Testing for Weak Form Efficiency of Stock Markets

Jonathan B. Hill$^1$  Kaiji Motegi$^2$

$^1$University of North Carolina at Chapel Hill  
$^2$Kobe University

Department of Statistics and Actuarial Science  
The University of Hong Kong

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Testing for Weak Form Efficiency of Stock Markets
= Testing for Unpredictability of Stock Returns (cf. Fama, 1970)

Many applications
 cf. Lim and Brooks (2009)

Only few applications

IID MDS White Noise

REASON: It is hard to establish a formal white noise test.

BREAKTHROUGH: Shao’s (2010, 2011) dependent wild bootstrap.

(Another option: Zhu and Li’s (2015) block-wise random weighting bootstrap)

This paper tests for the white noise hypothesis of stock returns, using the dependent wild bootstrap.
Consider a univariate time series \( \{y_t\} \) with

\[
E[y_t] = 0, \quad E[y_t^2] = \gamma(0) < \infty.
\]

There are four well-known layers of unpredictability:

1. **IID**: \( E[f(y_t) \mid y_{t-1}, y_{t-2}, \ldots] = E[f(y_t)] \) for any \( f(\cdot) \).
   - e.g. IID \( N(0,1) \).

2. **MDS**: \( E[y_t \mid y_{t-1}, y_{t-2}, \ldots] = 0 \).
   - e.g. GARCH(1,1).

3. **White Noise**: \( E[y_t y_{t-h}] = 0 \) for any \( h \in \mathbb{Z} \).
   - e.g. bilinear.

4. **Covariance Stationarity**: \( E[y_t y_{t-h}] = \gamma(h) \) for any \( j \in \mathbb{Z} \).
   - e.g. stationary AR(1).
Introduction: Simulated $\{y_t\}$

(i) IID $N(0, 1)$

(ii) GARCH(1,1)

(iii) Bilinear

(iv) AR(1)

- White noise test is supposed to accept (i)-(iii) and reject (iv).

- A rejection of the white noise hypothesis might serve as helpful information for arbitragers, because it indicates the presence of non-zero autocorrelations at some lags.
Introduction: Simulated $\{y_t^2\}$

1. IID $N(0, 1)$
2. GARCH(1,1)
3. Bilinear
4. AR(1)
Introduction

Adaptive Market Hypothesis (cf. Lo, 2004; Lo, 2005)

Dependent Wild Bootstrap (Shao, 2010; Shao, 2011)

Full Sample

Shao (2010): Temperature
Shao (2011): Stock returns

Rolling Window

New!

PERIODICITY IN CONFIDENCE BANDS

REASON: Fixed block size $b_n$ produces similar bootstrapped autocorrelations in every $b_n$ windows.

REMEDIY: Randomizing a block size across bootstrap samples and windows.
White noise hypothesis is **accepted** in most time periods.

Hence the weak form efficiency likely holds.
Introduction

White noise hypothesis is often rejected during Iraq War and the subprime crisis. We observe significantly negative autocorrelations during crisis periods. 

cf. Fama and French (1988) 
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Consider **full sample** analysis as a benchmark.

Consider univariate, covariance stationary \( \{y_1, \ldots, y_n\} \).

Assume \( E[y_t] = 0 \) for notational simplicity.

Population quantities:

\[
\gamma(0) = E[y_t^2], \quad \gamma(h) = E[y_ty_{t-h}], \quad \rho(h) = \frac{\gamma(h)}{\gamma(0)}.
\]

Sample quantities:

\[
\hat{\gamma}_n(0) = \frac{1}{n} \sum_{t=1}^{n} y_t^2, \quad \hat{\gamma}_n(h) = \frac{1}{n} \sum_{t=h+1}^{n} y_ty_{t-h}, \quad \hat{\rho}_n(h) = \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)}.
\]
White noise hypothesis: $\rho(h) = 0$ for all $h \geq 1$.

Consider an asymptotic Q-test:

$$\hat{Q}_n = n \sum_{h=1}^{L} w_n(h) \hat{\rho}_n^2(h) \xrightarrow{d} \chi^2_L.$$ 

- $w_n(h) = 1$ in Box and Pierce (1970).
- $w_n(h) = (n+2)/(n-h)$ in Ljung and Box (1978).

Strictly, the asymptotic Q-test is not a white noise test. 

