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Appendix to a review article for The Tokyo Foundation for Policy Research

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1. Simulation results

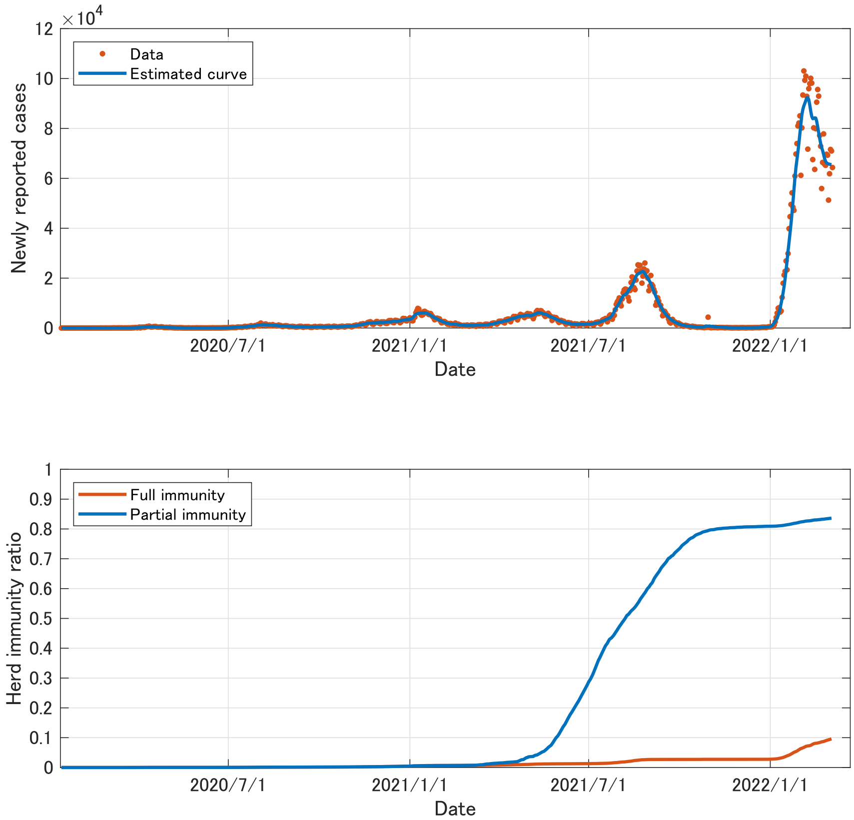

Figure 1: Time variation of newly reported cases (top) and estimated herd immunity ratio (bottom) for COVID-19 in Japan (2020/1/14 - 2022/3/6).

2. Symbols

Parameter	Description	Value
S	Susceptible population (unvaccinated)	-
E	Exposed population (unvaccinated)	-
I	Infectious population (unvaccinated)	-
R	Removed population (unvaccinated)	-
S_1	Susceptible population (vaccinated once)	-
E_1	Exposed population (vaccinated once)	-
I_1	Infectious population (vaccinated once)	-
R_1	Removed population (vaccinated once)	-
S_2	Susceptible population (vaccinated more than twice)	-
E_2	Exposed population (vaccinated more than twice)	-
I_2	Infectious population (vaccinated more than twice)	-
R_2	Removed population (vaccinated more than twice)	-
t	Time	-
a	Class age (time elapsed since the vaccination)	-
β	Infection rate	Estimated using data in [9]
ε	Onset rate	0.2 (incubation period $1/\varepsilon = 5$ days) [4]
γ	Removal rate	0.1 (infection period $1/\gamma = 10$ days) [1]
λ	Force of infection	Equation (1)
$1-\sigma$	Efficacy of one time vaccination	0.46 [6]
v	Vaccination rate (first)	Estimated using data in [7]
w	Vaccination rate (second)	Estimated using data in [7]
$q(a)$	Vaccination rate (third) at class age a	Equation (2)
u	Vaccination rate (third)	Estimated using data in [7]
T	Duration between the second and third vaccination	180 days
$1-p(a)$	Efficacy of full vaccination at class age a	$0.8e^{-0.003a}$ (estimated using data in [6])
δ	Detection rate	0.5 (estimated using data in [5])
N	Total population in Japan	1.26×10^8 [8]

3. Model

Before vaccination (January 14, 2020 - February 16, 2021).

$$\begin{aligned}
&S'(t) = -\beta S(t)I(t),\\
&E'(t) = \beta S(t)I(t) - \varepsilon E(t),\\
&I'(t) = \varepsilon E(t) - \gamma I(t),\\
&R'(t) = \gamma I(t).
\end{aligned}$$

After vaccination (February 17, 2021 - March 6, 2022).

