Testing for Weak Form Efficiency of Stock Markets

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A rejection of the white noise hypothesis might serve as helpful information for arbitragers, because it indicates the presence of non-zero autocorrelations at some lags.

**Testing for Weak Form Efficiency of Stock Markets**

= Testing for **Unpredictability** of Stock Returns (cf. Fama, 1970)

Many applications

cf. Lim and Brooks (2009)

Only few applications

**REASON**: It is hard to establish a formal white noise test.

**BREAKTHROUGH**: Shao’s (2010, 2011) **dependent wild bootstrap**.

This paper tests for the **white noise** hypothesis of stock returns, using the **dependent wild bootstrap**.

A rejection of the white noise hypothesis might serve as helpful information for arbitragers, because it indicates the presence of non-zero autocorrelations at some lags.
**Introduction**

**Dependent Wild Bootstrap**
(Shao, 2010; Shao, 2011)

**Adaptive Market Hypothesis**
(cf. Lo, 2004; Lo, 2005)

### Full Sample

- **Shao (2010):** Temperature
- **Shao (2011):** Stock returns

### Rolling Window

**New!**

**Reason:** Fixed block size $b_n$ produces similar bootstrapped autocorrelations in every $b_n$ windows.

**Remedy:** Randomizing a block size across bootstrap samples and windows.
Introduction

S&P 500

White noise hypothesis is often rejected during Iraq War and the subprime crisis.

We observe significantly negative autocorrelations during crisis periods.

cf. Fama and French (1988)
Campbell et al. (1993)
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Review of Dependent Wild Bootstrap

- Consider **full sample** analysis as a benchmark.
- Consider covariance stationary \( \{y_1, \ldots, y_n\} \).
- Assume \( E[y_t] = 0 \) for notational simplicity.
- Population quantities:

\[
\gamma(0) = E[y_t^2], \quad \gamma(h) = E[y_t y_{t-h}], \quad \rho(h) = \frac{\gamma(h)}{\gamma(0)}.
\]

- Sample quantities:

\[
\hat{\gamma}_n(0) = \frac{1}{n} \sum_{t=1}^{n} y_t^2, \quad \hat{\gamma}_n(h) = \frac{1}{n} \sum_{t=h+1}^{n} y_t y_{t-h}, \quad \hat{\rho}_n(h) = \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)}.
\]
White noise hypothesis: $\rho(h) = 0$ for all $h \geq 1$.

As a starting point, fix $h$ and consider testing for $\rho(h) = 0$.

How can we construct a confidence band for $\hat{\rho}_n(h)$, assuming little more than serial uncorrelatedness?
Review of Dependent Wild Bootstrap

1. Set a block size $b_n$ (typically $b_n = \sqrt{n}$).

2. Generate iid $\{\xi_1, \xi_2, \ldots, \xi_{n/b_n}\}$. Define an auxiliary variable:

   $\omega = \begin{bmatrix} \xi_1, \ldots, \xi_1, & \xi_2, \ldots, \xi_2, & \ldots, & \xi_{n/b_n}, \ldots, \xi_{n/b_n} \end{bmatrix}'.$

   $b_n$ terms $b_n$ terms $b_n$ terms

3. Compute a bootstrapped autocorrelation:

   $\hat{\rho}_{n,(dw)}(h) = \frac{1}{\hat{\gamma}_n(0)} \times \frac{1}{n} \sum_{t=h+1}^{n} \omega_t [y_{t}y_{t-h} - \hat{\gamma}_n(h)].$

4. Repeat Steps 2-3 $M$ times and sort $\hat{\rho}_{n,(1)}(h) < \cdots < \hat{\rho}_{n,(M)}(h)$.

5. The 95% confidence band is $C(h) = [\hat{\rho}_{n,(0.025M)}(h), \hat{\rho}_{n,(0.975M)}(h)]$.

6. If $\hat{\rho}_n(h) \in C(h)$, then we do not reject $\rho(h) = 0$.
   If $\hat{\rho}_n(h) \not\in C(h)$, then we reject $\rho(h) = 0$. 
Now consider **rolling window** analysis.

Suppose that window size is $n = 60$ and block size is $b_n = 3$.

