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Appendix to a review article for The Tokyo Foundation for Policy Research

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1. Simulation results

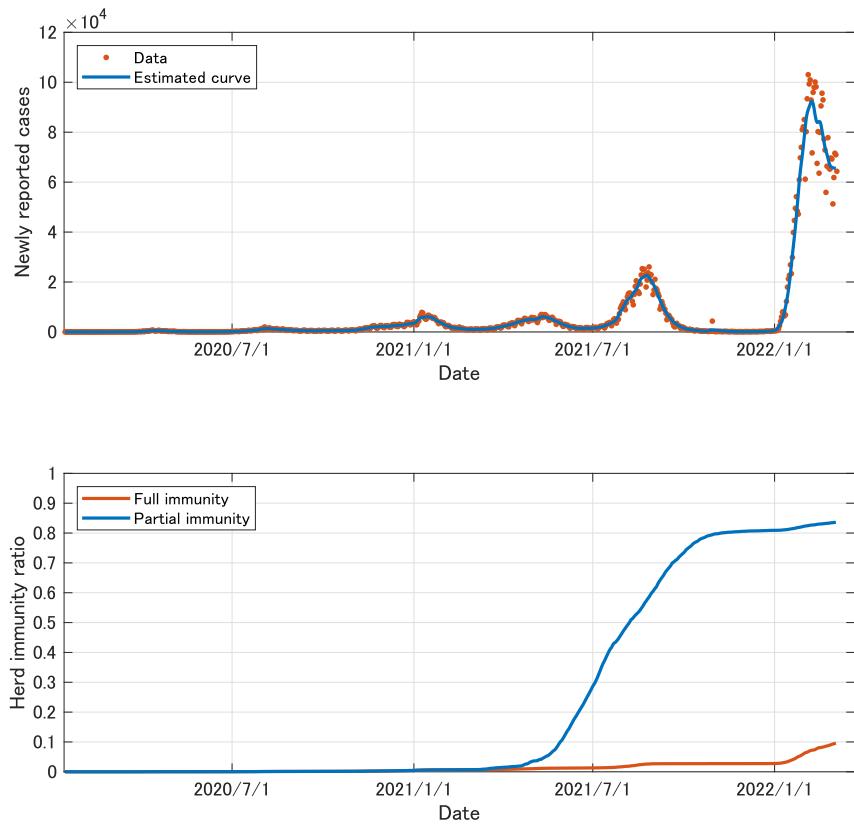


Figure 1: Time variation of newly reported cases (top) and estimated herd immunity ratio (bottom) for COVID-19 in Japan (2020/1/14 - 2022/3/6).

2. Symbols

Parameter	Description	Value
S	Susceptible population (unvaccinated)	-
E	Exposed population (unvaccinated)	-
I	Infectious population (unvaccinated)	-
R	Removed population (unvaccinated)	-
S_1	Susceptible population (vaccinated once)	-
E_1	Exposed population (vaccinated once)	-
I_1	Infectious population (vaccinated once)	-
R_1	Removed population (vaccinated once)	-
S_2	Susceptible population (vaccinated more than twice)	-
E_2	Exposed population (vaccinated more than twice)	-
I_2	Infectious population (vaccinated more than twice)	-
R_2	Removed population (vaccinated more than twice)	-
t	Time	-
a	Class age (time elapsed since the vaccination)	-
β	Infection rate	Estimated using data in [9]
ε	Onset rate	0.2 (incubation period $1/\varepsilon = 5$ days) [4]
γ	Removal rate	0.1 (infection period $1/\gamma = 10$ days) [1]
λ	Force of infection	Equation (1)
$1 - \sigma$	Efficacy of one time vaccination	0.46 [6]
v	Vaccination rate (first)	Estimated using data in [7]
w	Vaccination rate (second)	Estimated using data in [7]
$q(a)$	Vaccination rate (third) at class age a	Equation (2)
u	Vaccination rate (third)	Estimated using data in [7]
T	Duration between the second and third vaccination	180 days
$1 - p(a)$	Efficacy of full vaccination at class age a	$0.8e^{-0.003a}$ (estimated using data in [6])
δ	Detection rate	0.5 (estimated using data in [5])
N	Total population in Japan	1.26×10^8 [8]

3. Model

Before vaccination (January 14, 2020 - February 16, 2021).

$$\begin{aligned} S'(t) &= -\beta S(t)I(t), \\ E'(t) &= \beta S(t)I(t) - \varepsilon E(t), \\ I'(t) &= \varepsilon E(t) - \gamma I(t), \\ R'(t) &= \gamma I(t). \end{aligned}$$

After vaccination (February 17, 2021 - March 6, 2022).

