A Max-Correlation White Noise Test for Weakly Dependent Time Series

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A primary goal of time series analysis lies in prediction. It is of interest to test whether a time series is unpredictable. Definition of unpredictability matters.

Consider a covariance stationary time series \( \{y_t\} \) with

\[
\begin{align*}
\mu &= E[y_t], \\
\gamma(0) &= E[(y_t - \mu)^2] < \infty, \\
\gamma(h) &= E[(y_t - \mu)(y_{t-h} - \mu)], \\
\rho(h) &= \gamma(h)/\gamma(0).
\end{align*}
\]

Serial independence: \( E_{t-1}[f(y_{t+h_1}, \ldots, y_{t+h_k})] = E[f(y_{t+h_1}, \ldots, y_{t+h_k})] \) for any \( f : \mathbb{R}^k \to \mathbb{R}, \ h_1, \ldots, h_k \geq 0, \) and \( k \in \mathbb{N}. \)

Martingale difference sequence: \( E_{t-1}[y_t] = E[y_t] = \mu. \)

White noise: \( \gamma(h) = E[(y_t - \mu)(y_{t-h} - \mu)] = 0 \) for any \( h \geq 1. \)
Let $\nu_t \sim \text{i.i.d. } N(0,1)$.

Well-known examples of each notion include the following.

- Independence: $y_t = \nu_t$.
- MDS: GARCH(1,1) $y_t = \sigma_t \nu_t$, $\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2$.
- White noise: Bilinear $y_t = b\nu_{t-1}y_{t-2} + \nu_t$.
- Covariance stationary: AR(1) $y_t = \phi y_{t-1} + \nu_t$.

Independence $\subset$ MDS $\subset$ White noise $\subset$ Cov. stat.

Economic and financial time series belong to various classes.

A key characteristic of the middle classes is conditional heteroskedasticity, which is observed for most asset returns.
Introduction

NID

GARCH(1,1)

Bilinear

AR(1)
Introduction

NID$^2$

GARCH(1,1)$^2$

Bilinear$^2$

AR(1)$^2$
Testing the white noise hypothesis is of practical use:

1. Testing the **weak form efficiency** of stock markets. A rejection of the white noise hypothesis of stock returns would serve as a signal of arbitrage opportunity (Hill and Motegi, 2019).

2. **Residual diagnostics** of time series regressions. If a model fits data well, then the resulting residual should be white noise.

It is relatively easy to test the independence or MDS hypothesis, since the null hypothesis $H_0$ is strong enough to establish asymptotic theory (Box and Pierce, 1970).

It is challenging to test the white noise hypothesis, since $H_0$ is weak (only serial uncorrelatedness) and there are infinitely many zero restrictions.
Define

\[ \hat{\mu}_n = \frac{1}{n} \sum_{t=1}^{n} y_t, \quad \hat{\gamma}_n(0) = \frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{\mu}_n)^2, \]

\[ \hat{\gamma}_n(h) = \frac{1}{n} \sum_{t=1}^{n} (y_t - \hat{\mu}_n)(y_{t-h} - \hat{\mu}_n), \quad \hat{\rho}_n(h) = \frac{\hat{\gamma}_n(h)}{\hat{\gamma}_n(0)}. \]

Consider the classical Q-test:

\[ \hat{Q}_n = n \sum_{h=1}^{\mathcal{L}} w_n(h) \hat{\rho}_n^2(h) \xrightarrow{d} \chi_\mathcal{L}^2. \]

- \( w_n(h) = 1 \) in Box and Pierce (1970).
- \( w_n(h) = (n + 2)/(n - h) \) in Ljung and Box (1978).
Recall:

\[
\hat{Q}_n = n \sum_{h=1}^{L} w_n(h) \hat{\rho}_n(h) \xrightarrow{d} \chi^2_L.
\]

Two reasons why the Q-test is not a white noise test:

1. Asymptotic \( \chi^2 \) property requires the asymptotic independence of \( \{\hat{\rho}_n(1), \ldots, \hat{\rho}_n(L)\} \), which holds only when \( \{y_t\} \) is **serially independent**.

