Non-Perturbative symplectic manifolds
and
Non-Commutative algebras

Noumi 60

RIMS Kyoto 2015

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Some references: arXiv: 0806, 1307, 1501

Van den Bergh's spaces

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with moment map $\mu(a,b) = (ab, -ba)$

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Thm (Van den Bergh '04) $\mathrm{Rep}^*(\Gamma, V)$ is a "multiplicative" (or "quasi") Hamiltonian H-space with group valued moment map $\mu(a,b) = (1+ab, (1+ba)^{-1}) \in H$

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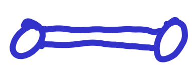
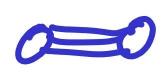
<u>Qn</u> Suppose $\Gamma =$  or  etc

then what is $\mathrm{Rep}^*(\Gamma, V)$?

SPECIMEN ALGORITHMI SINGVLARIS.

Auctore

L. EVLERO.

1.

Consideratio fractionum continuarum, quarum vsum vberrimum per totam Analysin iam aliquoties ostendi, deduxit me ad quantitates certo quodam modo ex indicibus formatas, quarum natura ita est comparata, vt singularem algorithmum requirat. Cum igitur summa Analyseos inuenta maximam partem algorithmo ad certas quasdam quantitates accommodato

6. Haec ergo teneatur definitio signorum ( ), inter quae indices ordine a sinistra ad dextram scribere constitui; atque indices hoc modo clausulis inclusi inposterum denotabunt numerum ex istis indicibus formatum. Ita a simplicissimis casibus inchoando, habebimus:

$$
\begin{aligned}
&(a) && = a \\
&(a,b) && = ab + 1 \\
&(a,b,c) && = abc + c + a \\
&(a,b,c,d) && = abcd + cd + ad + ab + 1 \\
&(a,b,c,d,e) && = abcde + cde + ade + abe + abc + e + c + a
\end{aligned}
$$

etc.

ex

"Euler's continuant polynomials"

How to define "multiplicative version"?

complex Lie group G $\Longrightarrow$ Lie algebra $\mathfrak{g} = T_e G$

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(generating series is perturbative expansion about trivial connection of connection matrix $0 \leftrightarrow 1$)

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$$\left(\frac{A}{z^2} + \frac{B}{z}\right)dz \quad \Longrightarrow \quad \text{Poisson Lie group underlying } U_q\mathfrak{g}$$

$$\mathbb{C}[\mathfrak{g}^*] = \mathrm{Sym}\,\mathfrak{g} \underset{\text{quantize}}{\rightsquigarrow} U\mathfrak{g}$$

(deform multiplication)

$$
\begin{tikzcd}
 & U_q\mathfrak{g} \\
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"quantize" (deform comultiplication)

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$$

Thm (2001) G^* is the space of monodromy/Stokes data of connections $\left(\frac{A}{z^2} + \frac{B}{z}\right)dz \Big|_{\text{unit disc}}$ $\quad A \in \mathfrak{t}_{reg}$ fixed, $B \in \mathfrak{g} \cong \mathfrak{g}^*$

and the desired nonlinear Poisson structure appears this way

Cartoon

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Hamiltonian geometry

$\mathcal{O} \subset \mathfrak{g}^*$, T^*G

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Hamiltonian geometry

$\mathcal{O} \subset \mathfrak{g}^*$, $\quad T^*G$

$$\Big\downarrow \; \mu^{-1}(0)/G$$

Additive symplectic geometry

$\mathcal{O}_1 \times \cdots \times \mathcal{O}_m /\!/ G$

Cartoon

∞-d Ham^{n} geometry

e.g connections on C^{∞} bundles / Riemann surfaces

$\cup$

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Betti spaces, character varieties

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$\mathcal{C} \subset G$, $D = G \times G$

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$\mathcal{O}_1 \times \cdots \times \mathcal{O}_m /\!\!/ G$
$\mathcal{M}^*$

$\xRightarrow{RH}$

Multiplicative symplectic geometry
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$\mathcal{M}_B$

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$\Big\downarrow \mu^{-1}(0)/G$

Additive symplectic geometry

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$\xRightarrow{\text{RHB}}$

quasi-Hamiltonian geometry

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Multiplicative symplectic geometry

Betti spaces, wild character varieties

$\mathcal{M}_B$

Wild Character Varieties

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Fix G (e.g $GL_n(\mathbb{C})$)

symplectic variety

Σ compact Riemann Surface $\Longrightarrow$ $\mathcal{M}_B = \mathrm{Hom}(\pi_1(\Sigma), G)/G$

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$\mathcal{M}_{DR} = \{\text{Alg. connections on } G\text{-bundles on } \Sigma\}/\text{isom}$

Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

symplectic variety

Σ compact Riemann Surface with marked points $\underset{\sim}{a} = (a_1, \ldots, a_m)$ $\Longrightarrow$ $\mathcal{M}_B = \mathrm{Hom}(\pi_1(\Sigma), G)/G$

$\cong$ RH

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Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

Σ compact Riemann Surface with marked points $\underset{\sim}{a} = (a_1, \dots, a_m)$

$\Sigma^\circ = \Sigma \setminus \underset{\sim}{a}$

Poisson variety

$$\Longrightarrow \quad \mathcal{M}_B^{tame} = \mathrm{Hom}(\pi_1(\Sigma^\circ), G)/G$$

$$\Big\| \wr \; RH$$

$$\mathcal{M}_{DR} = \left\{ \begin{array}{c} \text{Alg. connections on } G\text{-bundles on } \Sigma^\circ \\ \text{with reg. sing.s} \end{array} \right\} \Big/ \text{isom}$$

Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

Poisson scheme (∞-type)

Σ compact Riemann Surface with marked points $\underset{\sim}{a} = (a_1, \dots, a_m)$ $\implies$ $\mathcal{M}_B$

$\cong$ RHB

$\Sigma^\circ = \Sigma \setminus \underset{\sim}{a}$

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Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

Poisson variety

Σ compact Riemann Surface with marked points $\underset{\sim}{a} = (a_1, \dots, a_m)$ and irregular types $\underset{\sim}{Q} = Q_1, \dots, Q_m$ $\implies$ $\mathcal{M}_B$

$\mathcal{M}_B \overset{RHB}{\cong} \mathcal{M}_{DR}$

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cartan subalg.

$Q_i \in \mathfrak{t}_i \subset \mathfrak{g}((z_i))$

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$$\mathcal{M}_B \overset{\text{RHB}}{\cong} \mathcal{M}_{DR} = \{\text{Alg. connections on } G\text{-bundles on } \Sigma^\circ \text{ with irreg. types } \underset{\sim}{Q}\} / \text{isom}$$

$\nabla \cong dQ_i + \Lambda_i \frac{dz_i}{z_i} + \text{holom.}$

e.g. $Q_i \in \mathfrak{t}((z_i)) \subset \mathfrak{g}((z_i))$, $\mathfrak{t} \subset \mathfrak{g}$ Cartan subalg.

Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

Wild Riemann surface $(\Sigma, \underset{\sim}{a}, \underset{\sim}{Q}) \implies$ wild character variety

Σ compact Riemann Surface with marked points $\underset{\sim}{a} = (a_1, \dots, a_m)$ and irregular types $\underset{\sim}{Q} = Q_1, \dots, Q_m$ $\implies$ $\mathcal{M}_B$

$$\Big\| \wr \; RHB$$

$$\mathcal{M}_{DR} = \{ \text{Alg. connections on } G\text{-bundles on } \Sigma^\circ \text{ with irreg. types } \underset{\sim}{Q} \} / \text{isom}$$

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e.g. $\quad Q_i \in \underbrace{\mathfrak{t}}_{\text{cartan subalg.}}((z_i)) \subset \mathfrak{g}((z_i)) \qquad \mathfrak{t} \subset \mathfrak{g}$

Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

E.g. $(\text{Disc}, 0, Q)$ $G = GL_2(\mathbb{C})$

$Q = A/z^k$, $A = \begin{pmatrix} a & \\ & b \end{pmatrix}$ $a \neq b$

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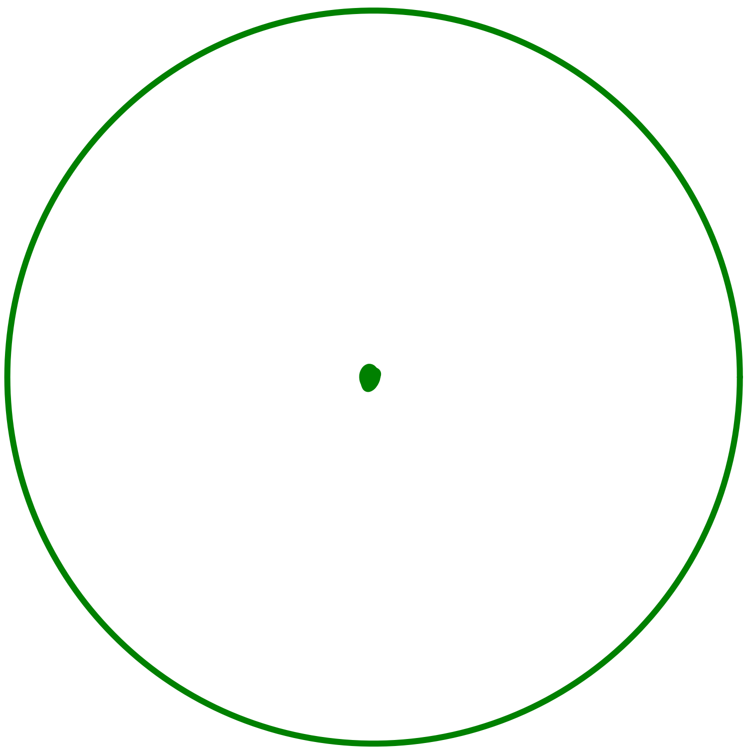

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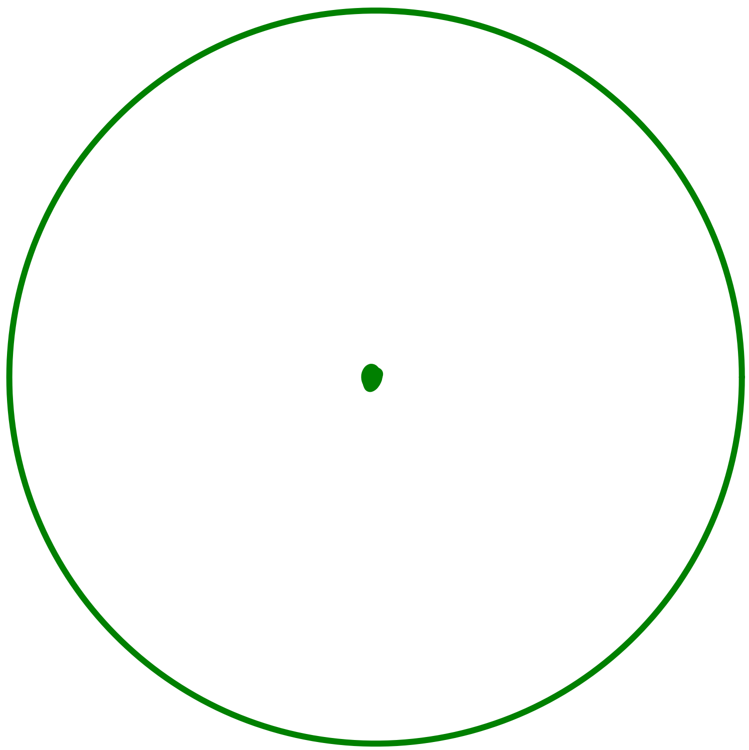

$Q \Rrightarrow$

- centraliser group $H = T = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \subset G$
  $C_G(Q)$

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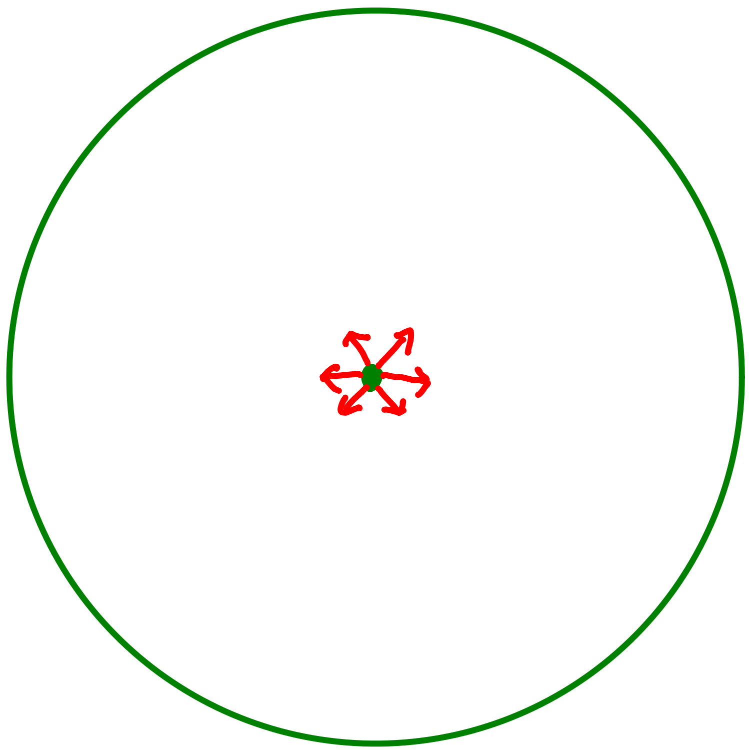

$Q \Rightarrow$

- centraliser group $C_G(Q)$ $H = T = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \subset G$
- singular directions $\mathbb{A}$