- Unvaccinated population:

$$\begin{aligned} S'(t) &= -\lambda(t)S(t) - vS(t), \\ E'(t) &= \lambda(t)S(t) - (\varepsilon + v)E(t), \\ I'(t) &= \varepsilon E(t) - (\gamma + v)I(t), \\ R'(t) &= \gamma I(t) - vR(t). \end{aligned}$$

- Population vaccinated once:

$$\begin{aligned} S'_1(t) &= vS(t) - \sigma\lambda(t)S_1(t) - wS_1(t), \\ E'_1(t) &= vE(t) + \sigma\lambda(t)S_1(t) - (\varepsilon + w)E_1(t), \\ I'_1(t) &= \varepsilon E_1(t) + \sigma\lambda(t)S_1(t) - (\gamma + w)I_1(t), \\ R'_1(t) &= vR(t) + \gamma I_1(t) - wR_1(t). \end{aligned}$$

- Population vaccinated more than twice:

$$\begin{aligned} S_2(t, 0) &= wS_1(t) + \int_0^\infty q(a)S_2(t, a)da, \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) S_2(t, a) &= -p(a)\lambda(t)S_2(t, a) - q(a)S_2(t, a), \\ E_2(t, 0) &= wE_1(t) + \int_0^\infty q(a)E_2(t, a)da, \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) E_2(t, a) &= p(a)\lambda(t)S_2(t, a) - [\varepsilon + q(a)]E_2(t, a), \\ I_2(t, 0) &= wI_1(t) + \int_0^\infty q(a)I_2(t, a)da, \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) I_2(t, a) &= \varepsilon E_2(t, a) - [\gamma + q(a)]I_2(t, a), \\ R_2(t, 0) &= wR_1(t) + \int_0^\infty q(a)R_2(t, a)da, \\ \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) R_2(t, a) &= \gamma I_2(t, a) - q(a)R_2(t, a). \end{aligned}$$

- Force of infection:

$$\lambda(t) = \beta \left[I(t) + I_1(t) + \int_0^\infty I_2(t, a) da \right]. \quad (1)$$

- Vaccination rate (third) at time a passed since the second vaccination:

$$q(a) = \begin{cases} 0, & a < T, \\ u, & \text{otherwise.} \end{cases} \quad (2)$$

- Efficacy of full vaccination at class age a : $1 - p(a) = 0.8e^{-0.003a}$, which is fitted to the data in [6] as shown in Figure 2.

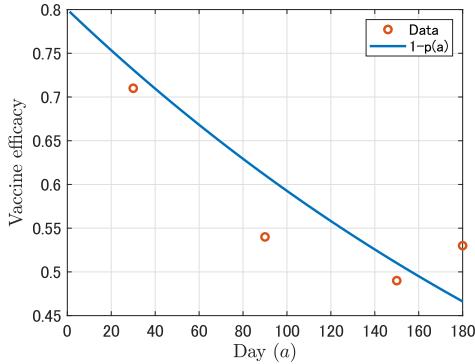


Figure 2:

- Full immunity at time t : $1 - S(t) - S_1(t) - \int_0^\infty S_2(t, a) da$.
- Partial immunity at time t : $1 - S(t)$.

How to estimate $\beta = \beta(t)$

Let the unit time be 1 day. Newly reported cases per day: let $A(t)$ be the actual data collected from [9] and let

$$\begin{aligned} Y(t) := & (\text{detection rate } \delta) \\ & \times (\text{newly removed } \gamma \left[I(t) + I_1(t) + \int_0^\infty I_2(t, a) da \right]) \\ & \times (\text{total population } N), \end{aligned}$$

be the simulation data. For $t \leq 7$, we assume $\beta = 0.25$ so that the basic reproduction number $\mathcal{R}_0 = \beta/\gamma = 2.5$ [2]. For $t > 7$ ($t \in \mathbb{Z}$), we seek $\beta = \beta^*$ that minimizes the least square error

$$L(\beta) := \sum_{s=1}^7 [Y(t-s) - A(t-s)]^2$$

and let $\beta(t) = \beta^*$. In other words, we repeatedly estimate $\beta(t)$ using data in previous 1 week. Simulation is carried out using the above model with initial condition

$$Y(0) = 1, \quad I(0) = \frac{Y(0)}{\delta\gamma N}, \quad S(0) = 1 - I(0),$$

and other variables are zero.