2. The Q-test cannot capture autocorrelations beyond lag \( L \).
Research question: How can we establish a formal white noise test?

1. How to formulate a test statistic?
2. How to choose a lag length?
3. How to compute a p-value?

Proposed solution: A bootstrapped max-correlation test with automatic lag selection.

1. The test statistic is $\hat{T}_n(L^*_n) = \sqrt{n} \times \max_{1 \leq h \leq L^*_n} |\hat{\rho}_n(h)|$.
3. An approximate p-value of Shao’s (2011) dependent wild bootstrap.
Table of contents

1. Introduction
2. Methodology
   - Max-correlation test statistic
   - P-value computation
   - Data-driven lag selection
3. Monte Carlo simulation
4. Empirical application
5. Conclusion
Consider a general time series model:

\[ y_t = f(x_{t-1}, \phi) + u_t, \quad u_t = \epsilon_t \sigma_t(\theta), \]

where \( \{y_t, x_t\} \) are strictly stationary; \( E[\epsilon_t] = 0; E[\epsilon_t^2] < \infty \).

This specification includes AR, MA, bilinear, GARCH, AR-GARCH models, etc.

We want to test whether \( \{\epsilon_t\} \) is white noise:

\[
\begin{align*}
H_0 : E[\epsilon_t \epsilon_{t-h}] &= 0 \quad \forall h \geq 1, \\
H_1 : \exists h \geq 1 \text{ such that } E[\epsilon_t \epsilon_{t-h}] \neq 0.
\end{align*}
\]

A contribution of the present paper is that the general filter above is allowed.

For the purpose of exposition, however, the present slides assume that there is not a filter (i.e., \( y_t = \epsilon_t \)).
Methodology: Max-correlation test

- We propose a **max-correlation** test statistic:

\[
\hat{T}_n = \sqrt{n} \times \max \left\{ |\hat{\rho}_n(1)|, \ldots, |\hat{\rho}_n(\mathcal{L}_n)| \right\},
\]

where \( \mathcal{L}_n \to \infty \) as \( n \to \infty \) and \( \mathcal{L}_n = o(n) \).

- Selection of \( \mathcal{L}_n \) shall be discussed later.

- An advantage of the **maximum** relative to the **sum of squares** is that the maximum leads to sharper performance when there exist autocorrelations at **remote lags** (e.g., seasonality).

- Max-type test statistics are studied in Berman (1964), Hannan (1974), and Xiao and Wu (2014).

- They require that the test variable should be observed, and they impose rather strong assumptions to derive a tractable asymptotic distribution (i.e., Gumbel distribution).
Methodology: Max-correlation test

- Contrary to the previous literature, we sidestep the extreme theory by utilizing a bootstrap.
- The standard wild bootstrap is valid for serially independent series and MDS, but invalid for white noise.
- Shao (2011) proposed the dependent wild bootstrap (DWB), which is valid for white noise.
- We use DWB to compute an approximate p-value.

**Remark #1**: Shao’s (2011) test statistic is the Cramér-von Mises statistic which uses all $\mathcal{L}_n = n - 1$ lags available.

**Remark #2**: Shao (2011) proved the validity of DWB by imposing moment contraction properties, while we prove it by imposing the near epoch dependence. The latter approach is technically more general than the former.
Methodology: Dependent wild bootstrap

1. Set a block size $b_n$ (typically $b_n = \sqrt{n}$).
2. Generate iid $\{\xi_1, \xi_2, \ldots, \xi_{n/b_n}\}$. Define an auxiliary variable:

$$\omega = \left[ \begin{array}{cccc}
\xi_1, \ldots, \xi_1, & \xi_2, \ldots, \xi_2, & \ldots, & \xi_{n/b_n}, \ldots, \xi_{n/b_n} \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\text{ } & \text{ } & \text{ } & \\
\end{array} \right]'.$$

$b_n$ terms $b_n$ terms $b_n$ terms

3. Compute bootstrapped autocorrelations:

$$\hat{\rho}_{n,(dw)}(h) = \frac{1}{\hat{\gamma}_n(0)} \times \frac{1}{n} \sum_{t=h+1}^{n} \omega_t [y_t y_{t-h} - \hat{\gamma}_n(h)], \quad h = 1, \ldots, \mathcal{L}_n,$$

and

$$\hat{T}_{n,(dw)} = \sqrt{n} \times \max_{1 \leq h \leq \mathcal{L}_n} |\hat{\rho}_{n,(dw)}(h)|.$$

