Footstep and Timing Adaptation for Humanoid Robots Utilizing Pre-computation of Capture Regions

Yuichi Tazaki (Kobe University)

Existing studies on robust walking and balance recovery

- ✔ **Footstep and timing adaptation** **[Khadiv et al. 2016][Yamamoto et al. 2020]**
- ✔ **Center-of-mass height variation** **[Caron et al. 2018]**
- ✔ **Angular momentum variation** **[Park et al. 2020]**

Multi-step v.s. single-step lookahead

- **Multi-step lookahead is clearly beneficial for expanding the stabilizable basin**
- **Real-time implementation without oversimplification is still an open issue**

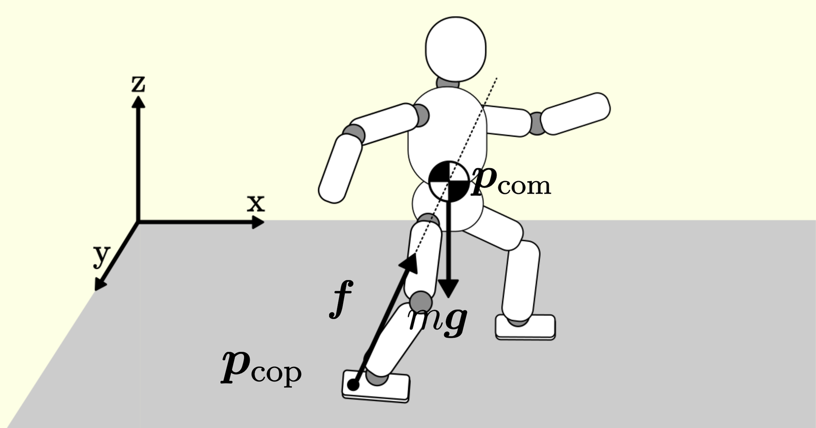
Our proposal

- ✔ **Fall-avoidance control based on multi-step lookahead while retaining sufficient level-of-detail of kinematic and dynamic information**
- ✔ **Precomputation of N-step capture basin**
- ✔ **Generation of fall-avoiding movement based on multi-step lookahead with small-enough computation cost for real-time use**

LOW-DIMENSIONAL DYNAMICAL MODEL FOR CAPTURABILITY ANALYSIS

Our analysis is based on the following **2D (x, y) centroidal dynamics**:

$$\dot{\boldsymbol{p}}_{\rm icp} = \frac{1}{T}(\boldsymbol{p}_{\rm icp} - \boldsymbol{p}_{\rm cop}),$$
$$\dot{\boldsymbol{p}}_{\rm com} = -\frac{1}{T}(\boldsymbol{p}_{\rm com} - \boldsymbol{p}_{\rm icp})$$



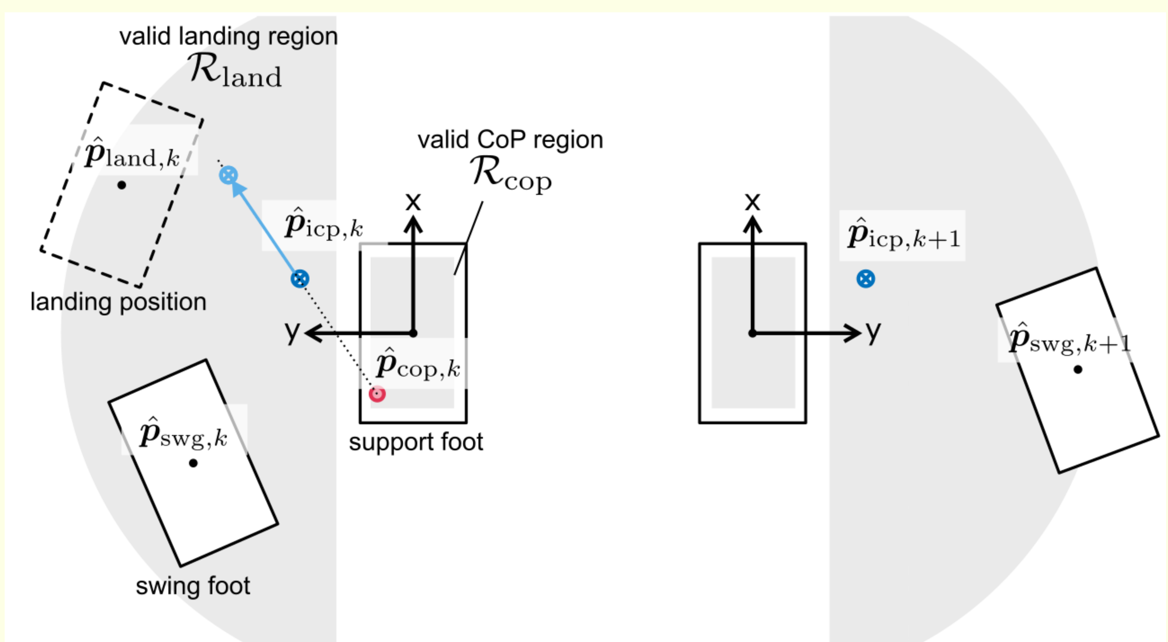
$\boldsymbol{p}_{\rm com}$ Center-of-mass

$\boldsymbol{p}_{\rm cop}$ Center-of-pressure (aka ZMP)