How to estimate v , w and u

Note that $v \times [S(t) + E(t) + I(t) + R(t)] \times N$ is the number of first vaccination at time t . Hence, we estimate $v = v(t)$ as

$$v(t) = \frac{(\text{number of first vaccination at time } t [7])}{[S(t) + E(t) + I(t) + R(t)] \times N}.$$

In a similar manner, we estimate $w = w(t)$ and $u = u(t)$ as

$$w(t) = \frac{(\text{number of second vaccination at time } t [7])}{[S_1(t) + E_1(t) + I_1(t) + R_1(t)] \times N},$$

and

$$u(t) = \frac{(\text{number of third vaccination at time } t [7])}{\int_T^\infty [S_2(t, a) + E_2(t, a) + I_2(t, a) + R_2(t, a)] da \times N},$$

respectively.

How to estimate δ

To estimate δ , we used the data in [5]. Taking average of each data in [5], we obtain the herd immunity ratio at June 2020 as 0.1%; at December 2020 as 0.662%; at December 2021 as 2.158%. We estimated $\delta = 0.5$ by fitting the curve to the data as shown in Figure 3.

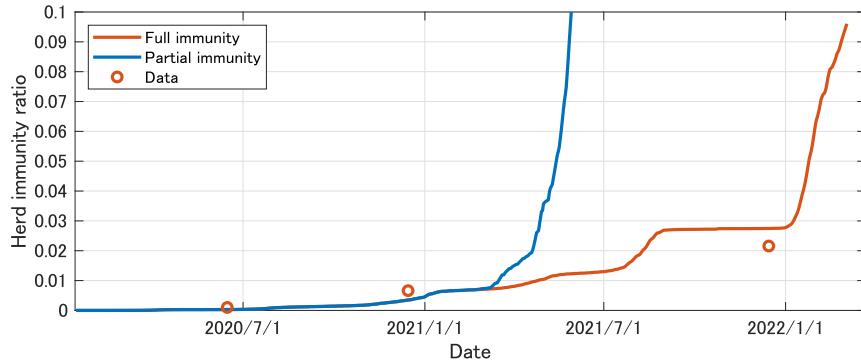


Figure 3: Time variation of herd immunity ratio for COVID-19 in Japan (2020/1/14 - 2022/3/6).

4. Limitations

The following factors are not explicitly considered in our model.

1. Seasonality;
2. Virus mutation;
3. Behavior change;
4. Different susceptibility;
5. Space structure;
6. (Chronological) age structure;
7. Stochasticity.

Since we estimated β for each day, effects of 1-3 on the infection rate may be implicitly considered in our model. However, effect of 2 on the herd immunity is not considered and this may lead to the overestimation of the herd immunity. On the other hand, considering 4 may decrease the critical proportion of immunization [3]. Considering 5-7 could improve the model but the estimation may become more complex.

References

- [1] R.M. Anderson, H. Heesterbeek, D. Klinkenberg, T.D. Hollingsworth, How will country-based mitigation measures influence the course of the COVID-19 epidemic?, *The Lancet* 395 (2020) 21–27.
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- [3] 稲葉寿, 感染症数理モデルと COVID-19, <https://www.covid19-jma-medical-expert-meeting.jp/topic/3925>, accessed on March 8, 2022.
- [4] N.M. Linton, T. Kobayashi, Y. Yang, et al., Incubation period and other epidemiological characteristics of 2019 novel coronavirus infections with right truncation: a statistical analysis of publicly available case data, *J. Clin. Med.* 9 (2020) 538.
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- [6] NIID 国立感染症研究所, 新型コロナワクチンの有効性を検討した症例対照研究の暫定報告（第三報）, <https://www.niid.go.jp/niid/ja/2019-ncov/2484-idsc/10966-covid19-71.html>, accessed on March 7, 2022.
- [7] Prime Minister of Japan and His Cabinet, <https://www.kantei.go.jp/jp/headline/kansensho/vaccine.html>, accessed on March 8, 2022.
- [8] Statistics Bureau Japan, Population estimates monthly report, <https://www.stat.go.jp/english/data/jinsui/tsuki/index.html>, accessed on March 7, 2022.
- [9] WHO Coronavirus (COVID-19) Dashboard, <https://covid19.who.int/>, accessed on March 8, 2022.