4. Repeat Steps 2-3 $M$ times and compute the bootstrapped p-value

$$\hat{p}_{n,M}^{(dw)} = (1/M) \sum_{i=1}^{M} I(\hat{T}_{n,i,(dw)} \geq \hat{T}_n).$$

5. If $\hat{p}_{n,M}^{(dw)} < \alpha$, then reject $H_0$ at the $100\alpha\%$ level. Otherwise accept $H_0$. 

Hill & Motegi (UNC & Kobe)  Max-Correlation White Noise Test  30th January, 2019  14 / 35
If $b_n = 1$, then DWB reduces to the standard wild bootstrap (up to centering).

To account for weak dependence, set $1 \leq b_n < n$, $b_n \to \infty$ as $n \to \infty$, and $b_n = o(n)$.

DWB is valid under weak dependence (e.g., GARCH, bilinear):

- Under $H_0$, $P(\hat{p}_{n,M}^{(dw)} < \alpha) \to \alpha$ (asymptotically correct size).
- Under $H_1$, $P(\hat{p}_{n,M}^{(dw)} < \alpha) \to 1$ (consistency).

An intuitive reason why DWB is valid:

- Weak dependence within each block is preserved.
- Weak dependence across blocks is eliminated.
Methodology: Data-driven lag selection

- Practical question: How should we choose lag length $\mathcal{L}_n$?
- We know $\mathcal{L}_n = o(n)$, but that is not informative enough.
- We utilize the automatic lag selection of Escanciano and Lobato (2009).
- Intuitively, we want to add a lag $h$ if and only if $|\hat{\rho}_n(h)|$ is "important" enough.
- More specifically, we should add a lag $h$ if and only if $|\hat{\rho}_n(h)|$ is large enough to offset a certain penalty.
Methodology: Data-driven lag selection

1. Choose the upper bound $\tilde{L}_n = o(n/\ln n)$ and a tuning parameter $q > 0$.

2. For each candidate $L \in \{1, \ldots, \tilde{L}_n\}$, compute the penalized max-correlation test statistic:

$$\hat{T}_n^P(L) = \hat{T}_n(L) - \mathcal{P}_n(L),$$

where

$$\mathcal{P}_n(L) = \begin{cases} \sqrt{(\ln n)L} & \text{if } \hat{T}_n(L) \leq \sqrt{q \ln n}, \\ \sqrt{2L} & \text{if } \hat{T}_n(L) > \sqrt{q \ln n}. \end{cases}$$

3. The optimal lag length $L_n^*$ is given by

$$L_n^* = \min \left\{ \arg \max_{L \in \{1, \ldots, \tilde{L}_n\}} \hat{T}_n^P(L) \right\}.$$
Methodology: Data-driven lag selection

Theorem

1. Under $H_0$, $P(\mathcal{L}_n^* = 1) \rightarrow 1$.

2. Under $H_1$, $\mathcal{L}_n^* \xrightarrow{p} h^*$, where

$$h^* = \min \left\{ \arg \max_{h \geq 1} |\rho(h)| \right\}.$$  

- A large (small) value of $q$ promotes a selection of the BIC (AIC) penalty.
- Our extensive Monte Carlo simulations suggest that using $q = 3.25$ leads to strong results in terms of empirical size and size-adjusted power.
Methodology: Data-driven lag selection

- **Size**: \( y_t \overset{i.i.d.}{\sim} N(0, 1) \) and mean filter.

- **Power**: \( y_t = 0.3y_{t-1} - 0.15y_{t-2} + \nu_t, \nu_t \overset{i.i.d.}{\sim} N(0, 1) \), and AR(1) filter \( y_t = \phi y_{t-1} + u_t \).