$\boldsymbol{p}_{\rm icp} = \boldsymbol{p}_{\rm com} + T\dot{\boldsymbol{p}}_{\rm com}$ Instantaneous capture point (aka DCM)

One-step dynamics expressed in the support-foot local coordinate

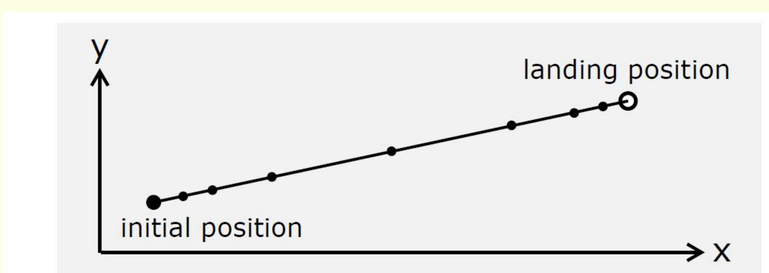
$$\hat{\boldsymbol{p}}'_{\rm swg} = -SR(\hat{\theta}_{\rm land})^{\mathsf{T}}\hat{\boldsymbol{p}}_{\rm land}$$
$$\hat{\theta}'_{\rm swg} = \hat{\theta}_{\rm land}$$
$$\hat{\boldsymbol{p}}'_{\rm icp} = SR(\hat{\theta}_{\rm land})^{\mathsf{T}}(e^{\frac{\tau}{T}}(\hat{\boldsymbol{p}}_{\rm icp} - \hat{\boldsymbol{p}}_{\rm cop}) + \hat{\boldsymbol{p}}_{\rm cop} - \hat{\boldsymbol{p}}_{\rm land})$$



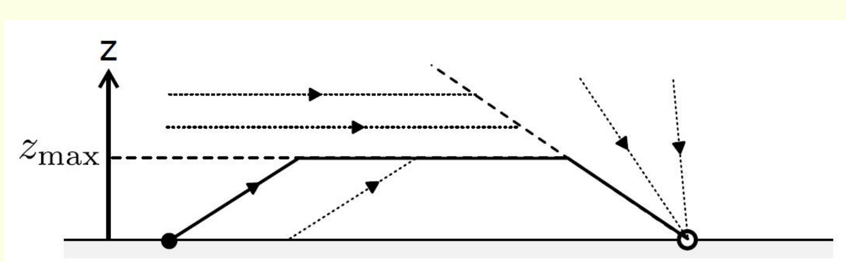
Step duration is lower-bounded by the travel distance of the swing foot (cannot step too quick)

$$\tau_k \geq \frac{3}{2}\max\left(\frac{\|\hat{\boldsymbol{p}}_{{\rm swg},k} - \hat{\boldsymbol{p}}_{{\rm land},k}\|}{v_{\max}}, \frac{|\hat{\theta}_{{\rm swg},k} - \hat{\theta}_{{\rm land},k}|}{\omega_{\max}}\right)$$

Horizontal movement is cubic spline



Vertical movement is parametrized by the horizontal movment



CAPTURABILITY ANALYSIS METHOD BASED ON DISCRETIZATION OF STATES

Variables

$$\boldsymbol{x} = \begin{bmatrix}\hat{\boldsymbol{q}}_{\rm swg} \\ \hat{\boldsymbol{p}}_{\rm icp}\end{bmatrix}$$

$\hat{\boldsymbol{q}}_{\rm swg}$ .... Swing-foot pose (x, y, and angle)

$\hat{\boldsymbol{p}}_{\rm icp}$ .... ICP position (x, y)

- Both expressed in the support-foot local coordinate

τ .... Step duration

Definition of N-step viable capture basin

0-step

$$\mathcal{P}_0 = \mathcal{R}_{\rm swg} \times \mathcal{R}_{\rm cop}$$

($\mathcal{R}_{\rm swg}$: Swing foot inside allowable range; $\mathcal{R}_{\rm cop}$: ICP inside support region)