Reason: $\chi^2$ property requires the asymptotic independence of $\{\hat{\rho}_n(1), \ldots, \hat{\rho}_n(L)\}$, which holds when $\{y_t\}$ is IID.
Review of Dependent Wild Bootstrap

- We need bootstrap to construct a formal white noise test.
- As a starting point, fix $h$ and consider testing for $\rho(h) = 0$.
- How can we construct a confidence band for $\hat{\rho}_n(h)$, assuming little more than serial uncorrelatedness?
1. Set a block size $b_n$ (typically $b_n = \sqrt{n}$).

2. Generate iid \{\(\xi_1, \xi_2, \ldots, \xi_{n/b_n}\)\}. Define an auxiliary variable:

\[
\omega = \begin{bmatrix}
\xi_1, \ldots, \xi_1, & \xi_2, \ldots, \xi_2, & \ldots, & \xi_{n/b_n}, \ldots, \xi_{n/b_n}
\end{bmatrix}.
\]

3. Compute a bootstrapped autocorrelation:

\[
\hat{\rho}_{n,(d,w)}(h) = \frac{1}{\hat{\gamma}_n(0)} \times \frac{1}{n} \sum_{t=h+1}^{n} \omega_t [y_t y_{t-h} - \hat{\gamma}_n(h)].
\]

4. Repeat Steps 2-3 \(M\) times and sort $\hat{\rho}_{n,(1)}(h) < \cdots < \hat{\rho}_{n,(M)}(h)$.

5. The 95% confidence band is $C(h) = [\hat{\rho}_{n,(0.025M)}(h), \hat{\rho}_{n,(0.975M)}(h)]$.

6. If $\hat{\rho}_n(h) \in C(h)$, then we do not reject $\rho(h) = 0$.

If $\hat{\rho}_n(h) \notin C(h)$, then we reject $\rho(h) = 0$. 

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Review of Dependent Wild Bootstrap

Generate i.i.d. random numbers $\xi_1, \ldots, \xi_{20} \sim N(0, 1)$

- Block 1: $Y_1Y_2, Y_2Y_3 
  \times \xi_1$
- Block 2: $Y_3Y_4, Y_4Y_5, Y_5Y_6 
  \times \xi_2$
- Block 3: $Y_6Y_7, Y_7Y_8, Y_8Y_9 
  \times \xi_3$
- ... Block 20: $Y_{57}Y_{58}, Y_{58}Y_{59}, Y_{59}Y_{60} 
  \times \xi_{20}$

Compute bootstrapped autocorrelation $\hat{\rho}_n^{(dw)}(1)$

- Repeat M times
- Compute bootstrapped p-value

Example:
h = 1; n = 60; $b_n = 3$.  

- Preserved dependence within each block.
- No dependence across different blocks.
- Asymptotically correct size and consistency under weak dependence (e.g. GARCH, bilinear).
Now consider **rolling window** analysis.

Suppose that window size is $n = 60$ and block size is $b_n = 3$.

In window #1 $\{y_1, \ldots, y_{60}\}$, we have

\[
\begin{align*}
\left[ \begin{array}{c}
  y_1 y_2 y_3, \\
  y_3 y_4 y_5 y_6, \\
  y_6 y_7 y_8 y_9, \\
  \vdots, \\
  y_{57} y_{58} y_{59} y_{60}
\end{array} \right].
\end{align*}
\]

- Block 1 ($\times \xi_1$)
- Block 2 ($\times \xi_2$)
- Block 3 ($\times \xi_3$)
- Block 20 ($\times \xi_{20}$)

In window #4 $\{y_4, \ldots, y_{63}\}$, we have

\[
\begin{align*}
\left[ \begin{array}{c}
  y_4 y_5 y_6, \\
  y_6 y_7 y_8 y_9, \\
  \vdots, \\
  y_{60} y_{61} y_{62} y_{63}
\end{array} \right].
\end{align*}
\]

- Block 1 ($\times \xi_1$)
- Block 2 ($\times \xi_2$)
- Block 19 ($\times \xi_{19}$)
- Block 20 ($\times \xi_{20}$)

Similar bootstrapped autocorrelations appear in windows #1, #4, #7, \ldots

$\implies$ **Periodicity** with $b_n = 3$ cycles.
Hidden Pitfall: Periodic Confidence Bands

- $y_1, \ldots, y_{71} \overset{i.i.d.}{\sim} N(0, 1)$.
- Window size is $n = 60$. There are $71 - 60 + 1 = 12$ windows.
- Block size is $b = 3$.
- We plot $\hat{\rho}_n(1)$ and 95% confidence band for each window.
Block size is \( b_n = \lfloor c\sqrt{n} \rfloor \).