- Setting \( q = 3.25 \) results in the best performance in terms of size and size-adjusted power.
1. Compute a data-driven lag length $\mathcal{L}^*_n$ and an actual test statistic $\hat{T}_n(\mathcal{L}^*_n) = \sqrt{n} \times \max_{1 \leq h \leq \mathcal{L}^*_n} |\hat{\rho}_n(h)|$.

2. Generate the $i^{th}$ bootstrap sample, compute bootstrapped autocorrelations $\{\hat{\rho}_{n,i}^{(dw)}(1), \ldots, \hat{\rho}_{n,i}^{(dw)}(\mathcal{L}_n)\}$, determine a lag length $\mathcal{L}^*_{n,i}$, and compute a bootstrapped test statistic $\hat{T}^{(dw)}_{n,i}(\mathcal{L}^*_{n,i}) = \sqrt{n} \times \max_{1 \leq h \leq \mathcal{L}^*_{n,i}} |\hat{\rho}_{n,i}^{(dw)}(h)|$.

3. Repeat Step 2 $M$ times and compute the bootstrapped p-value $\hat{p}_{n,M}^{(dw)} = (1/M) \sum_{i=1}^{M} I(\hat{T}^{(dw)}_{n,i}(\mathcal{L}^*_{n,i}) \geq \hat{T}_n(\mathcal{L}^*_n))$.

4. If $\hat{p}_{n,M}^{(dw)} < \alpha$, then reject $H_0$ at the $100\alpha\%$ level. Otherwise accept $H_0$. 

Hill & Motegi (UNC & Kobe)  
Max-Correlation White Noise Test  
30th January, 2019  
20 / 35
Monte Carlo simulation: Design

- $\hat{T}^{dw}(L^*) =$ the proposed max-correlation test with DWB and automatic lag selection.

- $CvM^{dw} =$ Shao’s (2011) Cramér-von Mises [CvM] test with DWB, where the test statistic is

$$C_n = n \int_0^\pi \left[ \sum_{h=1}^{n-1} \hat{\gamma}_n(h) \frac{\sin(h\lambda)}{h\pi} \right]^2 d\lambda.$$

- Our paper considers many alternative tests such as Ljung and Box’ (1978) Q-test, Andrews and Ploberger’s (1996) sup-LM test, Hong’s (1996) spectral test, and Zhu and Li’s (2015) CvM test with the block-wise random weighting bootstrap.

- $CvM^{dw}$ turned out to be the strongest competitor to $\hat{T}^{dw}(L^*)$. 
Monte Carlo simulation: Design

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Equation</th>
<th>$H_0$</th>
<th>$H_1$ (adjacent)</th>
<th>$H_1$ (remote)</th>
<th>$-\ $</th>
</tr>
</thead>
<tbody>
<tr>
<td>IID $e_t = \nu_t$</td>
<td>$H_0$</td>
<td>$H_1$ (adjacent)</td>
<td>$H_1$ (remote)</td>
<td>$-\ $</td>
<td></td>
</tr>
<tr>
<td>Simple $y_t = e_t$</td>
<td>$H_0$</td>
<td>$H_1$ (adjacent)</td>
<td>$H_1$ (remote)</td>
<td>$-\ $</td>
<td></td>
</tr>
<tr>
<td>Bilin $y_t = 0.5e_{t-1}y_{t-2} + e_t$</td>
<td>$H_0$</td>
<td>$H_1$ (adjacent)</td>
<td>$H_1$ (remote)</td>
<td>$-\ $</td>
<td></td>
</tr>
<tr>
<td>MA(6) $y_t = e_t + 0.25e_{t-6}$</td>
<td>$H_1$ (remote)</td>
<td>$H_1$ (remote)</td>
<td>$H_1$ (remote)</td>
<td>$-\ $</td>
<td></td>
</tr>
</tbody>
</table>

- $\nu_t \sim i.i.d. N(0, 1)$.
- Sample size is $n \in \{100, 250, 500, 1000\}$.
- For $\hat{T}^{dw}(\mathcal{L}^*)$, we set $\bar{L}_n = \lfloor 1.5 \times n/(\ln n)^{4/3} \rfloor$ so that $\bar{L}_n \in \{19, 38, 65, 114\}$ for $n \in \{100, 250, 500, 1000\}$.
- Nominal size is $\alpha \in \{0.01, 0.05, 0.10\}$.
- The number of bootstrap samples is $M = 500$.
- The number of Monte Carlo iterations is $J = 1000$. 
### Monte Carlo simulation: Results