- Unvaccinated population:

$$\begin{aligned}
&S'(t) = -\lambda(t)S(t) - vS(t),\\
&E'(t) = \lambda(t)S(t) - (\varepsilon + v)E(t),\\
&I'(t) = \varepsilon E(t) - (\gamma + v)I(t),\\
&R'(t) = \gamma I(t) - vR(t).
\end{aligned}$$

- Population vaccinated once:

$$\begin{aligned}
&S_1'(t) = vS(t) - \sigma\lambda(t)S_1(t) - wS_1(t),\\
&E_1'(t) = vE(t) + \sigma\lambda(t)S_1(t) - (\varepsilon + w)E_1(t),\\
&I_1'(t) = vI(t) + \varepsilon E_1(t) - (\gamma + w)I_1(t),\\
&R_1'(t) = vR(t) + \gamma I_1(t) - wR_1(t).
\end{aligned}$$

- Population vaccinated more than twice:

$$\begin{aligned}
&S_2(t,0) = wS_1(t) + \int_0^\infty q(a)S_2(t,a)da,\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) S_2(t,a) = -p(a)\lambda(t)S_2(t,a) - q(a)S_2(t,a),\\
&E_2(t,0) = wE_1(t) + \int_0^\infty q(a)E_2(t,a)da,\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) E_2(t,a) = p(a)\lambda(t)S_2(t,a) - [\varepsilon + q(a)]E_2(t,a),\\
&I_2(t,0) = wI_1(t) + \int_0^\infty q(a)I_2(t,a)da,\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) I_2(t,a) = \varepsilon E_2(t,a) - [\gamma + q(a)]I_2(t,a),\\
&R_2(t,0) = wR_1(t) + \int_0^\infty q(a)R_2(t,a)da,\\
&\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) R_2(t,a) = \gamma I_2(t,a) - q(a)R_2(t,a).
\end{aligned}$$

- Force of infection:

$$\lambda(t) = \beta \left[I(t) + I_1(t) + \int_0^\infty I_2(t,a)da \right]. \tag{1}$$

- Vaccination rate (third) at time a passed since the second vaccination:

$$q(a) = \begin{cases} 0, & a < T, \\ u, & \text{otherwise}. \end{cases} \tag{2}$$

- Efficacy of full vaccination at class age a: $1 - p(a) = 0.8e^{-0.003a}$, which is fitted to the data in [6] as shown in Figure 2.

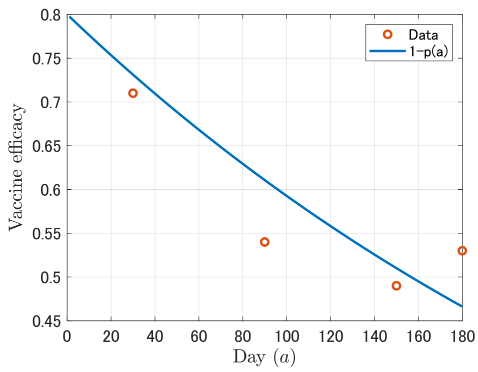

Figure 2:

- Full immunity at time t: $1 - S(t) - S_1(t) - \int_0^\infty S_2(t,a)da$.
- Partial immunity at time t: $1 - S(t)$.

How to estimate $\beta = \beta(t)$

Let the unit time be 1 day. Newly reported cases per day: let $A(t)$ be the actual data collected from [9] and let

$$\begin{aligned} Y(t) :=& (\text{detection rate } \delta) \\ & \times (\text{newly removed } \gamma \left[I(t) + I_1(t) + \int_0^\infty I_2(t,a)da \right]) \\ & \times (\text{total population } N), \end{aligned}$$

be the simulation data. For $t \leq 7$, we assume $\beta = 0.25$ so that the basic reproduction number $\mathcal{R}_0 = \beta/\gamma = 2.5$ [2]. For $t > 7$ ($t \in \mathbb{Z}$), we seek $\beta = \beta^*$ that minimizes the least square error

$$L(\beta) := \sum_{s=1}^{7} [Y(t-s) - A(t-s)]^2$$

and let $\beta(t) = \beta^*$. In other words, we repeatedly estimate $\beta(t)$ using data in previous 1 week. Simulation is carried out using the above model with initial condition

$$Y(0) = 1, \quad I(0) = \frac{Y(0)}{\delta\gamma N}, \quad S(0) = 1 - I(0),$$

and other variables are zero.

