Testing for Weak Form Efficiency of Stock Markets

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A rejection of the white noise hypothesis might serve as helpful information for arbitragers, because it indicates the presence of non-zero autocorrelations at some lags.

Many applications

**REASON:** It is hard to establish a formal white noise test.

**BREAKTHROUGH:** Shao’s (2010, 2011) dependent wild bootstrap.

This paper tests for the white noise hypothesis of stock returns, using the dependent wild bootstrap.

A rejection of the white noise hypothesis might serve as helpful information for arbitragers, because it indicates the presence of non-zero autocorrelations at some lags.
Adaptive Market Hypothesis (cf. Lo, 2004; Lo, 2005)
Dependent Wild Bootstrap (Shao, 2010; Shao, 2011)

**Full Sample**
- Shao (2010): Temperature
- Shao (2011): Stock returns

**Rolling Window**
- New!

**REASON:** Fixed block size $b_n$ produces similar bootstrapped autocorrelations in every $b_n$ windows.

**REMEDY:** Randomizing a block size across bootstrap samples and windows.
White noise hypothesis is often rejected during Iraq War and the subprime crisis.

We observe significantly negative autocorrelations during crisis periods.

cf. Fama and French (1988)
Campbell et al. (1993)
1) Introduction
2) Dependent Wild Bootstrap
3) Stock Price Data (S&P 500)
4) Empirical Results
5) Conclusions
Consider **full sample** analysis as a benchmark.

Consider covariance stationary \( \{y_1, \ldots, y_n\} \).

Assume \( E[y_t] = 0 \) for notational simplicity.

Population quantities:

\[
\gamma(0) = E[y_t^2], \quad \gamma(h) = E[y_t y_{t-h}], \quad \rho(h) = \frac{\gamma(h)}{\gamma(0)}.
\]

Sample quantities:

\[
\hat{\gamma}_n(0) = \frac{1}{n} \sum_{t=1}^{n} y_t^2, \quad \hat{\gamma}_n(h) = \frac{1}{n} \sum_{t=h+1}^{n} y_t y_{t-h}, \quad \hat{\rho}_n(h) = \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)}.
\]
White noise hypothesis is that $\rho(h) = 0$ for all $h \geq 1$.

**Naïve approach:** Box and Pierce’s (1970) asymptotic $\chi^2$ test

$$n \sum_{h=1}^{L} \hat{\rho}_n^2(h) \xrightarrow{d} \chi^2_L.$$

This is **not** a white noise test but an IID test, because the asymptotic $\chi^2$ property requires an IID assumption.

**Example:** Box-Pierce test rejects GARCH, a well-known example of non-IID MDS, with probability 1 asymptotically.

**We do not want to reject GARCH, because it is white noise.**
A formal white noise test requires Shao’s (2010, 2011) dependent wild bootstrap.

To describe how it works, fix lag length $h$ and consider testing for $\rho(h) = 0$.

How can we construct a confidence band for $\hat{\rho}_n(h)$?
1. Set a block size $b_n$ (typically $b_n = \sqrt{n}$).

2. Generate iid $\{\xi_1, \xi_2, \ldots, \xi_{n/b_n}\}$. Define an auxiliary variable:

$$\omega = \begin{bmatrix} \xi_1, \ldots, \xi_1, & \xi_2, \ldots, \xi_2, & \ldots, & \xi_{n/b_n}, \ldots, \xi_{n/b_n} \end{bmatrix}' .$$

3. Compute a bootstrapped autocorrelation:

$$\hat{\rho}_{n}^{(dw)}(h) = \frac{1}{\hat{\gamma}_n(0)} \times \frac{1}{n} \sum_{t=h+1}^{n} \omega_{t} [y_{t} y_{t-h} - \hat{\gamma}_n(h)] .$$

4. Repeat Steps 2-3 $M$ times and sort $\hat{\rho}_{n,(1)}^{(dw)}(h) < \cdots < \hat{\rho}_{n,(M)}^{(dw)}(h)$.

5. The 95% confidence band is $C(h) = [\hat{\rho}_{n,(0.025M)}^{(dw)}(h), \hat{\rho}_{n,(0.975M)}^{(dw)}(h)]$.

6. If $\hat{\rho}_n(h) \in C(h)$, then we do not reject $\rho(h) = 0$.

If $\hat{\rho}_n(h) \notin C(h)$, then we reject $\rho(h) = 0$. 
Review of Dependent Wild Bootstrap

Generate i.i.d. random numbers $\xi_1, \ldots, \xi_{20} \sim N(0, 1)$

- Block 1: $Y_1Y_2, Y_2Y_3 \times \xi_1$
- Block 2: $Y_3Y_4, Y_4Y_5, Y_5Y_6 \times \xi_2$
- Block 3: $Y_6Y_7, Y_7Y_8, Y_8Y_9 \times \xi_3$
- ... Blocks 20: $Y_{57}Y_{58}, Y_{58}Y_{59}, Y_{59}Y_{60} \times \xi_{20}$

→ Compute bootstrapped autocorrelation $\hat{\rho}_n^{(dw)}(1)$

→ Repeat M times
→ Construct confidence band

- Preserved dependence within each block.
- No dependence across different blocks.
- Asymptotically correct size and consistency under weak dependence (e.g. GARCH, bilinear).