#### Empirical size (simple $y_t = e_t$ & IID $e_t = \nu_t$)

<table>
<thead>
<tr>
<th></th>
<th>$\hat{T}_{dw}(L^*)$</th>
<th>$C_{VM}^{dw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>0.011</td>
<td>0.058</td>
</tr>
<tr>
<td>$n = 250$</td>
<td>0.010</td>
<td>0.050</td>
</tr>
<tr>
<td>$n = 500$</td>
<td>0.010</td>
<td>0.050</td>
</tr>
<tr>
<td>$n = 1000$</td>
<td>0.008</td>
<td>0.058</td>
</tr>
</tbody>
</table>

- Both tests have fairly accurate empirical size.
- $\hat{T}_{dw}(L^*)$ yields the sharper size than $C_{VM}^{dw}$ in small samples.
Monte Carlo simulation: Results

Empirical size (bilinear $y_t = 0.5y_{t-1}y_{t-2} + e_t$ & IID $e_t = \nu_t$)

<table>
<thead>
<tr>
<th></th>
<th>$\hat{T}^{dw}(\mathcal{L}^*)$</th>
<th>$CvM^{dw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>.009</td>
<td>.060</td>
</tr>
<tr>
<td>$n = 250$</td>
<td>.013</td>
<td>.054</td>
</tr>
<tr>
<td>$n = 500$</td>
<td>.008</td>
<td>.030</td>
</tr>
<tr>
<td>$n = 1000$</td>
<td>.011</td>
<td>.041</td>
</tr>
</tbody>
</table>

- Both tests have fairly accurate empirical size.
- $\hat{T}^{dw}(\mathcal{L}^*)$ yields the sharper size than $CvM^{dw}$ in small samples.
Monte Carlo simulation: Results

Empirical power (simple $y_t = e_t$ & AR(1) $e_t = 0.7e_{t-1} + \nu_t$)

<table>
<thead>
<tr>
<th></th>
<th>$\hat{T}^{dw}(\mathcal{L}^*)$</th>
<th>$CvM^{dw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%  5%  10%</td>
<td>1%  5%  10%</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>.496  .756  .848</td>
<td>.925  .996  1.00</td>
</tr>
<tr>
<td>$n = 250$</td>
<td>.882  .976  .993</td>
<td>.999  1.00  1.00</td>
</tr>
<tr>
<td>$n = 500$</td>
<td>.997  1.00  1.00</td>
<td>1.00  1.00  1.00</td>
</tr>
<tr>
<td>$n = 1000$</td>
<td>1.00  1.00  1.00</td>
<td>1.00  1.00  1.00</td>
</tr>
</tbody>
</table>

$\hat{T}^{dw}(\mathcal{L}^*)$ is less powerful than $CvM^{dw}$ in small samples.
Monte Carlo simulation: Results

Emp. power (bilin. $y_t = 0.5e_{t-1}y_{t-2} + e_t$ & AR(1) $e_t = 0.7e_{t-1} + \nu_t$)

<table>
<thead>
<tr>
<th></th>
<th>$\hat{T}^{dw}(L^*)$</th>
<th>$CvM^{dw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
<td>5%</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>.507</td>
<td>.646</td>
</tr>
<tr>
<td>$n = 250$</td>
<td>.686</td>
<td>.784</td>
</tr>
<tr>
<td>$n = 500$</td>
<td>.731</td>
<td>.837</td>
</tr>
<tr>
<td>$n = 1000$</td>
<td>.732</td>
<td>.822</td>
</tr>
</tbody>
</table>

$\hat{T}^{dw}(L^*)$ is more powerful than $CvM^{dw}$ in any sample size.
Empirical power (remote MA(6) $y_t = e_t + 0.25e_{t-6} \& \text{IID } e_t = \nu_t$)