How to estimate v, w and u

Note that $v \times [S(t) + E(t) + I(t) + R(t)] \times N$ is the number of first vaccination at time t. Hence, we estimate $v = v(t)$ as

$$v(t) = \frac{(number\ of\ first\ vaccination\ at\ time\ t\ [7])}{[S(t) + E(t) + I(t) + R(t)] \times N}.$$

In a similar manner, we estimate $w = w(t)$ and $u = u(t)$ as

$$w(t) = \frac{(number\ of\ second\ vaccination\ at\ time\ t\ [7])}{[S_1(t) + E_1(t) + I_1(t) + R_1(t)] \times N},$$

and

$$u(t) = \frac{(number\ of\ third\ vaccination\ at\ time\ t\ [7])}{\int_T^\infty [S_2(t,a) + E_2(t,a) + I_2(t,a) + R_2(t,a)] da \times N},$$

respectively.

How to estimate δ

To estimate δ, we used the data in [5]. Taking average of each data in [5], we obtain the herd immunity ratio at June 2020 as 0.1%; at December 2020 as 0.662%; at December 2021 as 2.158%. We estimated $\delta = 0.5$ by fitting the curve to the data as shown in Figure 3.

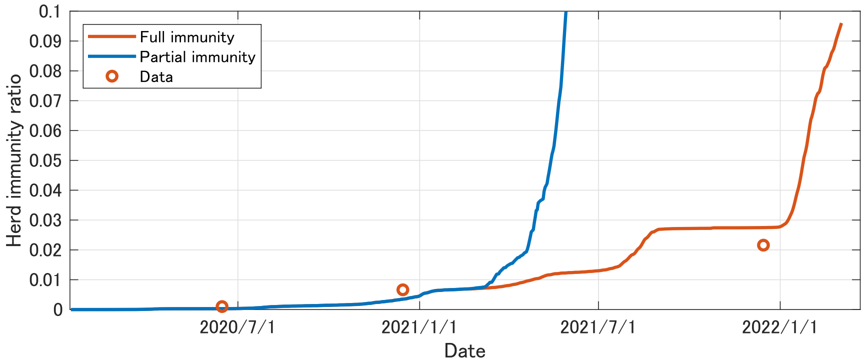

Figure 3: Time variation of herd immunity ratio for COVID-19 in Japan (2020/1/14 - 2022/3/6).

4. Limitations

The following factors are not explicitly considered in our model.

1. Seasonality;
2. Virus mutation;
3. Behavior change;
4. Different susceptibility;
5. Space structure;
6. (Chronological) age structure;
7. Stochasticity.

Since we estimated β for each day, effects of 1-3 on the infection rate may be implicitly considered in our model. However, effect of 2 on the herd immunity is not considered and this may lead to the overestimation of the herd immunity. On the other hand, considering 4 may decrease the critical proportion of immunization [3]. Considering 5-7 could improve the model but the estimation may become more complex.

References

[1] R.M. Anderson, H. Heesterbeek, D. Klinkenberg, T.D. Hollingsworth, How will country-based mitigation measures influence the course of the COVID-19 epidemic?, The Lancet 395 (2020) 21–27.

[2] N. Imai, A. Cori, I. Dorigatti, et al., Report 3: Transmissibility of 2019-nCoV (2020). https://www.imperial.ac.uk/media/imperial-college/medicine/mrc-gida/2020-01-25-COVID19-Report-3.pdf, accessed on March 8, 2022.

[3] 稲葉寿, 感染症数理モデルと COVID-19, https://www.covid19-jma-medical-expert-meeting.jp/topic/3925, accessed on March 8, 2022.

[4] N.M. Linton, T. Kobayashi, Y. Yang, et al., Incubation period and other epidemiological characteristics of 2019 novel coronavirus infections with right truncation: a statistical analysis of publicly available case data, J. Clin. Med. 9 (2020) 538.