MATLAB code for main simulation

- ‘cases.xlsx’ is an excel file collecting the newly reported cases of COVID-19 per day in Japan from 2020/1/14 to 2022/3/6.
- ‘vaccine.xlsx’ is an excel file collecting the newly number of vaccination against COVID-19 per day in Japan from 2020/1/14 to 2022/3/6.

```
1 clear
2 tic
3
4 A=xlsread('cases.xlsx');
5 B=xlsread('vaccine.xlsx');
6
7 gam=1/10;
8 eps=1/5;
9 N=1.26*10^8;
10 del=0.5;
11 sig=0.54;
12
13 pp=0.003;
14
15 dt=1;te=max(size(A))-1;nt=te/dt;
16 T=180;nT=T/dt;
17 da=1;ae=4*T;na=ae/da;
18
19 Y(1)=1;
20 I(1)=Y(1)/(del*N*gam);
21 S(1)=1-I(1);
22 E(1)=0;
23 R(1)=0;
24 S1(1)=0;
25 E1(1)=0;
26 I1(1)=0;
27 R1(1)=0;
28
29 for a=1:1:na
30     S2(1,a)=0;
31     E2(1,a)=0;
32     I2(1,a)=0;
33     R2(1,a)=0;
34 end
35 SS(1)=sum(S2(1,1:1:na))*da;
36 EE(1)=sum(E2(1,1:1:na))*da;
37 II(1)=sum(I2(1,1:1:na))*da;
38 RR(1)=sum(R2(1,1:1:na))*da;
39
40 db=0.01;be=4;nb=be/db;
41
42 t0=7/dt;
43 t1=401/dt; %2021/2/17 start of vaccination
44
45 for t=1:1:nt
46     if B(round(t*dt),2)==0
47         v(t)=0;
48     end
```

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49 if B(round(t*dt),3)==0
50     w(t)=0;
51 end
52 if B(round(t*dt),4)==0
53     u(t)=0;
54 end
55
56 if t < t0
57     bet=0.25;
58     S(t+1)=S(t)+dt*(-bet*S(t)*I(t));
59     E(t+1)=E(t)+dt*(bet*S(t)*I(t)-eps*E(t));
60     I(t+1)=I(t)+dt*(eps*E(t)-gam*I(t));
61     R(t+1)=R(t)+dt*gam*I(t);
62     Y(t+1)=del*N*gam*I(t+1);
63     S1(t+1)=0;
64     E1(t+1)=0;
65     I1(t+1)=0;
66     R1(t+1)=0;
67     for a=1:1:na
68         S2(t+1,a)=0;
69         E2(t+1,a)=0;
70         I2(t+1,a)=0;
71         R2(t+1,a)=0;
72     end
73 elseif t<t1
74     for b=1:1:nb
75         bet=b*db;
76         for s=1:1:t0
77             S(t+s-t0+1)=S(t+s-t0)+dt*(-bet*S(t+s-t0)*I(t+s-t0));
78             E(t+s-t0+1)=E(t+s-t0)+dt*(bet*S(t+s-t0)*I(t+s-t0)...
79             -eps*E(t+s-t0));
80             I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)-gam*I(t+s-t0));
81             R(t+s-t0+1)=R(t+s-t0)+dt*gam*I(t+s-t0);
82             Y(t+s-t0+1)=del*N*gam*I(t+s-t0+1);
83             Z(t+s-t0+1)=(Y(t+s-t0+1)-A(round((t+s-t0+1)*dt),1))^2;
84         end
85         L(b)=sum(Z(t-t0+2:1:t+1));
86     end
87     [L1 L2]=min(L);
88     bet=L2*db;
89
90     for s=1:1:t0+1
91         S(t+s-t0+1)=S(t+s-t0)+dt*(-bet*S(t+s-t0)*I(t+s-t0));
92         E(t+s-t0+1)=E(t+s-t0)+dt*(bet*S(t+s-t0)*I(t+s-t0)...
93             -eps*E(t+s-t0));
94         I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)-gam*I(t+s-t0));
95         R(t+s-t0+1)=R(t+s-t0)+dt*gam*I(t+s-t0);
96         Y(t+s-t0+1)=del*N*gam*I(t+s-t0+1);
97         S1(t+s-t0+1)=0;
98         E1(t+s-t0+1)=0;
99         I1(t+s-t0+1)=0;
100        R1(t+s-t0+1)=0;
101        for a=1:1:na
102            S2(t+s-t0+1,a)=0;
103            E2(t+s-t0+1,a)=0;
104            I2(t+s-t0+1,a)=0;
105            R2(t+s-t0+1,a)=0;
106        end
107        SS(t+s-t0+1)=sum(S2(t+s-t0+1,1:1:na))*da;

```

