Regression-Based Mixed Frequency Granger Causality Tests

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Aug. 21, 2015
Consider Granger causality from a high frequency variable $x_H$ to a low frequency variable $x_L$.

We have $m$ observations of $x_H$ in each low frequency time period $\tau_L$. 
Introduction

- Ghysels, Hill, and Motegi’s (2015, WP) mixed frequency Granger causality tests are a type of Wald tests:

\[ x_L(\tau_L) = \sum_{k=1}^{q} \alpha_k x_L(\tau_L - k) + \beta_1 x_H(\tau_L - 1, m + 1 - 1) + \cdots + \beta_h x_H(\tau_L - 1, m + 1 - h) + u(\tau_L). \]

You often need so many lags here!

- **How can we avoid poor statistical inference due to parameter proliferation?**
- We propose a new test called the **max test**.
- The max test achieves higher power in finite sample than the Wald test.
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Methodology: Max Tests

\[ \textbf{Model 1} \quad x_L(\tau_L) = \sum_{k=1}^{q} \alpha_{k,1} x_L(\tau_L - k) + \beta_1 x_H(\tau_L - 1, m + 1 - 1) + u_1(\tau_L). \]

\[ \textbf{Model 2} \quad x_L(\tau_L) = \sum_{k=1}^{q} \alpha_{k,2} x_L(\tau_L - k) + \beta_2 x_H(\tau_L - 1, m + 1 - 2) + u_2(\tau_L). \]

\[ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \]

\[ \textbf{Model } h \quad x_L(\tau_L) = \sum_{k=1}^{q} \alpha_{k,h} x_L(\tau_L - k) + \beta_h x_H(\tau_L - 1, m + 1 - h) + u_h(\tau_L). \]

Fit OLS for each model. Under \( H_0 : x_H \rightarrow x_L, \) all of \( \hat{\beta}_1, \ldots, \hat{\beta}_h \) converges to 0 in probability. This implies that \( \max\{\hat{\beta}_1^2, \ldots, \hat{\beta}_h^2\} \xrightarrow{P} 0. \) Using this property, we propose the \textbf{max test statistic}:

\[ T = \max\{ \sqrt{T_L} w_{T_L,1} \hat{\beta}_1^2, \ldots, \sqrt{T_L} w_{T_L,h} \hat{\beta}_h^2 \}, \]

where \( \{w_{T_L,1}, \ldots, w_{T_L,h}\} \) is a weighting scheme (typically equal weights).
Methodology: Main Theorems

- The limit distribution of the max test statistic $\mathcal{T}$ under $H_0: x_H \rightarrow x_L$ is non-standard but easy to simulate:

  $$\mathcal{T} \overset{d}{\rightarrow} \max\{\mathcal{N}_1^2, \ldots, \mathcal{N}_h^2\}, \quad \mathcal{N} \equiv [\mathcal{N}_1, \ldots, \mathcal{N}_h]' \sim N(0, \mathbf{V}),$$

  $$\hat{\mathbf{V}} \overset{p}{\rightarrow} \mathbf{V} \text{ can be constructed from sample.}$$

- $\mathbf{V}$ is a block matrix which consists of covariance matrices between all regressors in Models 1, 2, $\ldots$, $h$.

- Consistency $\mathcal{T} \overset{p}{\rightarrow} \infty$ is ensured, assuming lag length $h$ is larger than or equal to the truth.
Monte Carlo Simulations: Two Purposes

MF Max Test \leftrightarrow MF Wald Test

LF Max Test \leftrightarrow LF Wald Test
Monte Carlo Simulations: DGP

The DGP is Ghysels’ (2015, JE) structural MF-VAR(1):

\[
\begin{bmatrix}
1 & 0 & \ldots & \ldots & \ldots & \ldots & 0 \\
-0.8 & 1 & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & -0.8 & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & -0.8 & 1 & 0 \\
0 & 0 & \ldots & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_H(\tau_L, 1) \\
x_H(\tau_L, 12) \\
x_L(\tau_L)
\end{bmatrix}
= 
\begin{bmatrix}
x_H(\tau_L - 1, 1) \\
x_H(\tau_L - 1, 12) \\
x_L(\tau_L - 1)
\end{bmatrix}
+ 
\begin{bmatrix}
\eta_H(\tau_L, 1) \\
\eta_H(\tau_L, 12) \\
\eta_L(\tau_L)
\end{bmatrix}.
\]
Monte Carlo Simulations

- Week vs. quarter mixture \((m = 12\) approximately).
- Sample size is \(T_L = 80\) quarters, a reasonable sample size.
- \(b = [b_1, \ldots, b_{12}]\)' is a key parameter representing causal patterns of \(x_H\) on \(x_L\).

1. **Decaying Causality**: \(b = [0.3, -0.15, 0.1, \ldots, -0.025]'\).
   - Decaying impact with +/- signs.

2. **Lagged Causality**: \(b = [0, 0, \ldots, 0, 0.3]'\).
   - Delayed impact reflecting seasonality or lagged information transmission.