In window #1 $(y_1, \ldots, y_{60})$, we have

\[
\begin{bmatrix}
  y_1 y_2, y_2 y_3, \\
  \text{Block 1 (×$\xi_1$)}
\end{bmatrix}, \quad
\begin{bmatrix}
  y_3 y_4, y_4 y_5, y_5 y_6, \\
  \text{Block 2 (×$\xi_2$)}
\end{bmatrix}, \quad
\begin{bmatrix}
  y_6 y_7, y_7 y_8, y_8 y_9, \\
  \text{Block 3 (×$\xi_3$)}
\end{bmatrix}, \quad \ldots, \quad
\begin{bmatrix}
  y_{57} y_{58}, y_{58} y_{59}, y_{59} y_{60}, \\
  \text{Block 20 (×$\xi_{20}$)}
\end{bmatrix}.
\]

In window #4 $(y_4, \ldots, y_{63})$, we have

\[
\begin{bmatrix}
  y_4 y_5, y_5 y_6, \\
  \text{Block 1 (×$\xi_1$)}
\end{bmatrix}, \quad
\begin{bmatrix}
  y_6 y_7, y_7 y_8, y_8 y_9, \\
  \text{Block 2 (×$\xi_2$)}
\end{bmatrix}, \quad \ldots, \quad
\begin{bmatrix}
  y_{57} y_{58}, y_{58} y_{59}, y_{59} y_{60}, \\
  \text{Block 19 (×$\xi_{19}$)}
\end{bmatrix}, \quad
\begin{bmatrix}
  y_{60} y_{61}, y_{61} y_{62}, y_{62} y_{63}, \\
  \text{Block 20 (×$\xi_{20}$)}
\end{bmatrix}.
\]

Similar bootstrapped autocorrelations appear in windows #1, #4, #7, \ldots

$\implies$ **Periodicity** with $b_n = 3$ cycles.
$y_1, \ldots, y_{71} \sim \text{i.i.d. } N(0, 1)$.

Window size is $n = 60$. There are $71 - 60 + 1 = 12$ windows.

Block size is $b_n = 3$.

We plot $\hat{\rho}_n(1)$ and 95% confidence band for each window.
Remedy: Randomized Block Size

- Block size is $b_n = \lceil c\sqrt{n} \rceil$.
- Conventional choice that $c = 1$ produces periodicity.
- We propose to draw $c \sim U(1 - \delta, 1 + \delta)$ independently across rolling windows and bootstrap samples.
  - Randomness across **windows** removes periodicity.
  - Randomness across **bootstrap samples** reduces the volatility of confidence bands.
- We choose $\delta = 0.5$ (i.e. $c \sim U(0.5, 1.5)$).
Remedy: Randomized Block Size

**Illustrative Example:**

- \( y_1, y_2, \ldots, y_{400} \stackrel{i.i.d.}{\sim} N(0, 1). \)
- Window size is \( n = 240. \)
- There are \( 400 - 240 + 1 = 161 \) windows.
- Block size is \( b_n = \lfloor c \sqrt{n} \rfloor = \lfloor c \sqrt{240} \rfloor = \lfloor c \times 15.5 \rfloor. \)
- We choose either \( c = 1 \) or \( c \sim U(0.5, 1.5). \)
- We plot \( \hat{\rho}_n(1) \) and 95% confidence band for each window.
Remedy: Randomized Block Size

- When $c = 1$, confidence bands have clear periodicity.
- When $c \sim U(0.5, 1.5)$, the periodicity evaporates dramatically.
- January 1, 2003 – October 29, 2015 (3230 days).
Empirical Results: $\hat{\rho}_n(1)$ and Bands

- Window size is $n = 240$ (roughly a year).
- Block size is $b_n = [c\sqrt{n}] = [c \times 15.5]$.
- We have significantly **negative** autocorrelations during Iraq War and the subprime mortgage crisis.
Conclusions

1. Outline
   - Test for white noise hypothesis of stock returns
   - Perform rolling window analysis with dependent wild bootstrap

2. Contributions
   - Find that a fixed block size results in periodic confidence bands
   - Reveal that the periodicity stems from repeated block structures
   - Propose randomizing a block size to remove the periodicity

3. Empirical Finding
   - White noise hypothesis is rejected for S&P during crisis periods
     -- Significantly negative autocorrelations at lag 1
References