```

108    EE(t+s-t0+1)=sum(E2(t+s-t0+1,1:1:na))*da;
109    II(t+s-t0+1)=sum(I2(t+s-t0+1,1:1:na))*da;
110    RR(t+s-t0+1)=sum(R2(t+s-t0+1,1:1:na))*da;
111    end
112 else
113   for b=1:1:nb
114     bet=b*db;
115     for s=1:1:t0
116       if B(round((t+s-t0)*dt),2) ≠ 0
117         v(t+s-t0)=B(round((t+s-t0)*dt),2)/((S(t+s-t0) ...
118           +E(t+s-t0)+I(t+s-t0)+R(t+s-t0))*N);
119       else
120         v(t+s-t0)=0;
121       end
122       if B(round((t+s-t0)*dt),3) ≠ 0
123         w(t+s-t0)=B(round((t+s-t0)*dt),3)/((S1(t+s-t0) ...
124           +E1(t+s-t0)+I1(t+s-t0)+R1(t+s-t0))*N);
125       else
126         w(t+s-t0)=0;
127       end
128       if B(round((t+s-t0)*dt),4) ≠ 0
129         u(t+s-t0)=B(round((t+s-t0)*dt),4)/...
130           ((sum(S2(t+s-t0,nT:1:na))+sum(E2(t+s-t0,nT:1:na)) ...
131             +sum(I2(t+s-t0,nT:1:na))...
132               +sum(R2(t+s-t0,nT:1:na)))*da*N);
133       else
134         u(t+s-t0)=0;
135       end
136
137       QS=u(t+s-t0)*sum(S2(t+s-t0,nT:1:na-1))*da;
138       QE=u(t+s-t0)*sum(E2(t+s-t0,nT:1:na-1))*da;
139       QI=u(t+s-t0)*sum(I2(t+s-t0,nT:1:na-1))*da;
140       QR=u(t+s-t0)*sum(R2(t+s-t0,nT:1:na-1))*da;
141
142       lam(t+s-t0)=bet*(I(t+s-t0)+I1(t+s-t0) ...
143           +sum(I2(t+s-t0,1:1:na))*da);
144
145       S(t+s-t0+1)=S(t+s-t0)+dt*(-lam(t+s-t0)*S(t+s-t0) ...
146           -v(t+s-t0)*S(t+s-t0));
147       E(t+s-t0+1)=E(t+s-t0)+dt*(lam(t+s-t0)*S(t+s-t0) ...
148           -(eps+v(t+s-t0))*E(t+s-t0));
149       I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0) ...
150           -(gam+v(t+s-t0))*I(t+s-t0));
151       R(t+s-t0+1)=R(t+s-t0)+dt*(gam*I(t+s-t0) ...
152           -v(t+s-t0)*R(t+s-t0));
153
154       S1(t+s-t0+1)=S1(t+s-t0)+dt*(v(t+s-t0)*S(t+s-t0) ...
155           -sig*lam(t+s-t0)*S1(t+s-t0)-w(t+s-t0)*S1(t+s-t0));
156       E1(t+s-t0+1)=E1(t+s-t0)+dt*(v(t+s-t0)*E(t+s-t0) ...
157           +sig*lam(t+s-t0)*S1(t+s-t0)-(eps+w(t+s-t0))*E1(t+s-t0));
158       I1(t+s-t0+1)=I1(t+s-t0)+dt*(v(t+s-t0)*I(t+s-t0) ...
159           +eps*E1(t+s-t0)-(gam+w(t+s-t0))*I1(t+s-t0));
160       R1(t+s-t0+1)=R1(t+s-t0)+dt*(v(t+s-t0)*R(t+s-t0) ...
161           +gam*I1(t+s-t0)-w(t+s-t0)*R1(t+s-t0));
162
163       S2(t+s-t0+1,1)=w(t+s-t0)*S1(t+s-t0)+QS;
164       E2(t+s-t0+1,1)=w(t+s-t0)*E1(t+s-t0)+QE;
165       I2(t+s-t0+1,1)=w(t+s-t0)*I1(t+s-t0)+QI;
166       R2(t+s-t0+1,1)=w(t+s-t0)*R1(t+s-t0)+QR;

```

