Testing a large set of zero restrictions in regression models, with an application to mixed frequency Granger causality

Eric Ghysels  Jonathan B. Hill  Kaiji Motegi
UNC Chapel Hill  UNC Chapel Hill  Kobe University

1 Motivation

Consider a linear regression model

$$y_t = z_i^\top \alpha + x_i^\top \beta + u_t, \quad t = 1, \ldots, n,$$

where $z_i$ is assumed to have a small dimension while $x_i$ may have a large but finite dimension $h$. Consider testing $H_0 : \beta = 0$ against $H_1 : \beta \neq 0$. The classical Wald test, whether the asymptotic or bootstrapped p-value is used, may lead to imprecise inference when $h$ is large relative to $n$. This is a well-known problem of high dimensionality or parameter proliferation, and the present paper proposes an innovative solution to this problem.

2 Methodology

Instead of model (1), we use parsimonious regression models:

$$y_t = z_i^\top \alpha_i + \beta_i x_{it} + u_{it}, \quad i = 1, \ldots, h, \quad t = 1, \ldots, n.$$ Perform the least squares for each of the $h$ models and formulate a max test statistic:

$$\hat{T}_n = \max \left\{ (\sqrt{n} \hat{\beta}_{n1})^2, \ldots, (\sqrt{n} \hat{\beta}_{nh})^2 \right\}.$$ The limit distribution of $\hat{T}_n$ under $H_0$ is non-standard, but it is straightforward to draw from the limit distribution and hence an approximate p-value is readily available.

3 Main results

The max test has a correct size under $H_0$ and is consistent under $H_1$. Our simulation study indicates that the max test has sharp size and high power in finite samples due to the parsimonious structures of the models and test statistic. Using the max test, we test Granger causality from weekly yield spread to quarterly GDP growth in the U.S. We find that the yield spread causes the GDP until 1990s and the causality vanishes after that.