[5] m3.com, 自然感染による抗体保有率、東京 2.80%・大阪 3.78%に, https://www.m3.com/news/open/iryoishin/1017268, accessed on March 8, 2022.

[6] NIID 国立感染症研究所, 新型コロナワクチンの有効性を検討した症例対照研究の暫定報告（第三報）, https://www.niid.go.jp/niid/ja/2019-ncov/2484-idsc/10966-covid19-71.html, accessed on March 7, 2022.

[7] Prime Minister of Japan and His Cabinet, https://www.kantei.go.jp/jp/headline/kansensho/vaccine.html, accessed on March 8, 2022.

[8] Statistics Bureau Japan, Population estimates monthly report, https://www.stat.go.jp/english/data/jinsui/tsuki/index.html, accessed on March 7, 2022.

[9] WHO Coronavirus (COVID-19) Dashboard, https://covid19.who.int/, accessed on March 8, 2022.

MATLAB code for main simulation

- 'cases.xlsx' is an excel file collecting the newly reported cases of COVID-19 per day in Japan from 2020/1/14 to 2022/3/6.
- 'vaccine.xlsx' is an excel file collecting the newly number of vaccination against COVID-19 per day in Japan from 2020/1/14 to 2022/3/6.

```
clear
tic

A=xlsread('cases.xlsx');
B=xlsread('vaccine.xlsx');

gam=1/10;
eps=1/5;
N=1.26*10^8;
del=0.5;
sig=0.54;

pp=0.003;

dt=1;te=max(size(A))-1;nt=te/dt;
T=180;nT=T/dt;
da=1;ae=4*T;na=ae/da;

Y(1)=1;
I(1)=Y(1)/(del*N*gam);
S(1)=1-I(1);
E(1)=0;
R(1)=0;
S1(1)=0;
E1(1)=0;
I1(1)=0;
R1(1)=0;

for a=1:1:na
    S2(1,a)=0;
    E2(1,a)=0;
    I2(1,a)=0;
    R2(1,a)=0;
end
SS(1)=sum(S2(1,1:1:na))*da;
EE(1)=sum(E2(1,1:1:na))*da;
II(1)=sum(I2(1,1:1:na))*da;
RR(1)=sum(R2(1,1:1:na))*da;

db=0.01;be=4;nb=be/db;

t0=7/dt;
t1=401/dt; %2021/2/17 start of vaccination

for t=1:1:nt
    if B(round(t*dt),2)==0
        v(t)=0;
    end
```

```
    if B(round(t*dt),3)==0
        w(t)=0;
    end
    if B(round(t*dt),4)==0
        u(t)=0;
    end

    if t < t0
        bet=0.25;
        S(t+1)=S(t)+dt*(-bet*S(t)*I(t));
        E(t+1)=E(t)+dt*(bet*S(t)*I(t)-eps*E(t));
        I(t+1)=I(t)+dt*(eps*E(t)-gam*I(t));
        R(t+1)=R(t)+dt*gam*I(t);
        Y(t+1)=del*N*gam*I(t+1);
        S1(t+1)=0;
        E1(t+1)=0;
        I1(t+1)=0;
        R1(t+1)=0;
        for a=1:1:na
            S2(t+1,a)=0;
            E2(t+1,a)=0;
            I2(t+1,a)=0;
            R2(t+1,a)=0;
        end
    elseif t<t1
        for b=1:1:nb
            bet=b*db;
            for s=1:1:t0
               S(t+s-t0+1)=S(t+s-t0)+dt*(-bet*S(t+s-t0)*I(t+s-t0));
               E(t+s-t0+1)=E(t+s-t0)+dt*(bet*S(t+s-t0)*I(t+s-t0)...
                   -eps*E(t+s-t0));
               I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)-gam*I(t+s-t0));
               R(t+s-t0+1)=R(t+s-t0)+dt*gam*I(t+s-t0);
               Y(t+s-t0+1)=del*N*gam*I(t+s-t0+1);
               Z(t+s-t0+1)=(Y(t+s-t0+1)-A(round((t+s-t0+1)*dt),1))^2;
            end
            L(b)=sum(Z(t-t0+2:1:t+1));
        end
        [L1 L2]=min(L);
        bet=L2*db;

        for s=1:1:t0+1
            S(t+s-t0+1)=S(t+s-t0)+dt*(-bet*S(t+s-t0)*I(t+s-t0));
            E(t+s-t0+1)=E(t+s-t0)+dt*(bet*S(t+s-t0)*I(t+s-t0)...
                -eps*E(t+s-t0));
            I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)-gam*I(t+s-t0));
            R(t+s-t0+1)=R(t+s-t0)+dt*gam*I(t+s-t0);
            Y(t+s-t0+1)=del*N*gam*I(t+s-t0+1);
            S1(t+s-t0+1)=0;
            E1(t+s-t0+1)=0;
            I1(t+s-t0+1)=0;
            R1(t+s-t0+1)=0;
            for a=1:1:na
                S2(t+s-t0+1,a)=0;
                E2(t+s-t0+1,a)=0;
                I2(t+s-t0+1,a)=0;
                R2(t+s-t0+1,a)=0;
            end
            SS(t+s-t0+1)=sum(S2(t+s-t0+1,1:1:na))*da;
```