```

167   for a=2:1:nT
168     S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a) ...
169       -S2(t+s-t0,a-1))/da...
170       -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1));
171     E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a) ...
172       -E2(t+s-t0,a-1))/da...
173       +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1) ...
174       -eps*E2(t+s-t0,a-1));
175     I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a) ...
176       -I2(t+s-t0,a-1))/da...
177       +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1));
178     R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a) ...
179       -R2(t+s-t0,a-1))/da+gam*I2(t+s-t0,a-1));
180   end
181   for a=nT+1:1:na
182     S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a) ...
183       -S2(t+s-t0,a-1))/da...
184       -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1) ...
185       -u(t+s-t0)*S2(t+s-t0,a-1));
186     E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a) ...
187       -E2(t+s-t0,a-1))/da...
188       +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1) ...
189       -eps*E2(t+s-t0,a-1)-u(t+s-t0)*E2(t+s-t0,a-1));
190     I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a) ...
191       -I2(t+s-t0,a-1))/da...
192       +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1) ...
193       -u(t+s-t0)*I2(t+s-t0,a-1));
194     R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a) ...
195       -R2(t+s-t0,a-1))/da...
196       +gam*I2(t+s-t0,a-1)-u(t+s-t0)*R2(t+s-t0,a-1));
197   end
198
199   Y(t+s-t0+1)=del*N*gam*(I(t+s-t0+1)+I1(t+s-t0+1) ...
200     +sum(I2(t+s-t0+1,1:1:na))*da);
201   Z(t+s-t0+1)=(Y(t+s-t0+1)-A(round((t+s-t0+1)*dt),1))^2;
202 end
203 L(b)=sum(Z(t-t0+2:1:t+1));
204 end
205 [L1 L2]=min(L);
206 bet=L2*db;
207
208 for s=1:1:t0+1
209   if B(round((t+s-t0)*dt),2) ≠ 0
210     v(t+s-t0)=B(round((t+s-t0)*dt),2)/((S(t+s-t0)+E(t+s-t0) ...
211       +I(t+s-t0)+R(t+s-t0))*N);
212   else
213     v(t+s-t0)=0;
214   end
215   if B(round((t+s-t0)*dt),3) ≠ 0
216     w(t+s-t0)=B(round((t+s-t0)*dt),3)/((S1(t+s-t0)+E1(t+s-t0) ...
217       +I1(t+s-t0)+R1(t+s-t0))*N);
218   else
219     w(t+s-t0)=0;
220   end
221   if B(round((t+s-t0)*dt),4) ≠ 0
222     u(t+s-t0)=B(round((t+s-t0)*dt),4)/...
223       ((sum(S2(t+s-t0,nT:1:na))+sum(E2(t+s-t0,nT:1:na))...
224       +sum(I2(t+s-t0,nT:1:na))+sum(R2(t+s-t0,nT:1:na)))*da*N);
225   else

```