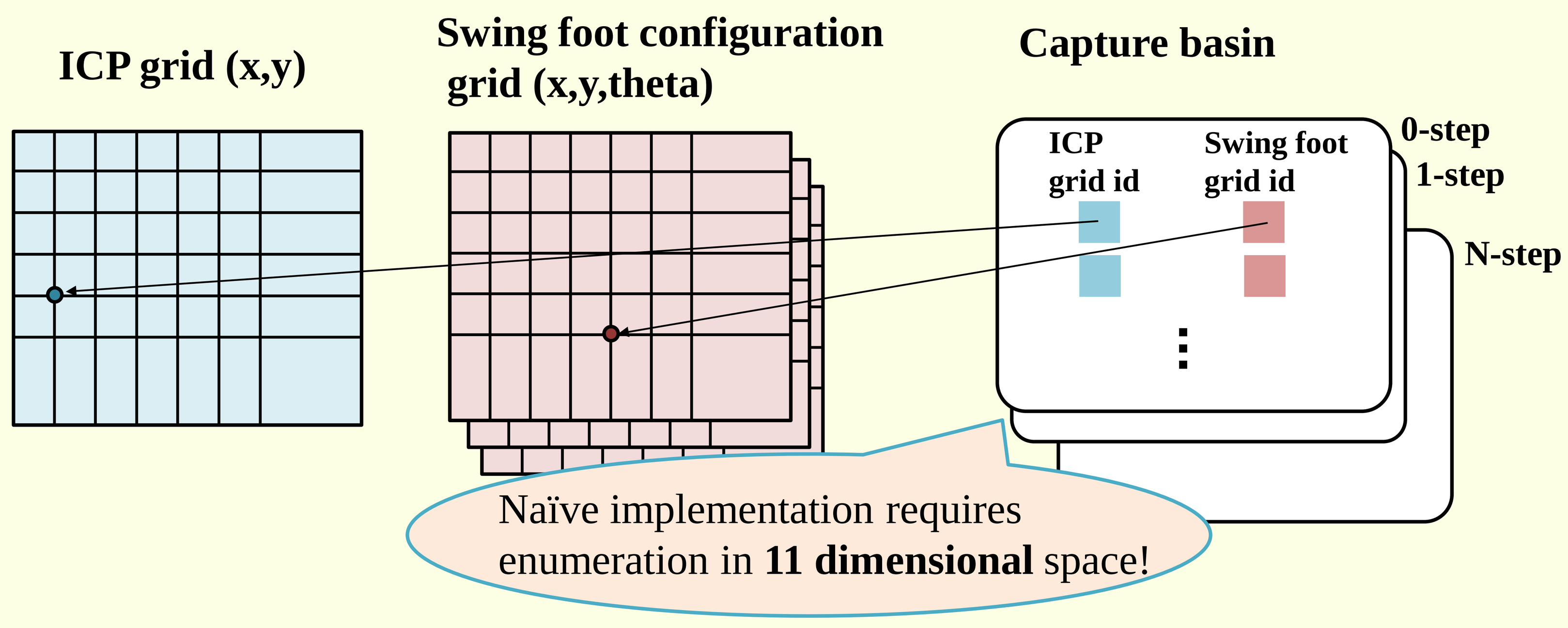
N-step (recursive definition)

$$\mathcal{P}_N = \left\{\boldsymbol{x} \,\middle|\, \boldsymbol{x} \notin \bigcup_{k=0}^{N-1}\mathcal{P}_k, \; \exists\tau, \boldsymbol{x}' \text{ s.t. } (\boldsymbol{x}, \tau, \boldsymbol{x}') \in \mathcal{F}, \boldsymbol{x}' \in \mathcal{P}_{N-1}\right\}$$

Not already included in 0 to (N-1)-step basins

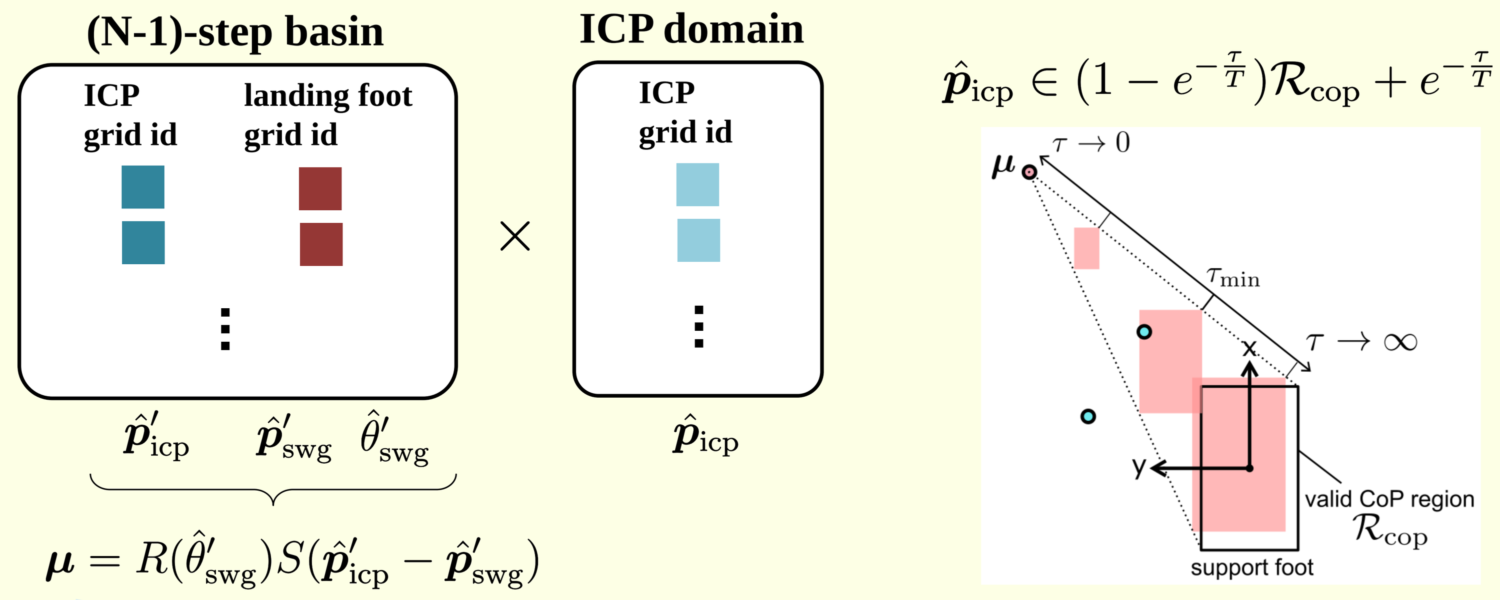
Existence of a feasible transition to the (N-1)-step basin