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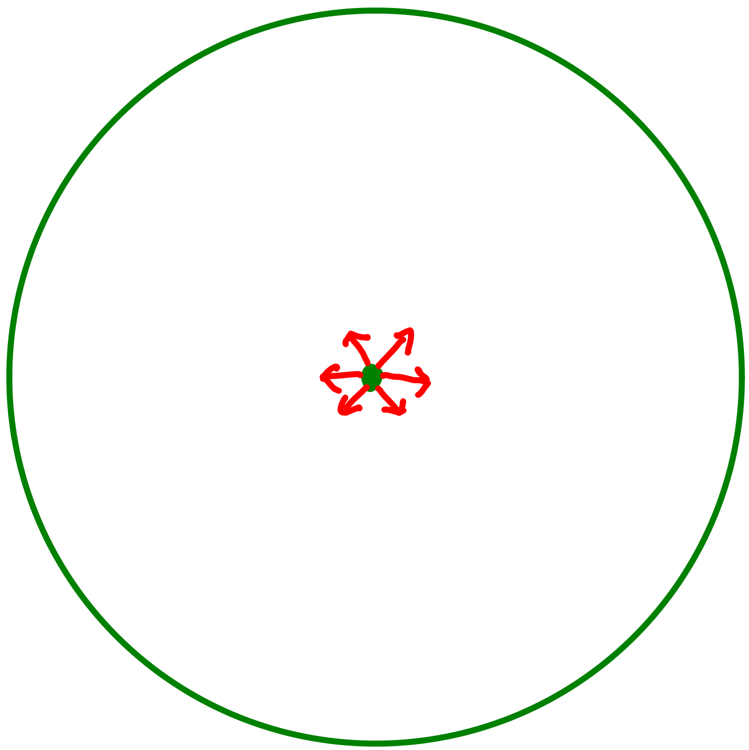

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- Stokes groups $\mathbb{S}\text{to}_d \subset G$ $\forall d \in \mathbb{A}$

  $\cong U_+$ or U_- here

  $\begin{pmatrix} 1 & * \\ & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & \\ * & 1 \end{pmatrix}$

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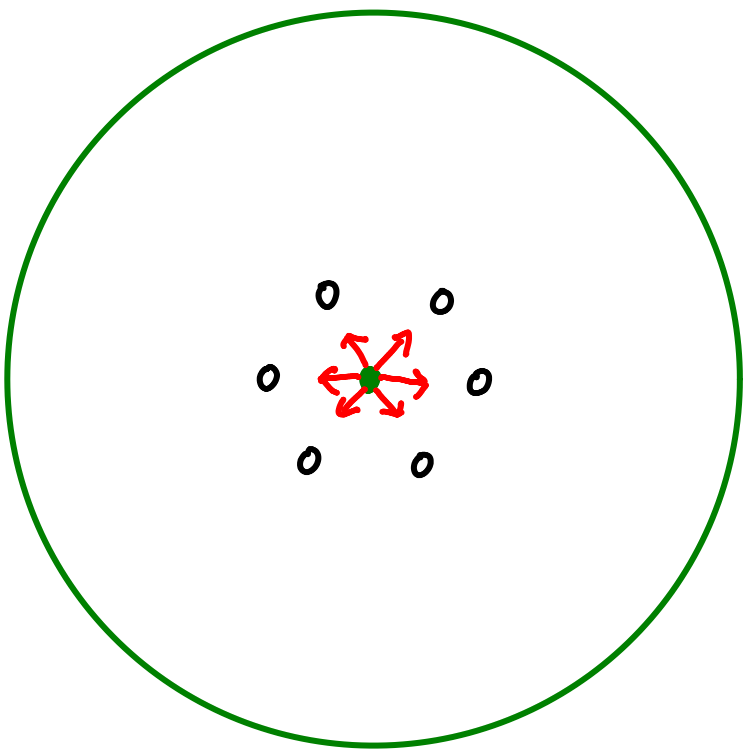

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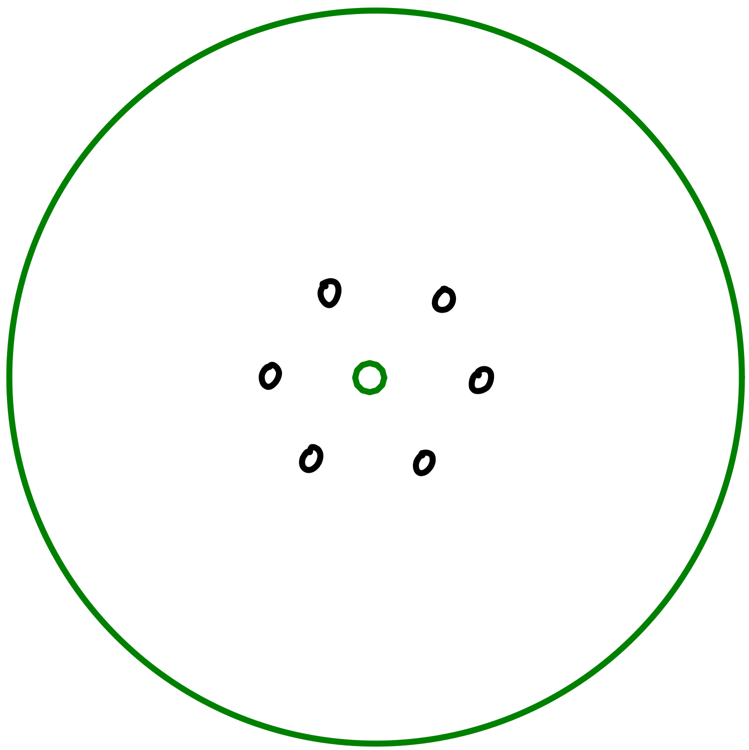

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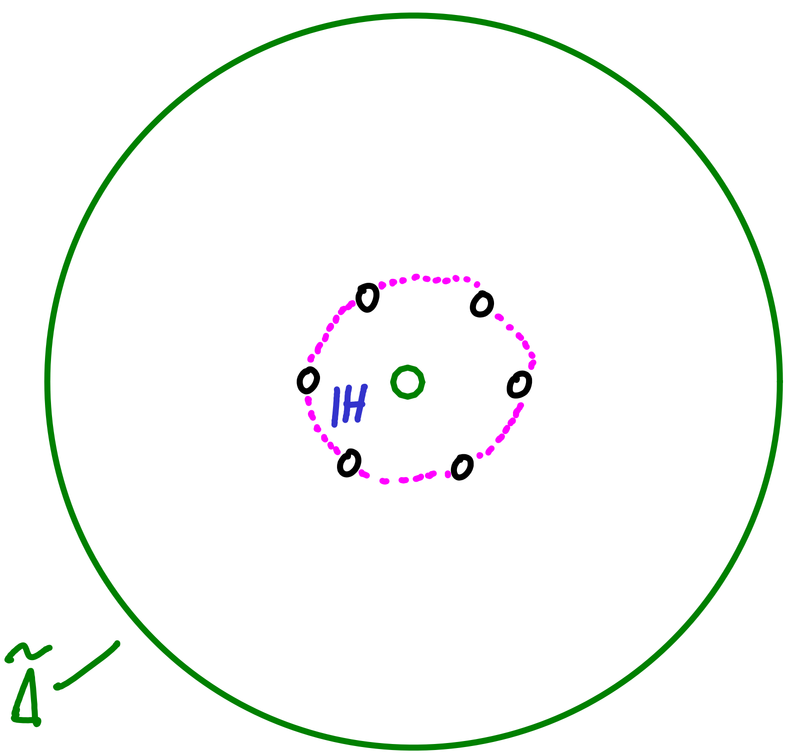
  $\cong U_+$ or U_- here
  $\begin{pmatrix} 1 & * \\ & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & \\ * & 1 \end{pmatrix}$

