1. Introduction

This paper tests for the white noise hypothesis of stock returns, using the dependent wild bootstrap.

2. Dependent Wild Bootstrap

Let $\rho(h)$ be the autocorrelation of stationary $\{y_t\}$ at lag $h$.

White noise hypothesis is $H_0: \rho(h) = 0$ for all $h \geq 1$.

1. Set a block size $b_n$. (Typically $b_n = \sqrt{n}$.)
2. Generate iid $\{\xi_1, \xi_2, \ldots, \xi_n/b_n\}$. Define
   $$\omega = \begin{bmatrix} \xi_1/b_n & \cdots & \xi_{b_n}/b_n \\ \cdots & \cdots & \cdots \\ \xi_{n-b_n+1}/b_n & \cdots & \xi_n/b_n \end{bmatrix}.'$$
3. Compute a bootstrapped autocorrelation:
   $$\hat{\rho}^{(dw)}(h) = \frac{1}{\hat{\gamma}(0)} \frac{1}{n} \sum_{t=h+1}^{n} \omega[y_t y_{t-h} - \hat{\gamma}(h)].$$
4. Repeat Steps 2-3 $M$ times and sort:
   $$\hat{\rho}^{(dw)}_{n,1}(h) < \cdots < \hat{\rho}^{(dw)}_{n,M}(h).$$
5. The 95% confidence band under $H_0$ is
   $$C(h) = \left[ \hat{\rho}^{(dw)}_{n,0.025M}(h), \hat{\rho}^{(dw)}_{n,0.975M}(h) \right].$$

3. Hidden Pitfall: Periodicity

Consider rolling window analysis (e.g. window size $n = 60$ and block size $b_n = 3$).

- In window #1 $(y_1, \ldots, y_{60})$, we have
  $$\hat{\rho}^{(dw)}_{1}(h) < \cdots < \hat{\rho}^{(dw)}_{20}(h).$$
- In window #4 $(y_4, \ldots, y_{63})$, we have
  $$\hat{\rho}^{(dw)}_{4}(h) < \cdots < \hat{\rho}^{(dw)}_{19}(h).$$
- Similar structures in windows #1, #4, #7, \ldots

$\implies$ Periodicity with $b_n = 3$ cycles.

4. Remedy: Randomized Block Size

Block size $b_n = [c \times \sqrt{n}]$. Picking $c = 1$ causes periodicity. We draw $c \sim U(0.5, 1.5)$ independently.

5. Empirical Application (S&P 500)

- White noise hypothesis is rejected due to large negative autocorrelations during Iraq War and the subprime mortgage crisis.