Capture basin representation based on the discretization of space



Efficient 2-phase computation method

Phase 1: For each possible value of ICP, enumerate all combinations of minimum step duration and landing configuration that leads to (N-1)-step basin. (enumeration in 7D space)



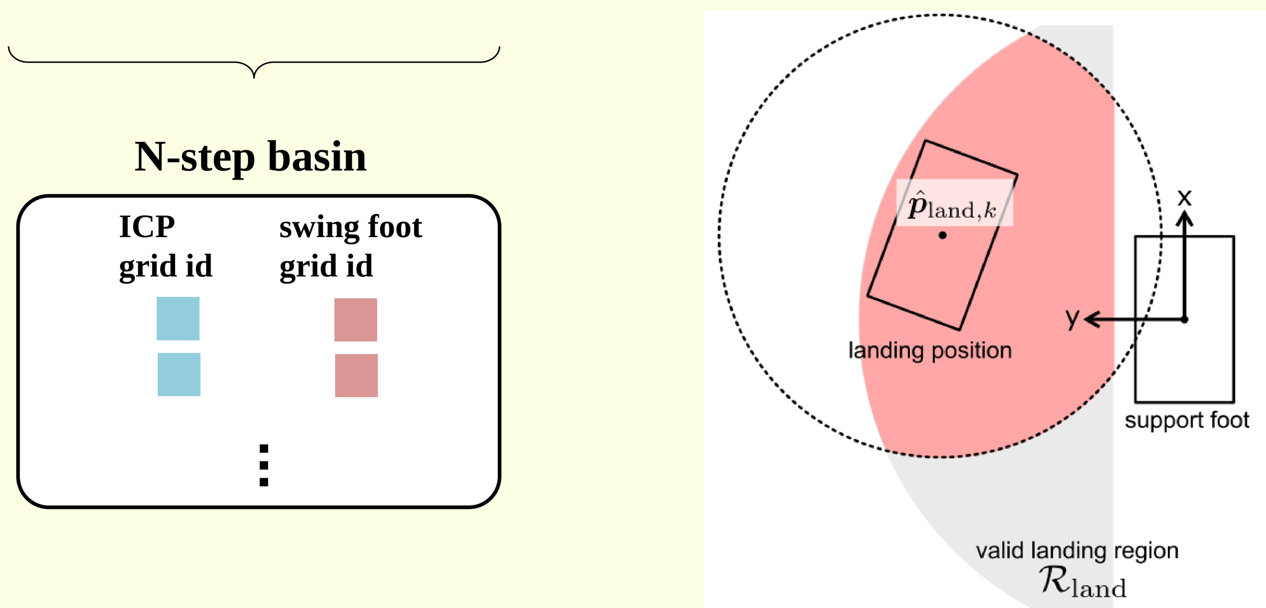
$$\boldsymbol{\mu} = R(\hat{\theta}'_{\rm swg})S(\hat{\boldsymbol{p}}'_{\rm icp} - \hat{\boldsymbol{p}}'_{\rm swg})$$

Position of ICP at landing (expressed in the current support-foot coordinate)

The relationship between $\hat{\boldsymbol{p}}_{\rm icp}$ and $\boldsymbol{\mu}$ determines a lower-bound on the step duration

ICP grid id	landing foot grid id	Minimum step duration
		0.31
		0.42
⋮	⋮	

Phase 2: For each tuple enumerated in Phase 1, enumerate all possible swing-foot configurations that are reachable to the landing configuration within the miminum step duration.



STEP ADAPTATION THAT UTILIZES PRE-COMPUTED CAPTURE BASINS

Input: Current state $\boldsymbol{x}(t)$
Time-to-landing τ
Planned next state $\boldsymbol{x}'$

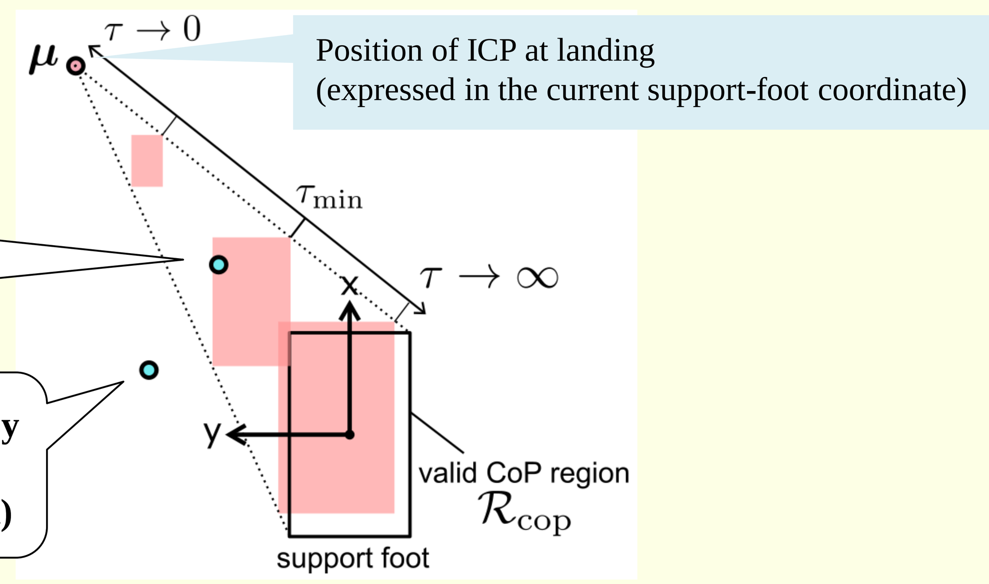
Is $\boldsymbol{x}(t)$ reachable to $\boldsymbol{x}'$ without timing adjustment?