Wild Character Varieties

Fix G (e.g $GL_n(\mathbb{C})$)

E.g. $(\text{Disc}, 0, Q)$ $\quad G = GL_2(\mathbb{C})$

$$Q = A/z^k, \quad A = \begin{pmatrix} a & \\ & b \end{pmatrix} \quad a \neq b$$



$Q \Rightarrow$

- centraliser group $C_G(Q)$ $\quad H = T = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix} \subset G$
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o $\quad e(d)$ extra punctures

IH $\quad$ halo/annulus

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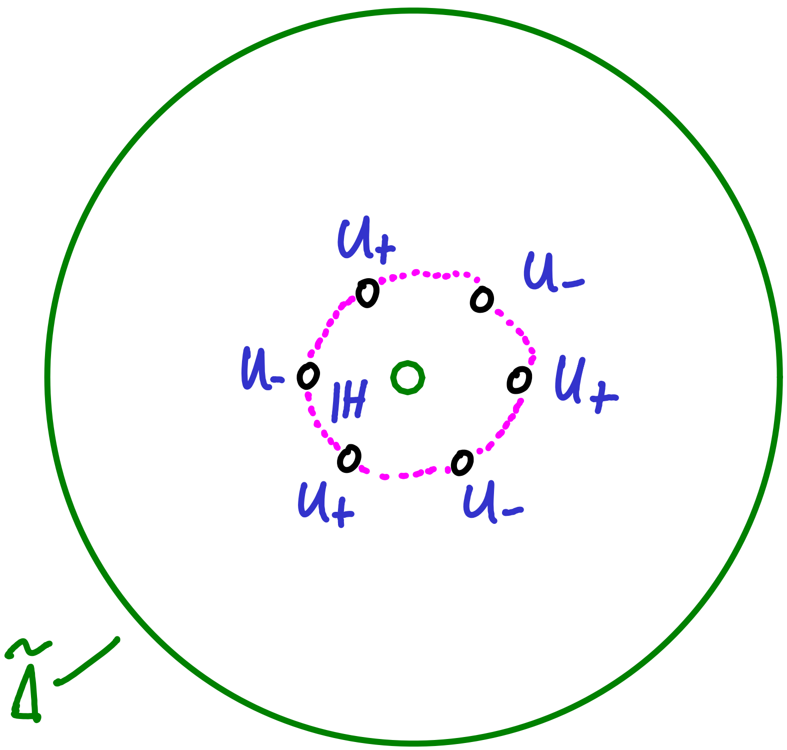

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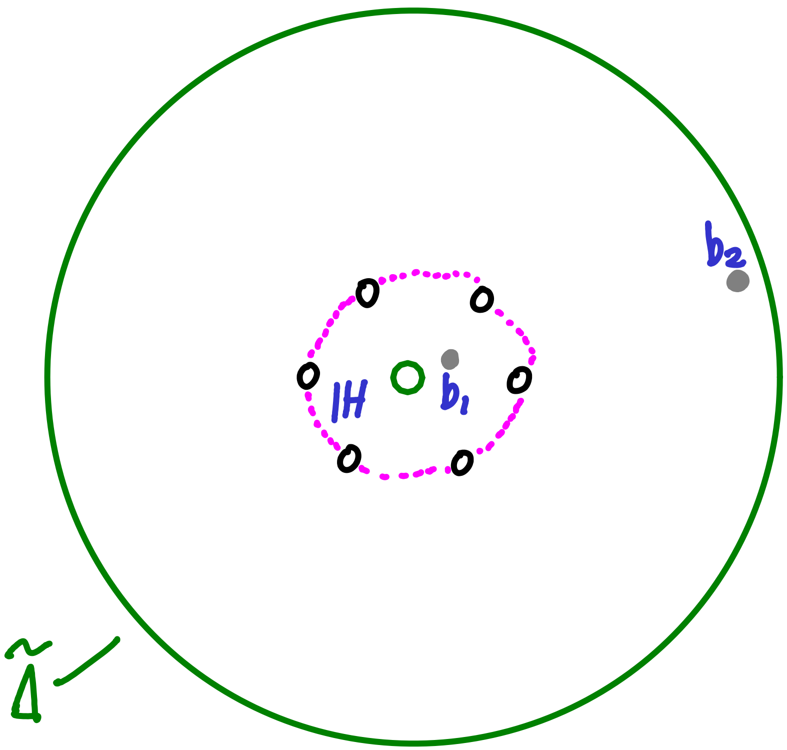

basepoints b_1, b_2

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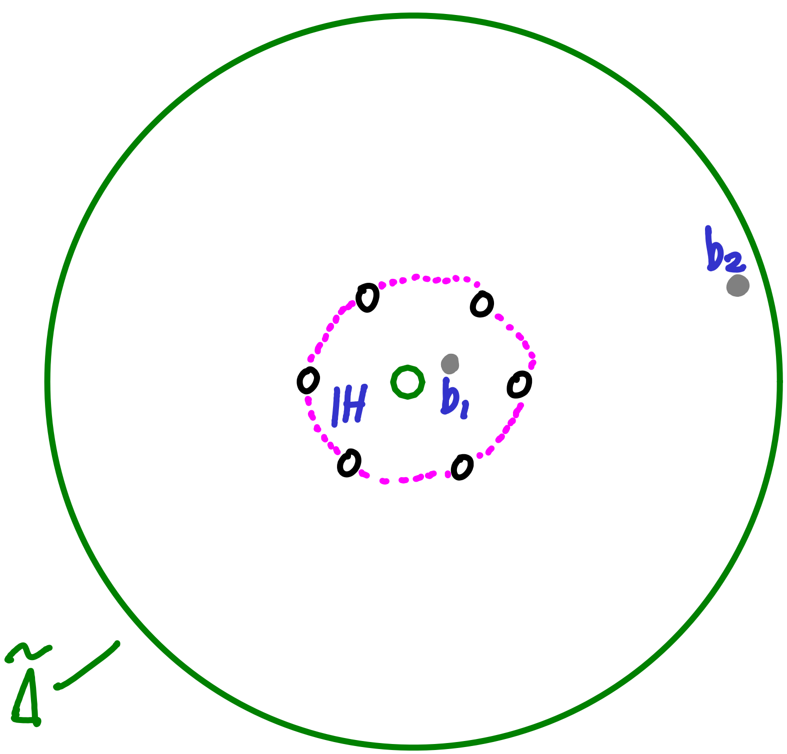