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            EE(t+s-t0+1)=sum(E2(t+s-t0+1,1:1:na))*da;
            II(t+s-t0+1)=sum(I2(t+s-t0+1,1:1:na))*da;
            RR(t+s-t0+1)=sum(R2(t+s-t0+1,1:1:na))*da;
        end
    else
        for b=1:1:nb
            bet=b*db;
            for s=1:1:t0
                if B(round((t+s-t0)*dt),2) ≠ 0
                    v(t+s-t0)=B(round((t+s-t0)*dt),2)/((S(t+s-t0)...
                        +E(t+s-t0)+I(t+s-t0)+R(t+s-t0))*N);
                else
                    v(t+s-t0)=0;
                end
                if B(round((t+s-t0)*dt),3) ≠ 0
                    w(t+s-t0)=B(round((t+s-t0)*dt),3)/((S1(t+s-t0)...
                        +E1(t+s-t0)+I1(t+s-t0)+R1(t+s-t0))*N);
                else
                    w(t+s-t0)=0;
                end
                if B(round((t+s-t0)*dt),4) ≠ 0
                    u(t+s-t0)=B(round((t+s-t0)*dt),4)/...
                        ((sum(S2(t+s-t0,nT:1:na))+sum(E2(t+s-t0,nT:1:na))...
                        +sum(I2(t+s-t0,nT:1:na))...
                        +sum(R2(t+s-t0,nT:1:na)))*da*N);
                else
                    u(t+s-t0)=0;
                end

                QS=u(t+s-t0)*sum(S2(t+s-t0,nT:1:na-1))*da;
                QE=u(t+s-t0)*sum(E2(t+s-t0,nT:1:na-1))*da;
                QI=u(t+s-t0)*sum(I2(t+s-t0,nT:1:na-1))*da;
                QR=u(t+s-t0)*sum(R2(t+s-t0,nT:1:na-1))*da;

                lam(t+s-t0)=bet*(I(t+s-t0)+I1(t+s-t0)...
                    +sum(I2(t+s-t0,1:1:na))*da);

                S(t+s-t0+1)=S(t+s-t0)+dt*(-lam(t+s-t0)*S(t+s-t0)...
                    -v(t+s-t0)*S(t+s-t0));
                E(t+s-t0+1)=E(t+s-t0)+dt*(lam(t+s-t0)*S(t+s-t0)...
                    -(eps+v(t+s-t0))*E(t+s-t0));
                I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)...
                    -(gam+v(t+s-t0))*I(t+s-t0));
                R(t+s-t0+1)=R(t+s-t0)+dt*(gam*I(t+s-t0)...
                    -v(t+s-t0)*R(t+s-t0));

                S1(t+s-t0+1)=S1(t+s-t0)+dt*(v(t+s-t0)*S(t+s-t0)...
                    -sig*lam(t+s-t0)*S1(t+s-t0)-w(t+s-t0)*S1(t+s-t0));
                E1(t+s-t0+1)=E1(t+s-t0)+dt*(v(t+s-t0)*E(t+s-t0)...
                    +sig*lam(t+s-t0)*S1(t+s-t0)-(eps+w(t+s-t0))*E1(t+s-t0));
                I1(t+s-t0+1)=I1(t+s-t0)+dt*(v(t+s-t0)*I(t+s-t0)...
                    +eps*E1(t+s-t0)-(gam+w(t+s-t0))*I1(t+s-t0));
                R1(t+s-t0+1)=R1(t+s-t0)+dt*(v(t+s-t0)*R(t+s-t0)...
                    +gam*I1(t+s-t0)-w(t+s-t0)*R1(t+s-t0));