- YES → $(\tau, \boldsymbol{x}')$
- NO → Reachable with timing adjustment only?
  - YES → Adjust timing → $(\tilde{\tau}, \boldsymbol{x}')$
  - NO → Find modified target state and timing
    Note: only this step uses precomputed capture basins
    - → $(\tilde{\tau}, \tilde{\boldsymbol{x}}')$
    - No capturable state found → ERROR!

How to adjust timing

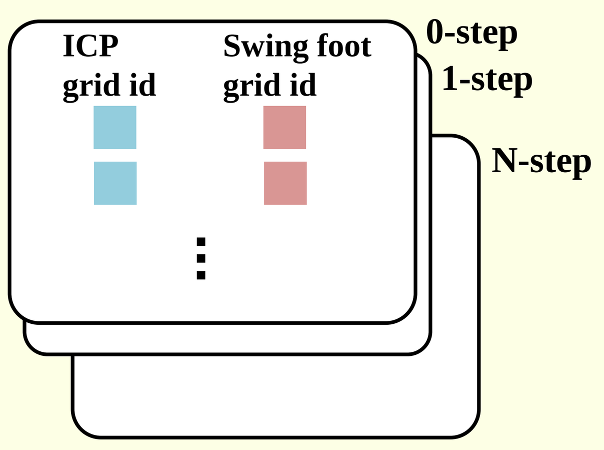
This ICP is capturable by timing adjustment, provided that the swing-foot can reach the landing position in time.

This ICP is uncapturable by timing adjustment alone (i.e., needs step adjustment)



How to modify the target state

Search the capture basin database for a next target state that is reachable from the current state and minimizes the following cost function.



$$J(\tilde{\boldsymbol{x}}', \tilde{\tau}') = w_{\mathrm{swg}} \|\tilde{\hat{\boldsymbol{q}}}'_{\mathrm{swg}} - \hat{\boldsymbol{q}}'_{\mathrm{swg}}\| + w_{\mathrm{icp}} \|\tilde{\hat{\boldsymbol{p}}}'_{\mathrm{icp}} - \hat{\boldsymbol{p}}'_{\mathrm{icp}}\| + w_{\tau} |\tilde{\tau} - \tau|$$

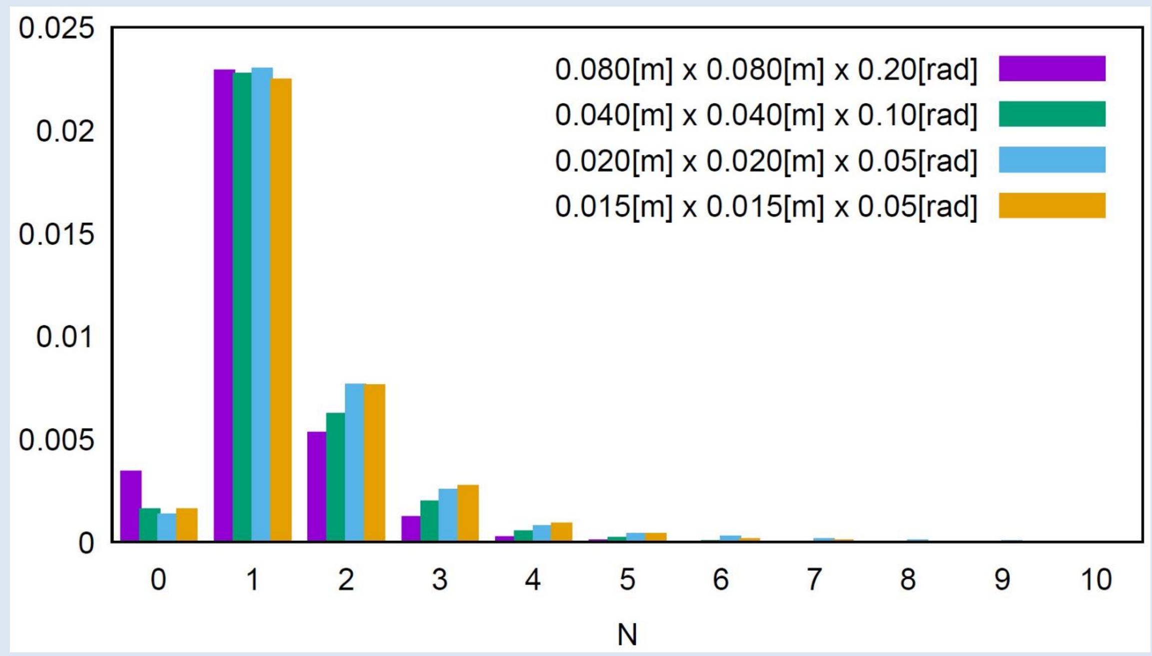
Error between modified and desired landing configuration