<table>
<thead>
<tr>
<th>$\hat{T}^{dw}(\mathcal{L}^*)$</th>
<th>$C_vM^{dw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% 5% 10%</td>
<td>1% 5% 10%</td>
</tr>
<tr>
<td>$n = 100$</td>
<td>.014 .066 .127</td>
</tr>
<tr>
<td>$n = 250$</td>
<td>.159 .261 .315</td>
</tr>
<tr>
<td>$n = 500$</td>
<td>.685 .746 .759</td>
</tr>
<tr>
<td>$n = 1000$</td>
<td>.998 .998 .999</td>
</tr>
</tbody>
</table>

- $\hat{T}^{dw}(\mathcal{L}^*)$ does capture the remote autocorrelation $\rho(6) = 0.235$ with probability approaching 1.
- This is essentially because $\mathcal{L}_n^* \xrightarrow{p} h^* = 6$ as desired.
- $C_vM^{dw}$ fails to capture the remote autocorrelation.
- $C_vM^{dw}$ uses all $\mathcal{L}_n = n - 1$ lags, but most weights are assigned on adjacent lags.
Empirical application

- Testing the white noise hypothesis of log returns of major stock price indices:
  - S&P 500 (SPX)
  - FTSE 100 (FTSE)
  - Nikkei 225 (N225)
  - SSE Composite Index (SSEC)

- Daily close-to-close log returns from 11th Jan. 2017 through 10th Jan. 2019 (two years) are used.

- The second half of the sample is analyzed separately as a subsample analysis.

- We perform $\hat{T}^{\text{dw}}(\mathcal{L}^*)$ with $q = 3.25$, $\bar{L}_n = [1.5 \times n/(\ln n)^{4/3}]$, and $M = 1000$.

- Data source: Realized Library (Oxford-Man Institute of Quantitative Finance).
Data: Normalized prices in recent 2 years

SPX

FTSE

N225

SSEC
Data: Log returns in recent 2 years

![Graphs of log returns for SPX, FTSE, N225, and SSEC from April 2017 to January 2019.]
Empirical results

<table>
<thead>
<tr>
<th></th>
<th>Recent 2 years</th>
<th></th>
<th>Recent 1 year</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n)</td>
<td>(L^*_n)</td>
<td>(\hat{\rho}_n(1))</td>
<td>(\hat{\rho}_{n,M}^{(dw)})</td>
</tr>
<tr>
<td>SPX</td>
<td>502</td>
<td>1</td>
<td>-0.025</td>
<td>0.951</td>
</tr>
<tr>
<td>FTSE</td>
<td>506</td>
<td>1</td>
<td>-0.092</td>
<td>0.263</td>
</tr>
<tr>
<td>N225</td>
<td>493</td>
<td>1</td>
<td>0.018</td>
<td>0.955</td>
</tr>
<tr>
<td>SSEC</td>
<td>488</td>
<td>1</td>
<td>-0.043</td>
<td>0.707</td>
</tr>
</tbody>
</table>

- For FTSE in the recent 1 year, the white noise hypothesis is **rejected** at the 10% level.
- It suggests that the recent UK stock market might be **inefficient** due to the large negative autocorrelation at lag 1.
- For other cases, weak form efficiency seems to hold.
Conclusion

- It is rather challenging to establish a formal white noise test due to the relatively weak null hypothesis and the infinitely many zero restrictions.

- Key issues are how to formulate a test statistic, how to choose a lag length, and how to compute a p-value.

- Our proposed solution is the bootstrapped max-correlation test with automatic lag selection.
  - The test statistic is based on the maximum of sample autocorrelations.
  - Escanciano and Lobato’s (2009) automatic lag selection is utilized.
  - Shao’s (2011) dependent wild bootstrap is used.
Conclusion

- Our test is asymptotically **valid** under the null hypothesis of white noise and **consistent** under the alternative hypothesis.

- Since our test allows for various filters such as AR and GARCH, it can be used for **residual diagnostics**.

- The simulation results highlight that the proposed test has sharp size and high power. In particular, it is remarkably powerful against **remote autocorrelations**.

- The empirical application on the weak form efficiency of stock markets suggests that the recent FTSE market might be **inefficient** due to the large negative autocorrelation at lag 1.
References