                S2(t+s-t0+1,1)=w(t+s-t0)*S1(t+s-t0)+QS;
                E2(t+s-t0+1,1)=w(t+s-t0)*E1(t+s-t0)+QE;
                I2(t+s-t0+1,1)=w(t+s-t0)*I1(t+s-t0)+QI;
                R2(t+s-t0+1,1)=w(t+s-t0)*R1(t+s-t0)+QR;
```

```
                for a=2:1:nT
                    S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
                        -S2(t+s-t0,a-1))/da...
                        -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1));
                    E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
                        -E2(t+s-t0,a-1))/da...
                        +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                        -eps*E2(t+s-t0,a-1));
                    I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
                        -I2(t+s-t0,a-1))/da...
                        +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1));
                    R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
                        -R2(t+s-t0,a-1))/da+gam*I2(t+s-t0,a-1));
                end
                for a=nT+1:1:na
                    S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
                        -S2(t+s-t0,a-1))/da...
                        -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                        -u(t+s-t0)*S2(t+s-t0,a-1));
                    E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
                        -E2(t+s-t0,a-1))/da...
                        +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                        -eps*E2(t+s-t0,a-1)-u(t+s-t0)*E2(t+s-t0,a-1));
                    I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
                        -I2(t+s-t0,a-1))/da...
                        +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1)...
                        -u(t+s-t0)*I2(t+s-t0,a-1));
                    R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
                        -R2(t+s-t0,a-1))/da...
                        +gam*I2(t+s-t0,a-1)-u(t+s-t0)*R2(t+s-t0,a-1));
                end

                Y(t+s-t0+1)=del*N*gam*(I(t+s-t0+1)+I1(t+s-t0+1)...
                    +sum(I2(t+s-t0+1,1:1:na))*da);
                Z(t+s-t0+1)=(Y(t+s-t0+1)-A(round((t+s-t0+1)*dt),1))^2;
            end
            L(b)=sum(Z(t-t0+2:1:t+1));
        end
        [L1 L2]=min(L);
        bet=L2*db;

        for s=1:1:t0+1
            if B(round((t+s-t0)*dt),2) ≠ 0
                v(t+s-t0)=B(round((t+s-t0)*dt),2)/((S(t+s-t0)+E(t+s-t0)...
                    +I(t+s-t0)+R(t+s-t0))*N);
            else
                v(t+s-t0)=0;
            end
            if B(round((t+s-t0)*dt),3) ≠ 0
                w(t+s-t0)=B(round((t+s-t0)*dt),3)/((S1(t+s-t0)+E1(t+s-t0)...
                    +I1(t+s-t0)+R1(t+s-t0))*N);
            else
                w(t+s-t0)=0;
            end
            if B(round((t+s-t0)*dt),4) ≠ 0
                u(t+s-t0)=B(round((t+s-t0)*dt),4)/...
                    ((sum(S2(t+s-t0,nT:1:na))+sum(E2(t+s-t0,nT:1:na))...
                    +sum(I2(t+s-t0,nT:1:na))+sum(R2(t+s-t0,nT:1:na)))*da*N);
            else
```

```
            u(t+s-t0)=0;
        end

        QS=u(t+s-t0)*sum(S2(t+s-t0,nT:1:na-1))*da;
        QE=u(t+s-t0)*sum(E2(t+s-t0,nT:1:na-1))*da;
        QI=u(t+s-t0)*sum(I2(t+s-t0,nT:1:na-1))*da;
        QR=u(t+s-t0)*sum(R2(t+s-t0,nT:1:na-1))*da;

        lam(t+s-t0)=bet*(I(t+s-t0)+I1(t+s-t0)+sum(I2(t+s-t0,1:1:na))*da);

        S(t+s-t0+1)=S(t+s-t0)+dt*(-lam(t+s-t0)*S(t+s-t0)...
            -v(t+s-t0)*S(t+s-t0));
        E(t+s-t0+1)=E(t+s-t0)+dt*(lam(t+s-t0)*S(t+s-t0)...
            -(eps+v(t+s-t0))*E(t+s-t0));
        I(t+s-t0+1)=I(t+s-t0)+dt*(eps*E(t+s-t0)...
            -(gam+v(t+s-t0))*I(t+s-t0));
        R(t+s-t0+1)=R(t+s-t0)+dt*(gam*I(t+s-t0)-v(t+s-t0)*R(t+s-t0));