Error between modified and desired ICP at landing

Error between modified and desired step duration

CAPTURE BASIN COMPUTATION RESULTS

Volume of capture basins

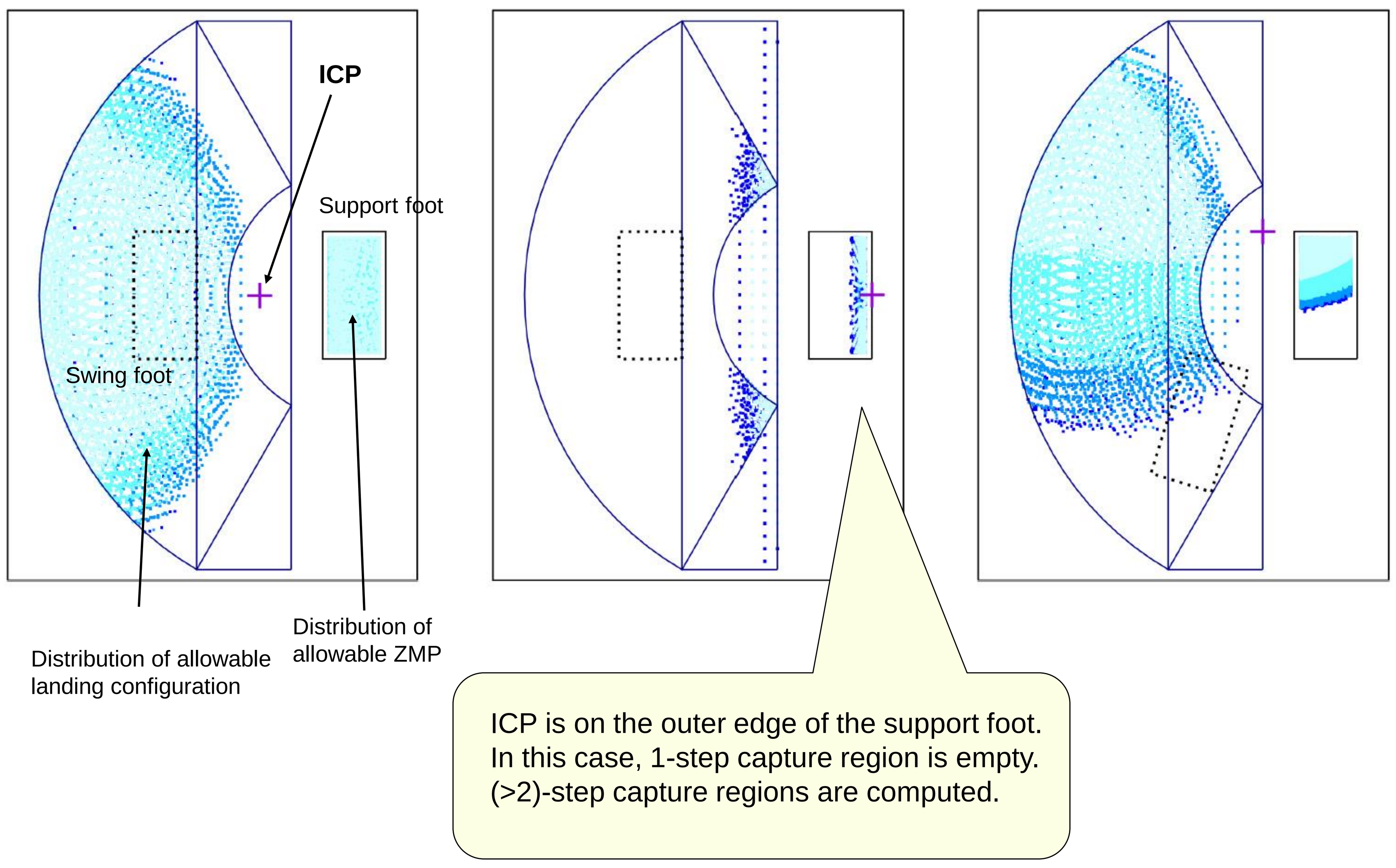


Difference in volume between ■ and ■ is negligible.
0.02[m] x 0.02[m] x 0.05[rad] is considered to have high enough resolution.

N		0	1	2	3	4	5	6	7	8
Data size [KB]		1,932	33,648	11,166	3,697	1,113	534	369	202	78
Computation time [ms]	phase 1	19	1,144	20,321	8,251	1,858	950	179	288	46
	phase 2		3,576	4,754	5,261	4,645	4,431	3,750	3,601	2,540

