A Max-Correlation White Noise Test for Weakly Dependent Time Series

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1. Introduction

Consider a covariance stationary time series \( \{y_t\} \) with mean zero. Define autocorrelations \( \rho(h) \equiv E[y_t y_{t-h}] \). Then, the white noise hypothesis is written as \( H_0: \rho(h) = 0 \) for all \( h \geq 1 \).

We use dependent wild bootstrap to allow for weak dependence under \( H_0 \).

Desired Properties of White Noise Tests
- Applicable to both observed and filtered data.
- Allow for weak dependence under \( H_0 \) (e.g. bilinear, GARCH).
- Allow for lag length \( L_n \to \infty \) with a fast rate \( L_n = o(n) \).
- Achieve sharp size and high power in finite sample.

We propose a new test that accomplishes ALL above: max-correlation test with dependent wild bootstrap.

2. Max-Correlation Test

We propose a max-correlation test statistic:

\[
T_n = \sqrt{n} \max_{1 \leq h \leq L_n} |\hat{\rho}_n(h)|.
\]

Previous tests include the Ljung-Box Q-test:

\[
Q_n = n \sum_{h=1}^{L_n} \frac{n+2}{n-h} \hat{\rho}_n(h).
\]

the generalized Andrews-Ploberger test:

\[
\text{AP}_n = \sup_{L \in (1, \infty)} \{ n(1 - \lambda^L) \left( \sum_{k=1}^{n} \lambda^{k-L} \hat{\rho}_n(h) \right)^2 \},
\]

and others (e.g. Shao’s spectral test, Hong’s standardized Q-test).

3. P-Value Computation

We use Shao’s dependent wild bootstrap (DWB) to compute p-values. This approach sidesteps extreme value theory (e.g. Xiao and Wu, 2014).

4. Simulation Study

4.1. Empirical Size (Nominal Size = 5%)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{A. } n = 100 & \text{i.i.d.} & \text{GARCH} & \text{Bilinear} \\
\hline
L = 5 & L = 21 & L = 5 & L = 21 & L = 5 & L = 21 \\
\hline
\text{Max} & .053 & .033 & .039 & .024 & .065 & .027 \\
Q & .057 & .017 & .021 & .006 & .044 & .013 \\
AP & .161 & .199 & .124 & .176 & .189 & .221 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{B. } n = 1000 & \text{i.i.d.} & \text{GARCH} & \text{Bilinear} \\
\hline
L = 5 & L = 144 & L = 5 & L = 144 & L = 5 & L = 144 \\
\hline
\text{Max} & .054 & .024 & .029 & .012 & .055 & .019 \\
Q & .044 & .020 & .020 & .000 & .041 & .002 \\
AP & .072 & .080 & .073 & .094 & .095 & .106 \\
\hline
\end{array}
\]

Max test achieves most accurate size.

- Q-test is seriously under-sized for large \( L \).
- AP test is seriously over-sized when \( n = 100 \).

4.2. Empirical Power

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{A. } n = 100 & \text{AR}(1) & \text{AR}(2) & \text{AR}(12) \\
\hline
L = 5 & L = 21 & L = 5 & L = 21 & L = 5 & L = 21 \\
\hline
\text{Max} & .271 & .121 & .593 & .354 & .082 & .166 \\
Q & .182 & .051 & .439 & .088 & .076 & .103 \\
AP & .159 & .139 & .300 & .275 & .172 & .220 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{B. } n = 1000 & \text{AR}(1) & \text{AR}(2) & \text{AR}(12) \\
\hline
L = 5 & L = 144 & L = 5 & L = 144 & L = 5 & L = 144 \\
\hline
\text{Max} & 1.00 & .996 & 1.00 & 1.00 & .063 & 1.00 \\
Q & 1.00 & .078 & 1.00 & .552 & .056 & .247 \\
AP & .970 & .954 & 1.00 & .999 & .084 & .086 \\
\hline
\end{array}
\]

Max test achieves highest power.

- In particular, the max test is powerful for remote autocorrelations (seasonality).

5. Conclusions

We establish a new white noise test based on the maximum autocorrelation.

We use dependent wild bootstrap to allow for weak dependence under \( H_0 \) (e.g. bilinear, GARCH).

Our test outperforms the Ljung-Box Q-test and the generalized Andrews-Ploberger test.

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