp/english/data/jinsui/tsuki/index.html`, accessed on March 7, 2022.

[8] WHO Coronavirus (COVID-19) Dashboard, `https://covid19.who.int/`, accessed on July 27, 2022.