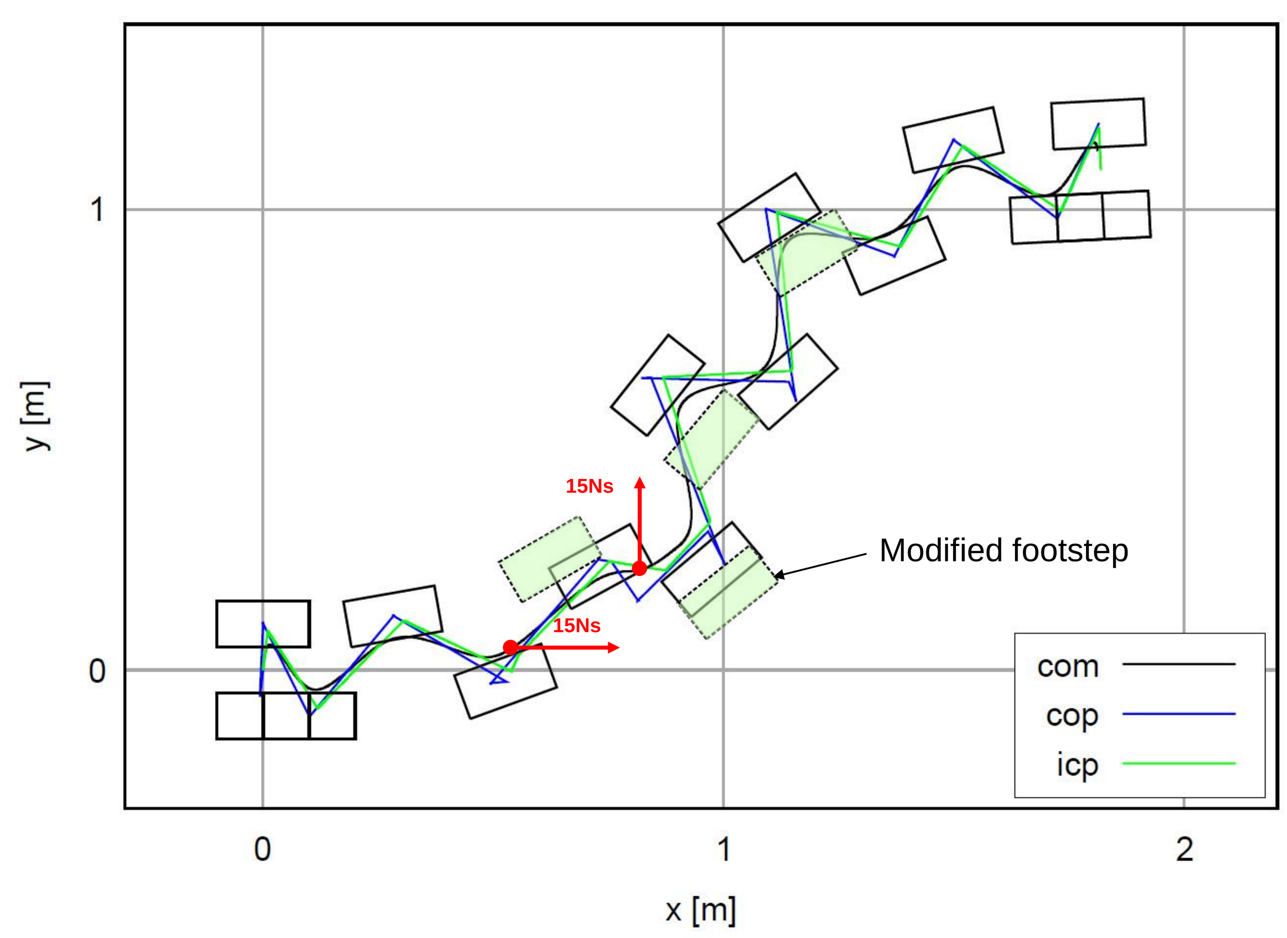
*AMD Ryzen 9 5950X 3.4GHz, single-core implementation

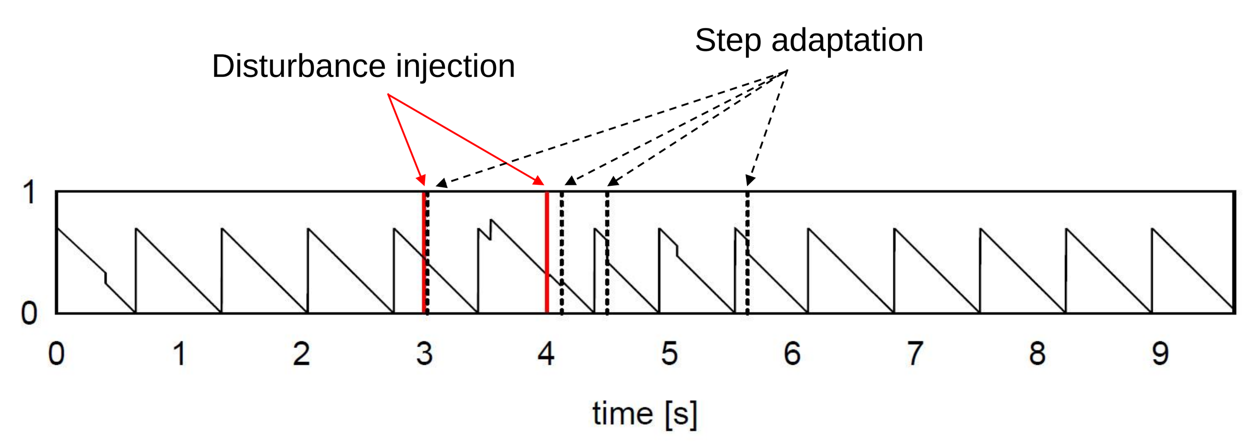
Visualization of N-step capturable regions