3. **Sporadic Causality**: \((b_3, b_7, b_{10}) = (0.2, 0.05, -0.3)\) and all others are 0.
   - No clear patterns in terms of signs and decaying structures.
Monte Carlo Simulations

- We implement the **max test** based on 12 parsimonious models:

\[ x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,1} x_L(\tau_L - k) + \beta_1 x_H(\tau_L - 1, 12 + 1 - 1) + u_1(\tau_L), \]

\[ x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,2} x_L(\tau_L - k) + \beta_2 x_H(\tau_L - 1, 12 + 1 - 2) + u_2(\tau_L), \]

\[ \vdots \quad \vdots \quad \vdots \quad \vdots \]

\[ x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,12} x_L(\tau_L - k) + \beta_{12} x_H(\tau_L - 1, 12 + 1 - 12) + u_{12}(\tau_L). \]

- For comparison, we implement the **Wald test** based on one large model:

\[ x_L(\tau_L) = \sum_{k=1}^{2} \alpha_k x_L(\tau_L - k) + \sum_{j=1}^{12} \beta_j x_H(\tau_L - 1, 12 + 1 - j) + u(\tau_L). \]
Monte Carlo Simulations: Low Frequency Tests

- Implement flow sampling \( x_H(\tau_L) = \frac{1}{12} \sum_{j=1}^{12} x_H(\tau_L, j) \) or stock sampling \( x_H(\tau_L) = x_H(\tau_L, 12) \).
- We implement the max test based on 3 parsimonious models:

\[
x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,1} x_L(\tau_L - k) + \beta_1 x_H(\tau_L - 1) + u_1(\tau_L),
\]

\[
x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,2} x_L(\tau_L - k) + \beta_2 x_H(\tau_L - 2) + u_2(\tau_L),
\]

\[
x_L(\tau_L) = \sum_{k=1}^{2} \alpha_{k,3} x_L(\tau_L - k) + \beta_3 x_H(\tau_L - 3) + u_3(\tau_L).
\]

- We implement the Wald test:

\[
x_L(\tau_L) = \sum_{k=1}^{2} \alpha_k x_L(\tau_L - k) + \sum_{k=1}^{3} \beta_k x_H(\tau_L - k) + u(\tau_L).
\]
**Simulation Results**

Rejection Frequencies (Empirical Size)

<table>
<thead>
<tr>
<th></th>
<th>MF</th>
<th>LF (flow)</th>
<th>LF (stock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>.055</td>
<td>.054</td>
<td>.058</td>
</tr>
<tr>
<td>Wald</td>
<td>.043</td>
<td>.039</td>
<td>.038</td>
</tr>
</tbody>
</table>

- All tests are correctly sized, so we can compare power meaningfully.
- The max test has accurate size due to parsimonious specifications.
- The Wald test has accurate size *after bootstrapping* (heavy computation).
Simulation Results: MF Max Test vs. MF Wald Test

Rejection Frequencies (Empirical Power)

<table>
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<tr>
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<th>MF Wald</th>
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<tr>
<td>Decaying Causality</td>
<td>.642</td>
<td>.480</td>
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<td>.931</td>
<td>.731</td>
</tr>
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<td>Sporadic Causality</td>
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<td>.740</td>
</tr>
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- MF max test is more powerful than MF Wald test for all causal patterns.
### Simulation Results: MF Tests vs. LF Tests

#### Rejection Frequencies (Empirical Power)

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<td>.740</td>
<td>.107</td>
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- Stock-sampling tests are more powerful than MF tests for Decaying Causality.
- Stock-sampling tests are roughly as powerful as MF tests for Lagged Causality.
- MF tests are much more powerful than LF tests for Sporadic Causality.
Simulation Results: Summary

- MF max test is more powerful than MF Wald test for all causal patterns.
- If there exist relatively simple causal patterns, LF tests may outperform MF tests due to parsimony (cfr. Decaying Causality and Lagged Causality).
- MF tests are more robust against complex causal patterns than LF tests (cfr. Sporadic Causality).
Empirical Applications

We test for Granger causality from weekly yield spread to quarterly GDP growth in the U.S.

Weekly Yield Spread and Quarterly GDP Growth in 1962 - 2013
Background Knowledge

- Historically, negative yield spread used to be a well-known predictor of immediate economic recession.
- More recently, there is a possibility that such a relationship became less apparent (cfr. Greenspan’s Conundrum).

Methodology

- To see how causal patterns from spread to GDP changed over time, we compute $p$-values for each 20-year rolling window.
Empirical Results

(a) MF Max Test

(b) MF Wald Test

(c) LF Max Test

(d) LF Wald Test

\( p \)-values for Causality from Weekly Yield Spread to Quarterly GDP Growth
Empirical Results

- MF Wald test says spread was not a valid predictor until recently.
  - 

- Counter-Intuitive.

- MF max test and LF tests say spread used to be a valid predictor of GDP.
  - Intuitive.

- MF max test produces longer periods of significant causality than LF tests.
  - High power.
Conclusions

- Mixed frequency Granger causality tests often involve many parameters.

- We have proposed the max test, which has higher power than the existing Wald test in small sample with many parameters.

- In the empirical application on weekly yield spread and quarterly GDP, the max test produces more intuitive results than the Wald test.