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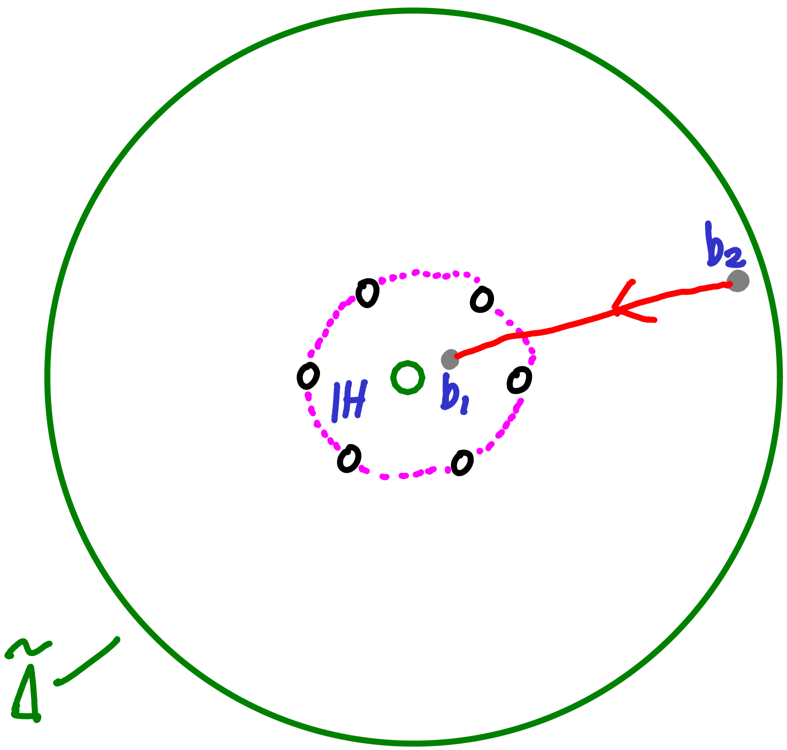

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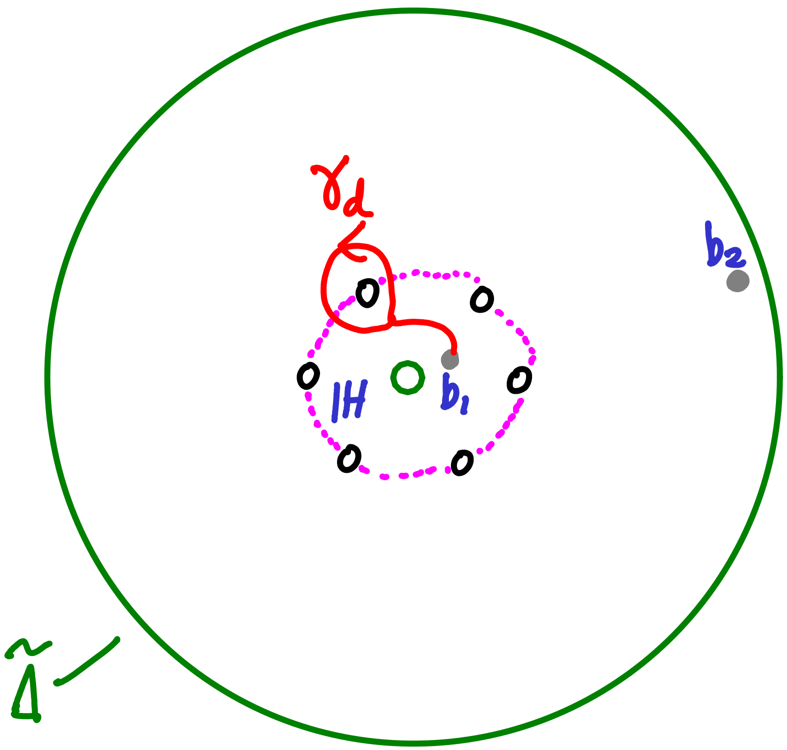

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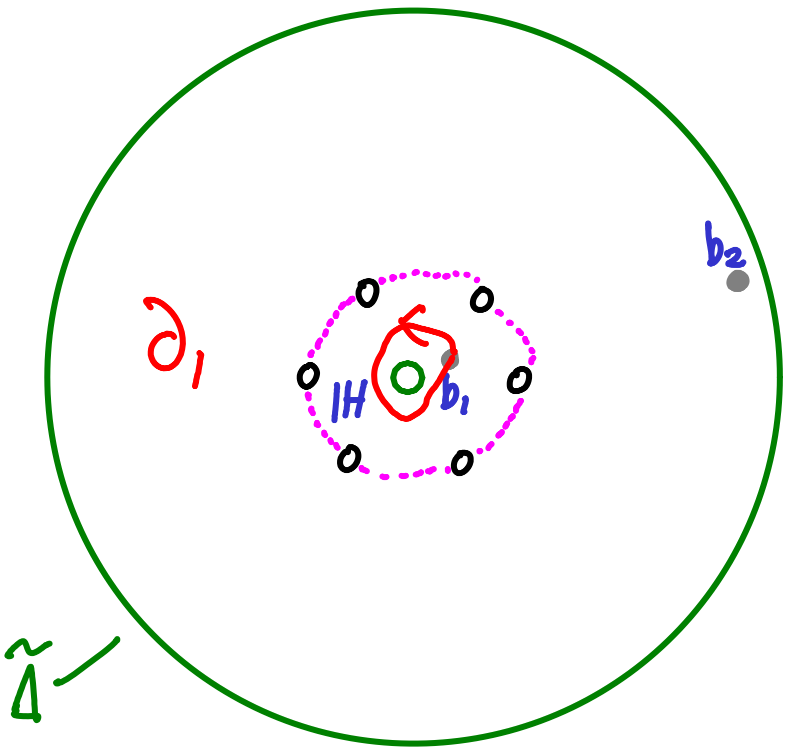

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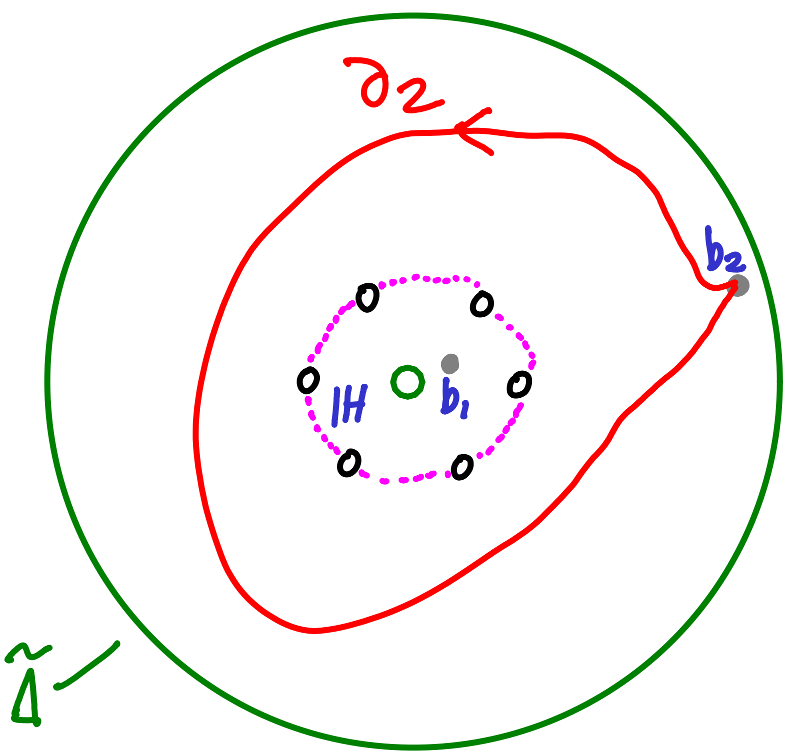