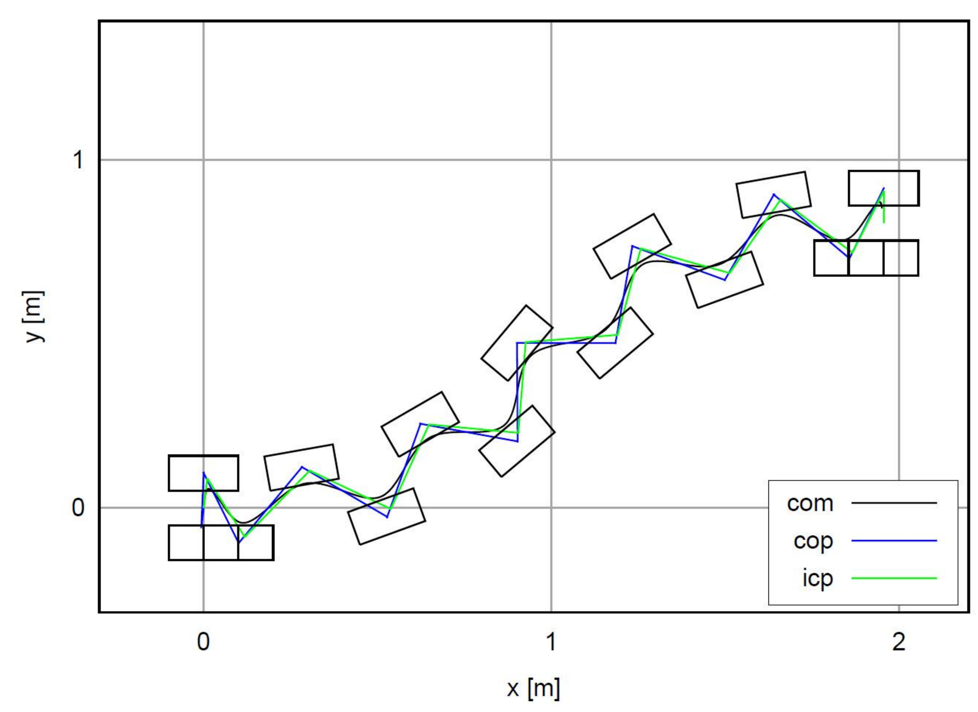
SIMULATION RESULTS

Impulsive disturbances during walking





Walking without distuabances



Run-time computation cost	
No adaptation	< 1 [us]
Timing adaptation only	< 1 [us]
Step & timing adaptation	15 to 30 [ms]

*AMD Ryzen 9 5950X 3.4GHz, single-core implementation

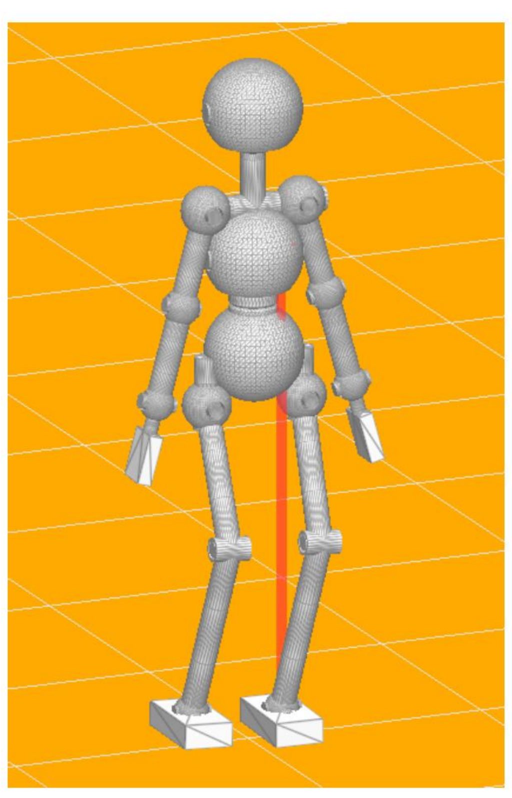
SIMULATION MODEL

Humanoid robot model used for simualtion

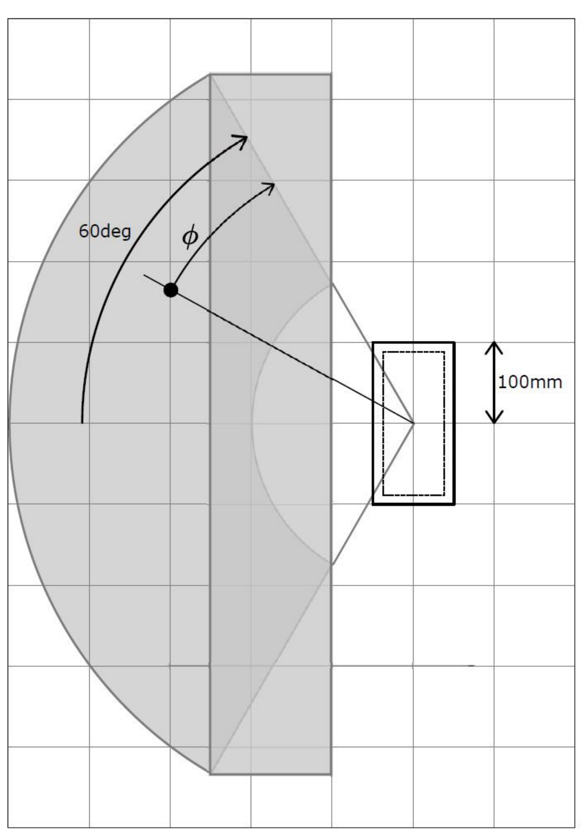
Multi-body model, 31 links, 30 joints

Head:	2 joints (not used)
Body:	2 joints (not used)
Arm:	7 joints (for additional arm swinging)
Leg:	6 joints (for walking)

Robot model



Feasible landing region



Acknowledgement This research was supported by Nagamori Foundation research grant..