Choosing \( c = 1 \) produces periodicity in every \( \sqrt{n} \) windows.

We propose to draw \( c \sim U(1 - \delta, 1 + \delta) \) independently across rolling windows and bootstrap samples.

- Randomness across windows removes periodicity.
- Randomness across bootstrap samples reduces the volatility of confidence bands.

We choose \( \delta = 0.5 \) (i.e. \( c \sim U(0.5, 1.5) \)).
Illustrative Example:

- $y_1, y_2, \ldots, y_{400} \overset{i.i.d.}{\sim} N(0, 1)$.
- Window size is $n = 240$.
- There are $400 - 240 + 1 = 161$ windows.
- Block size is $b_n = \lfloor c \sqrt{n} \rfloor = \lfloor c \sqrt{240} \rfloor = \lfloor c \times 15.5 \rfloor$.
- We choose either $c = 1$ or $c \sim U(0.5, 1.5)$.
- We plot $\hat{\rho}_n(1)$ and 95% confidence band for each window.
Remedy: Randomized Block Size

- When $c = 1$, there is clear periodicity in every 15 windows.
- When $c \sim U(0.5, 1.5)$, the periodicity evaporates dramatically.
We analyze daily log returns of four stock price indices:

1. Shanghai Composite Index (China).
2. Nikkei 225 (Japan).
3. FTSE 100 (UK).


Sample size is approximately 3200 (different across series).
Data: Shanghai Composite Index

a. Level

b. Log Return

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Data: Nikkei 225

(a) Level

(b) Log Return
Data: FTSE 100

a. Level

b. Log Return

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Data: S&P 500

a. Level

b. Log Return

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Empirical Results

- We plot $\hat{\rho}_n(1)$ and 95% confidence bands for each rolling window.

- Window size is $n = 240$ days (roughly a year).

- Block size is $b_n = \lfloor c \sqrt{n} \rfloor = \lfloor c \times 15.5 \rfloor$.

- We try both $c = 1$ and $c \sim U(0.5, 1.5)$ for comparison.
\[ H_0 : \rho(1) = 0 \text{ is not rejected for most windows.} \]

Hence we do not see an evidence against weak form efficiency.
Autocorrelations at Lag 1: Nikkei 225

\[ a. \quad c = 1 \]

\[ b. \quad c \sim U \]

- \( H_0 : \rho(1) = 0 \) is **not** rejected for most windows.
- Hence we do not see an evidence against weak form efficiency.
\( H_0 : \rho(1) = 0 \) is rejected in some periods due to large negative autocorrelations in some periods.

Those periods coincide with Iraq War and the subprime mortgage crisis.
\( H_0 : \rho(1) = 0 \) is rejected in some periods due to large negative autocorrelations in some periods.

Those periods coincide with Iraq War and the subprime mortgage crisis.
Cramér-von Mises White Noise Test

White noise requires that \( \rho(h) = 0 \) for all \( h \geq 1 \).

Following Shao (2011), we use the Cramér-von Mises statistic:

\[
C_n = n \int_0^\pi \left\{ \sum_{h=1}^{n-1} \hat{\gamma}_n(h) \psi_h(\lambda) \right\}^2 d\lambda, \quad \psi_h(\lambda) = \frac{\sin(h\lambda)}{h\pi}.
\]

Bootstrapped p-values are computed based on the dependent wild bootstrap (with a randomized block size).

We observe similar results after using Hill and Motegi’s (2017) max-correlation test and Andrews and Ploberger’s (1996) sup-LM test.
White noise hypothesis is **not** rejected for most windows.

Weak form efficiency likely holds for Shanghai and Nikkei.
White noise hypothesis is rejected during Iraq War and the subprime mortgage crisis.

Rejections stem from the significantly negative autocorrelations at lag 1.
Conclusions

1. Outline
- Test for white noise hypothesis of stock returns
- Perform rolling window analysis with dependent wild bootstrap

2. Contributions
- Find that a fixed block size results in periodic confidence bands
- Reveal that the periodicity stems from repeated block structures
- Propose randomizing a block size to remove the periodicity

3. Empirical Finding
- White noise hypothesis is accepted for most subsamples
  -- Weak form efficiency likely holds
- White noise hypothesis is rejected during crisis periods
  -- Significantly negative autocorrelations at lag 1


References