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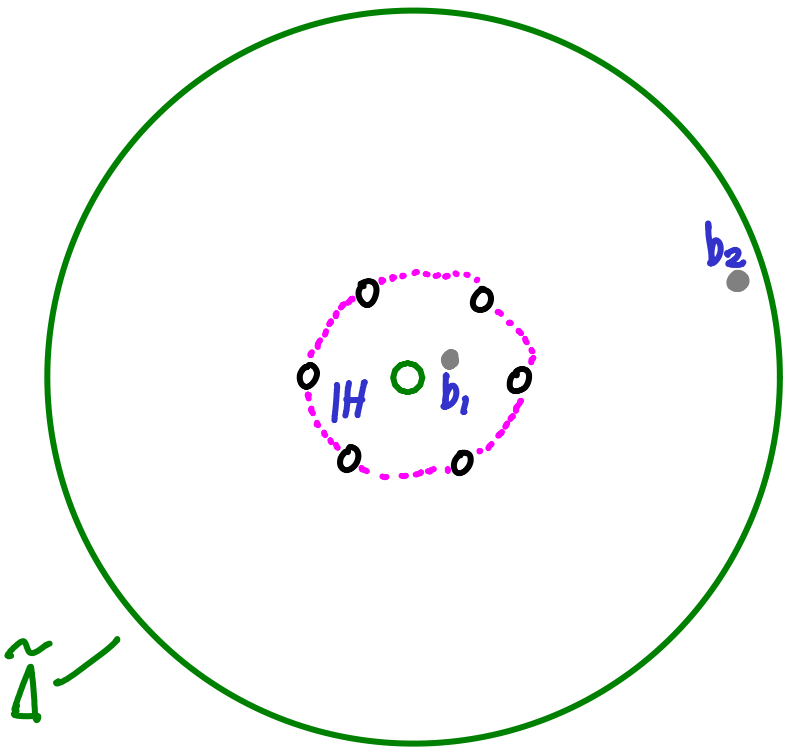

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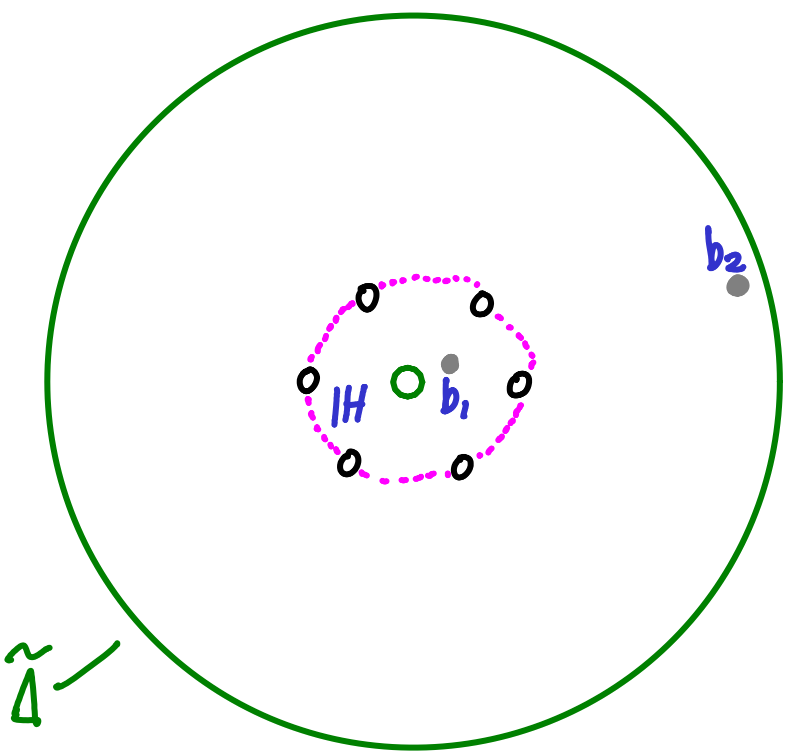

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$$= \left\{ \rho : \Pi \to G \;\middle|\; \begin{array}{l} \rho(\partial_1) \in H \\ \rho(\gamma_d) \in \mathrm{Sto}_d \quad \forall d \in \mathcal{A} \end{array} \right\}$$

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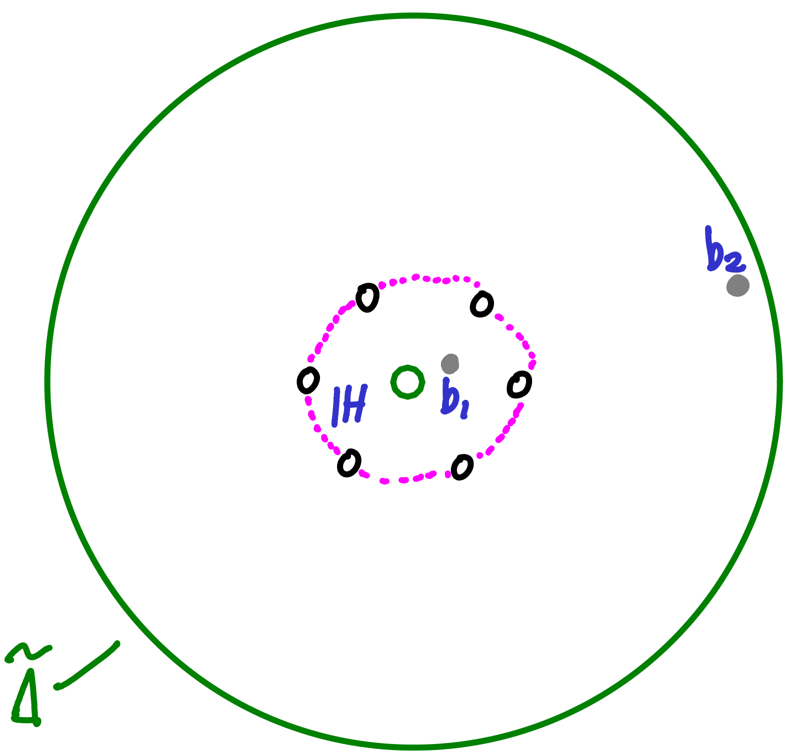

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Thm (arXiv 0203.*****)