Example:
h = 1; n = 60; $b_n = 3$. 

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Now consider **rolling window** analysis.

Suppose that window size is \( n = 60 \) and block size is \( b_n = 3 \).

In window \#1 \((y_1, \ldots, y_{60})\), we have

\[
\begin{align*}
&\{y_1 y_2, y_2 y_3\}, &\{y_3 y_4, y_4 y_5, y_5 y_6\}, &\{y_6 y_7, y_7 y_8, y_8 y_9\}, &\ldots, &\{y_{57} y_{58}, y_{58} y_{59}, y_{59} y_{60}\}.
\end{align*}
\]

In window \#4 \((y_4, \ldots, y_{63})\), we have

\[
\begin{align*}
&\{y_4 y_5, y_5 y_6\}, &\{y_6 y_7, y_7 y_8, y_8 y_9\}, &\ldots, &\{y_{57} y_{58}, y_{58} y_{59}, y_{59} y_{60}\}, &\{y_{60} y_{61}, y_{61} y_{62}, y_{62} y_{63}\}.
\end{align*}
\]

Similar bootstrapped autocorrelations appear in windows \#1, \#4, \#7, \ldots.

\[\implies \textbf{Periodicity} \text{ with } b_n = 3 \text{ cycles.}\]
Hidden Pitfall: Periodic Confidence Bands

- $y_1, \ldots, y_{71} \overset{i.i.d.}{\sim} N(0, 1)$.
- Window size is $n = 60$. There are $71 - 60 + 1 = 12$ windows.
- Block size is $b_n = 3$.
- We plot $\hat{\rho}_n(1)$ and 95% confidence band for each window.
Remedy: Randomized Block Size

- Block size is $b_n = \lfloor c\sqrt{n} \rfloor$.
- Conventional choice that $c = 1$ produces periodicity.
- We propose to draw $c \sim U(1 - \delta, 1 + \delta)$ independently across rolling windows and bootstrap samples.
  - Randomness across windows removes periodicity.
  - Randomness across bootstrap samples reduces the volatility of confidence bands.
- We naively choose $\delta = 0.5$ (i.e. $c \sim U(0.5, 1.5)$).
- Open question: "optimal" choice of $\delta$. 
Illustrative Example:

- $y_1, y_2, \ldots, y_{400} \overset{i.i.d.}{\sim} N(0, 1)$.
- Window size is $n = 240$.
- There are $400 - 240 + 1 = 161$ windows.
- Block size is $b_n = \lceil c \sqrt{n} \rceil = \lceil c \sqrt{240} \rceil = \lceil c \times 15.5 \rceil$.
- We choose either $c = 1$ or $c \sim U(0.5, 1.5)$.
- We plot $\hat{\rho}_n(1)$ and 95% confidence band for each window.
Remedy: Randomized Block Size

- When $c = 1$, confidence bands have clear periodicity.
- When $c \sim U(0.5, 1.5)$, the periodicity evaporates dramatically.
Daily data of S&P 500.

Autocorrelations at Lag 1

- Window size is $n = 240$ (roughly a year).
- Block size is $b_n = [c \sqrt{n}] = [c \times 15.5]$. 
White noise requires that $\rho(h) = 0$ for all $h \geq 1$.

Following Shao (2011), we use the Cramér-von Mises statistic:

$$C_n = n \int_0^\pi \left\{ \sum_{h=1}^{n-1} \hat{\gamma}_n(h) \psi_h(\lambda) \right\}^2 d\lambda, \quad \psi_h(\lambda) = \frac{\sin(h\lambda)}{h\pi}.$$ 

Bootstrapped p-values are computed based on the dependent wild bootstrap (with a randomized block size).
P-values of Cramér-von Mises Test

- P-values over rolling windows.
- S&P has **significant autocorrelations** during Iraq War and the subprime mortgage crisis.
Conclusions

1. Outline

- Test for white noise hypothesis of stock returns
- Perform rolling window analysis with dependent wild bootstrap

2. Contributions

- Find that a fixed block size results in periodic confidence bands
- Reveal that the periodicity stems from repeated block structures
- Propose randomizing a block size to remove the periodicity

3. Empirical Finding

- White noise hypothesis is rejected for S&P during crisis periods
  -- Significantly negative autocorrelations at lag 1
References


References