```

226      u(t+s-t0)=0;
227
228
229      QS=u(t+s-t0)*sum(S2(t+s-t0,nT:1:na-1))*da;
230      QE=u(t+s-t0)*sum(E2(t+s-t0,nT:1:na-1))*da;
231      QI=u(t+s-t0)*sum(I2(t+s-t0,nT:1:na-1))*da;
232      QR=u(t+s-t0)*sum(R2(t+s-t0,nT:1:na-1))*da;
233
234      lam(t+s-t0)=bet*(I(t+s-t0)+I1(t+s-t0)+sum(I2(t+s-t0,1:1:na))*da);
235
236      S(t+s-t0+1)=S(t+s-t0)+dt*(-lam(t+s-t0)*S(t+s-t0)...
237          -v(t+s-t0)*S(t+s-t0));
238      E(t+s-t0+1)=E(t+s-t0)+dt*(lam(t+s-t0)*S(t+s-t0)...
239          -(eps+v(t+s-t0))*E(t+s-t0));
240      I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)...
241          -(gam+v(t+s-t0))*I(t+s-t0));
242      R(t+s-t0+1)=R(t+s-t0)+dt*(gam*I(t+s-t0)-v(t+s-t0)*R(t+s-t0));
243
244      S1(t+s-t0+1)=S1(t+s-t0)+dt*(v(t+s-t0)*S(t+s-t0)...
245          -sig*lam(t+s-t0)*S1(t+s-t0)-w(t+s-t0)*S1(t+s-t0));
246      E1(t+s-t0+1)=E1(t+s-t0)+dt*(v(t+s-t0)*E(t+s-t0)...
247          +sig*lam(t+s-t0)*S1(t+s-t0)-(eps+w(t+s-t0))*E1(t+s-t0));
248      I1(t+s-t0+1)=I1(t+s-t0)+dt*(v(t+s-t0)*I(t+s-t0)...
249          +eps*E1(t+s-t0)-(gam+w(t+s-t0))*I1(t+s-t0));
250      R1(t+s-t0+1)=R1(t+s-t0)+dt*(v(t+s-t0)*R(t+s-t0)...
251          +gam*I1(t+s-t0)-w(t+s-t0)*R1(t+s-t0));
252
253      S2(t+s-t0+1,1)=w(t+s-t0)*S1(t+s-t0)+QS;
254      E2(t+s-t0+1,1)=w(t+s-t0)*E1(t+s-t0)+QE;
255      I2(t+s-t0+1,1)=w(t+s-t0)*I1(t+s-t0)+QI;
256      R2(t+s-t0+1,1)=w(t+s-t0)*R1(t+s-t0)+QR;
257      for a=2:1:nT
258          S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
259              -S2(t+s-t0,a-1))/da...
260              -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1));
261          E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
262              -E2(t+s-t0,a-1))/da...
263              +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
264              -eps*E2(t+s-t0,a-1));
265          I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
266              -I2(t+s-t0,a-1))/da...
267              +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1));
268          R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
269              -R2(t+s-t0,a-1))/da+gam*I2(t+s-t0,a-1));
270      end
271      for a=nT+1:1:na
272          S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
273              -S2(t+s-t0,a-1))/da...
274              -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
275              -u(t+s-t0)*S2(t+s-t0,a-1));
276          E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
277              -E2(t+s-t0,a-1))/da...
278              +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
279              -eps*E2(t+s-t0,a-1)-u(t+s-t0)*E2(t+s-t0,a-1));
280          I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
281              -I2(t+s-t0,a-1))/da+eps*E2(t+s-t0,a-1)...
282              -gam*I2(t+s-t0,a-1)-u(t+s-t0)*I2(t+s-t0,a-1));
283          R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
284              -R2(t+s-t0,a-1))/da...

```

```

285           +gam*I2(t+s-t0,a-1)-u(t+s-t0)*R2(t+s-t0,a-1));
286       end
287
288   Y(t+s-t0+1)=del*N*gam*(I(t+s-t0+1)+II(t+s-t0+1)...
289           +sum(I2(t+s-t0+1,1:1:na))*da);
290   SS(t+s-t0+1)=sum(S2(t+s-t0+1,1:1:na))*da;
291   EE(t+s-t0+1)=sum(E2(t+s-t0+1,1:1:na))*da;
292   II(t+s-t0+1)=sum(I2(t+s-t0+1,1:1:na))*da;
293   RR(t+s-t0+1)=sum(R2(t+s-t0+1,1:1:na))*da;
294       end
295   end
296 end
297
298 T1=1:1:nt*dt;
299 T2=1:1:nt-1;
300 plot(T1,A(T1,1), 'o', T2*dt, Y(T2), '-','Markersize',1.5,'Linewidth',2)
301 newcolors = {'#D95319', '#0072BD'};
302 colororder(newcolors)
303 toc

```