        S1(t+s-t0+1)=S1(t+s-t0)+dt*(v(t+s-t0)*S(t+s-t0)...
            -sig*lam(t+s-t0)*S1(t+s-t0)-w(t+s-t0)*S1(t+s-t0));
        E1(t+s-t0+1)=E1(t+s-t0)+dt*(v(t+s-t0)*E(t+s-t0)...
            +sig*lam(t+s-t0)*S1(t+s-t0)-(eps+w(t+s-t0))*E1(t+s-t0));
        I1(t+s-t0+1)=I1(t+s-t0)+dt*(v(t+s-t0)*I(t+s-t0)...
            +eps*E1(t+s-t0)-(gam+w(t+s-t0))*I1(t+s-t0));
        R1(t+s-t0+1)=R1(t+s-t0)+dt*(v(t+s-t0)*R(t+s-t0)...
            +gam*I1(t+s-t0)-w(t+s-t0)*R1(t+s-t0));

        S2(t+s-t0+1,1)=w(t+s-t0)*S1(t+s-t0)+QS;
        E2(t+s-t0+1,1)=w(t+s-t0)*E1(t+s-t0)+QE;
        I2(t+s-t0+1,1)=w(t+s-t0)*I1(t+s-t0)+QI;
        R2(t+s-t0+1,1)=w(t+s-t0)*R1(t+s-t0)+QR;
        for a=2:1:nT
            S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
                -S2(t+s-t0,a-1))/da...
                -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1));
            E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
                -E2(t+s-t0,a-1))/da...
                +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                -eps*E2(t+s-t0,a-1));
            I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
                -I2(t+s-t0,a-1))/da...
                +eps*E2(t+s-t0,a-1)-gam*I2(t+s-t0,a-1));
            R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
                -R2(t+s-t0,a-1))/da+gam*I2(t+s-t0,a-1));
        end
        for a=nT+1:1:na
            S2(t+s-t0+1,a)=S2(t+s-t0,a)+dt*(-(S2(t+s-t0,a)...
                -S2(t+s-t0,a-1))/da...
                -p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                -u(t+s-t0)*S2(t+s-t0,a-1));
            E2(t+s-t0+1,a)=E2(t+s-t0,a)+dt*(-(E2(t+s-t0,a)...
                -E2(t+s-t0,a-1))/da...
                +p(pp,(a-1)*da)*lam(t+s-t0)*S2(t+s-t0,a-1)...
                -eps*E2(t+s-t0,a-1)-u(t+s-t0)*E2(t+s-t0,a-1));
            I2(t+s-t0+1,a)=I2(t+s-t0,a)+dt*(-(I2(t+s-t0,a)...
                -I2(t+s-t0,a-1))/da+eps*E2(t+s-t0,a-1)...
                -gam*I2(t+s-t0,a-1)-u(t+s-t0)*I2(t+s-t0,a-1));
            R2(t+s-t0+1,a)=R2(t+s-t0,a)+dt*(-(R2(t+s-t0,a)...
                -R2(t+s-t0,a-1))/da...
```

```
                    +gam*I2(t+s-t0,a-1)-u(t+s-t0)*R2(t+s-t0,a-1));
            end

            Y(t+s-t0+1)=del*N*gam*(I(t+s-t0+1)+I1(t+s-t0+1)...
                +sum(I2(t+s-t0+1,1:1:na))*da);
            SS(t+s-t0+1)=sum(S2(t+s-t0+1,1:1:na))*da;
            EE(t+s-t0+1)=sum(E2(t+s-t0+1,1:1:na))*da;
            II(t+s-t0+1)=sum(I2(t+s-t0+1,1:1:na))*da;
            RR(t+s-t0+1)=sum(R2(t+s-t0+1,1:1:na))*da;
        end
    end
end

T1=1:1:nt*dt;
T2=1:1:nt-1;
plot(T1,A(T1,1),'o',T2*dt,Y(T2),'-','Markersize',1.5,'Linewidth',2)
newcolors = {'#D95319','#0072BD'};
colororder(newcolors)
toc
```