$\tilde{\mathcal{M}}_B$ is a quasi-Hamiltonian $G \times H$ space

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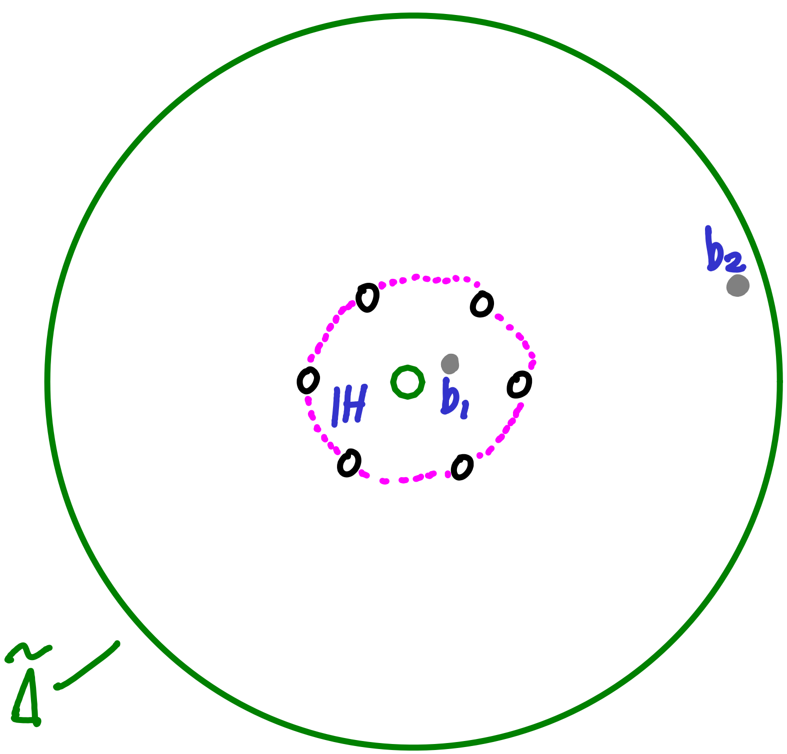

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$$\begin{array}{c} \Downarrow \\ (C, \underset{\sim}{S}, h) \end{array} \qquad \underset{\sim}{S} = (S_1, \dots, S_{2k}) \qquad S_{\text{odd/even}} \in U_{+/-}$$

Moment map $\quad \mu(C, \underset{\sim}{S}, h) = (C^{-1} h S_{2k} \cdots S_2 S_1 C, \ h^{-1}) \in G \times H$

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$$= \mu_G^{-1}(1)/G \qquad = \widetilde{\mathcal{M}}_B((\mathbb{P}^1, 0, Q))$$

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$$\cong \{ (\underset{\sim}{S}, h) \in (U_+ \times U_-)^k \times H \mid h S_{2k} \cdots S_2 S_1 = 1 \}$$

Wild Character Varieties

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$\{ (\underset{\sim}{S}, h) \in (U_+ \times U_-)^k \times H \mid h S_{2k} \cdots S_2 S_1 = 1 \}$ is a quasi-Hamiltonian H-space

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E.g. $k=2$ $\quad \left(\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix} \right)_{11} = 1 + ab$

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Lemma

$$\left(\begin{pmatrix} 1 & a_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ b_1 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & a_r \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ b_r & 1 \end{pmatrix} \right)_{11} = (a_1, b_1, \dots, a_r, b_r)$$

— Euler's continuants are group valued moment maps

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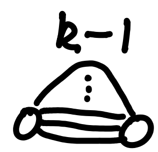
$$\cong \{ \underset{\sim}{a}, \underset{\sim}{b} \in \mathrm{Rep}(\Gamma, V) \mid (a_1, b_1, \dots, a_{k-1}, b_{k-1}) \neq 0 \}$$

$\Gamma =$ 

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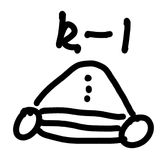
$$=: \mathrm{Rep}^*(\Gamma, V)$$

$\Gamma =$ 

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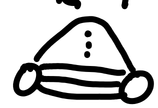
$\Gamma =$  $k-1$, $V = \mathbb{C} \oplus \mathbb{C}$

[Similarly for $V = V_1 \oplus V_2$ any dimension
(2009-2015) Γ any "fission graph"]

$$\mu(a_1, \dots, b_{k-1}) = \left((a_1, b_1, \dots, a_{k-1}, b_{k-1}), (b_{k-1}, \dots, b_1, a_1)^{-1} \right)$$

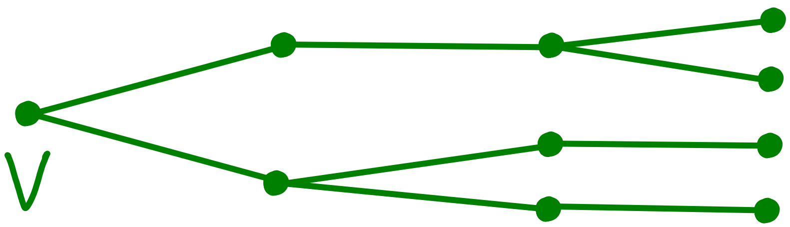
Fission graphs

$G = GL(V)$

$$Q = A_r/z^r + \cdots + A_1/z \qquad (A_i \in \mathfrak{t})$$

$$= A_r w^r + \cdots + A_1 w \qquad w = 1/z$$

$r = 3:$



"fission tree"

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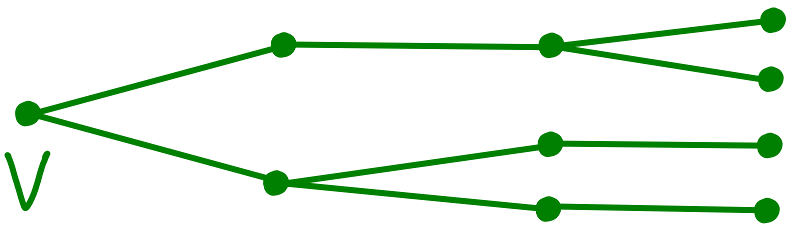

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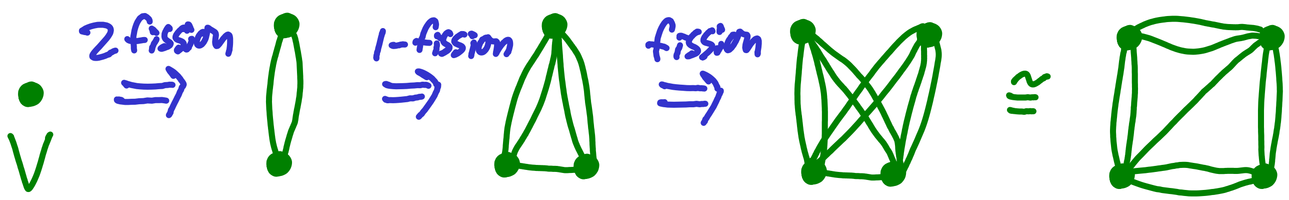
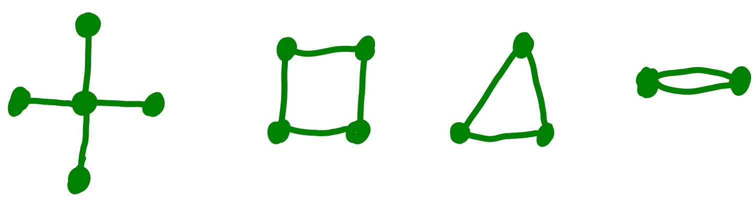


"fission graph" $\Gamma(Q)$

- $r=2$ get all complete k-partite graphs
- e.g.



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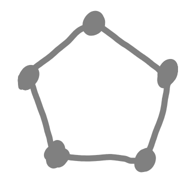

$Q = \operatorname{diag}(q_1, \ldots, q_n) \Rightarrow$ nodes $= \{1, \ldots, n\}$, # edges $i \leftrightarrow j$ $= \deg_w(q_i - q_j) - 1$

Wild Character Varieties

In this example $\left(\, (\mathbb{P}^1, 0, Q) \quad Q = A/z^k, \quad GL_2(\mathbb{C}) \right)$

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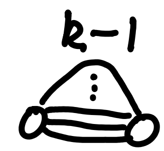
$\Gamma =$ 

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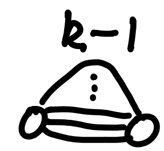
$\Gamma =$ 

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E.g. $k=3$ (Painlevé 2 Betti space)

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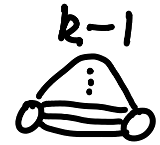
$\Gamma =$ , $V = \mathbb{C} \oplus \mathbb{C}$

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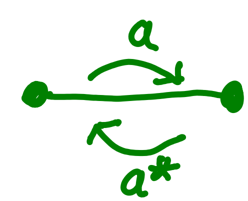

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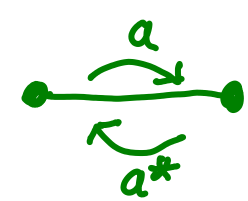

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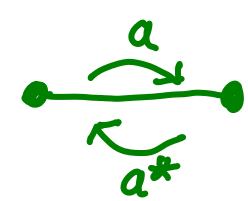

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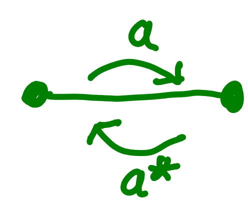

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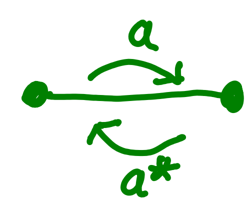

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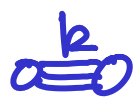
Eg. $\Gamma =$  $\quad q = (q_1, q_2) \in (\mathbb{C}^*)^I$

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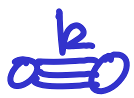
Eg. $\Gamma =$  $\qquad q = (q_1, q_2) \in (\mathbb{C}^*)^I$

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(more examples in arXiv:1307.****)

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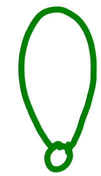
- From Okamoto expect Painlevé 1 Betti space $\sim \Gamma =$  $A_0^{(1)}$

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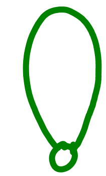
- From Okamoto expect Painlevé 1 Betti space $\sim \Gamma =$  $A_0^{(1)}$ $V = \mathbb{C}^d$

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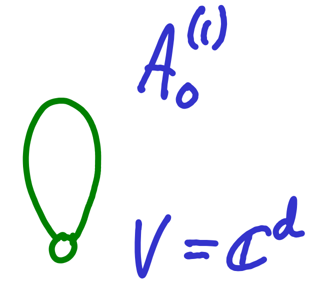

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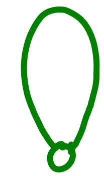
- From Okamoto expect Painlevé 1 Betti space $\sim \Gamma =$ 

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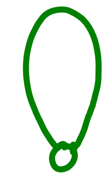
- From Okamoto expect Painlevé 1 Betti space $\sim \Gamma =$  $A_0^{(1)}$, $V = \mathbb{C}^d$

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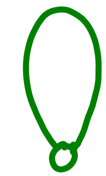
- From Okamoto expect Painlevé 1 Betti space $\sim \Gamma =$  $A_0^{(1)}$, $V = \mathbb{C}^d$

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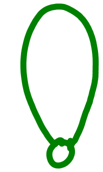
§3 Odd continuants (work with D. Yamakawa) $\Gamma =$  $V = \mathbb{C}^d$

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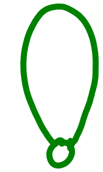
$\Gamma =$  $V = \mathbb{C}^d$

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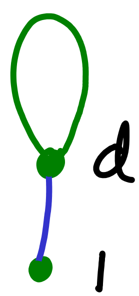
Other reductions:

$\mathrm{Rep}^*\Big($ 

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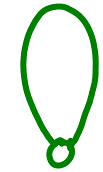
higher/hyperbolic/Hilbert Painlevé 1

§3 Odd continuants (work with D. Yamakawa) $\Gamma =$  $V = \mathbb{C}^d$

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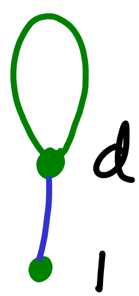
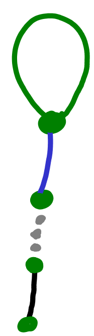
If a, c invertible then $\boxed{\mu = c a^{-1} c^{-1} a}$ If $d=1$ get $\mathcal{M}_B(\text{Painlevé } 1)$

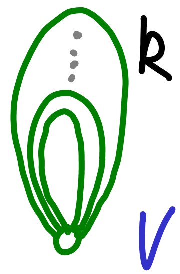
Other reductions:

$\mathrm{Rep}^*\Big($  $, \mathbb{C}^d \oplus \mathbb{C} \Big) /\!/_q H \cong \mathcal{M}_B(h P_1^{(d)})$ dim $2d$

higher/hyperbolic/Hilbert Painlevé 1

$\mathrm{Rep}^*\Big($  $, \oplus V_i \Big) /\!/_q H \cong \mathcal{M}_B(\text{matrix } P_1)$

§3 Odd continuants (work with D. Yamakawa)

More generally if $\Gamma =$ 

$V = \mathbb{C}^d$

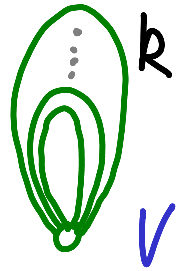
$(r = 2k+1)$

- **Thm** $\mathrm{Rep}^*(\Gamma, V) := \{ a_1, \dots, a_r \in \mathrm{End}(V) \mid (a_1, \dots, a_r) = 1 \}$

is a quasi-Hamiltonian $GL(V)$-space of dimension $2d^2k$

with moment map $\mu(a_1, \dots, a_r) = (a_r, \cdots, a_2, a_1)$

§3 Odd continuants (work with D. Yamakawa)

More generally if $\Gamma =$ 

$V = \mathbb{C}^d$

• **Thm** $\quad (r = 2k+1)$

$$\mathrm{Rep}^*(\Gamma, V) := \{ a_1, \dots, a_r \in \mathrm{End}(V) \mid (a_1, \dots, a_r) = 1 \}$$

is a quasi-Hamiltonian $GL(V)$-space of dimension $2d^2k$

with moment map $\quad \mu(a_1, \dots, a_r) = (a_r, \dots, a_2, a_1)$

– and similarly for any twisted irregular type Q $\quad (G = GL(V))$