

# Kobe-Lyon Summer School in Mathematics 2015

## On Quivers : Computational Aspects and Geometric Applications

July 21 - 31, 2015  
B301, Graduate School of Science, Kobe University

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| 1st week (21–24, July) |
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**Nobuki TAKAYAMA (Kobe)**

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### **1. Basics on ideals and Gröbner basis: Dickson's lemma, division or reduction, Buchberger's criterion, confluence.**

A "good" set of generators of an ideal  $I$  is called a Gröbner basis. It satisfies a property that a given polynomial  $f$  belongs to the ideal  $I$  if and only if  $f$  is reduced to 0 by divisions by the Gröbner basis. Gröbner bases can be constructed by the Buchberger algorithm. Reference: Cox, Little, O'Shea, Ideals, Varieties and Algorithms, Chapters 1 and 2.

### **2. Integration of $D$ -modules and an algorithm for it.**

The Gröbner basis theory of the ring of differential operators is analogous to that for the ring of polynomials as long as we consider left ideals. However, important constructions such as the integration functor on the direct image functor for left  $D$ -modules require to consider a sum of left and right ideals. We explain a fundamental idea to consider such sums in algorithmic way. We also illustrate applications to statistics. Reference: Hibi et al, Gröbner basis : statistics and software systems, Chapter 6.

**Rouchdi BAHLOUL (Lyon1)**

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### **Gröbner bases in $D$ -modules and application to Bernstein-Sato ideals**

#### **1. Standard bases in $D$ -modules.**

We will talk about standard bases for rings of differential operators. We shall treat the global and the local case (with polynomial and power series coefficients).

#### **2. Applications to Bernstein-Sato ideals.**

We shall introduce Bernstein-Sato ideals. We shall give algorithms to compute these ideals and show that these algorithms can also involve some more theoretical results.

**Philippe MALBOS (Lyon1)**

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### **Non-commutative Gröbner basis : applications and generalizations**

- 1. Introduction to rewriting and linear rewriting**
- 2. Generators, relations and syzygies.**
- 3. Construction of resolutions of algebras and path algebras**
- 4. Higher-dimensional linear rewriting**

The aim of these four lectures is to provide a summary of the theory of linear rewriting and the application of this theory to the construction of projective resolutions for algebras and path algebras. We review fundamental notions of rewriting theory and we introduce both linear rewriting systems and noncommutative Gröbner bases. We describe algorithmic ways to compute free resolutions using methods defined by Anick, Green and Kobayashi. Finally, we define linear polygraphs as higher dimensional linear rewriting systems. We will show how to construct polygraphic resolutions for algebras, starting from a convergent presentation, and how to relate these resolutions with the Koszul property.

**Satoshi AOKI (Kobe)**

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**Markov basis and Gröbner basis in Statistics**

Markov chain Monte Carlo method is used to calculate conditional  $p$  values for statistical testing problems. To construct a connected Markov chain over the given sample space, we can use the Markov basis, which is given as the Gröbner basis of the toric ideal. This is one of the applications of the Gröbner basis theory in statistics. Reference: Chapter 4 of Gröbner Bases, Statistics and Software Systems, T. Hibi (ed.), Springer, 2013.

**Nohra HAGE (Lyon1)**

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**Study of plactic monoids by rewriting methods.**

The structure of plactic monoids appears in various problems in algebra, representation theory and algebraic combinatorics. The plactic monoid can be defined for any finite dimensional complex simple Lie algebra. Using rewriting methods, we compute a coherent presentation of the plactic monoid of type A which allows us to compute a polygraphic resolution of it. Moreover, using a homotopical reduction procedure, we reduce this coherent presentation to a minimal one. In addition, we construct an explicit finite convergent presentation of the plactic monoid of type C by introducing admissible column generators.

**Cyrille CHENAVIER (Lyon1)**

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**Confluence algebras and acyclicity of the Koszul complex**

The Koszul complex of an  $N$ -homogeneous algebra was introduced by Berger to define the so-called  $N$ -Koszul algebras. In particular, Berger studied the property of  $N$ -Koszulness through computational approaches and proved that an algebra admitting a side-confluent presentation is  $N$ -Koszul if and only if the extra-condition holds. However, his proof does not provide an explicit contracting homotopy for the Koszul complex. After recalling the construction of the Koszul complex we will present a way to construct such a contracting homotopy. The property of side-confluence enables us to define specific representations of confluence algebras. These representations provide a candidate for the contracting homotopy. When the extra-condition holds, it turns out that this candidate works.

**Clément ALLEAUME (Lyon1)**

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**Study of monoidal linear categories by rewriting methods**

Monoidal linear categories play an increasingly important role in representation theory. We study those categories from a polygraphical point of view. A polygraph is a generalisation of a presentation by generators and relations studied in higher dimensional rewriting theory. After introducing the concept of  $(3,2)$ -linear polygraph, we will give some applications to monoidal categories such that the category of Bott-Samelson bimodules. Some algebraic aspects of this study will be given.

2nd week (27–31, July)

**Kenji IOHARA (Lyon1)**

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**Introduction to representations of quivers.**

I will introduce some basic notions on quivers and their representations. Providing several examples, I will explain Gabriel's theorem on a parametrization of indecomposable representations over a quiver of finite type. Some representations of a quiver of non finite type will be also treated.

**Antoine CARADOT (Lyon1)**

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**Deformations and resolutions of Kleinian singularities**

The Kleinian singularities have been studied since Felix Klein first described them in 1884. They are isolated singularities on surfaces in the complex 3-dimensional space, which can be obtained as quotients of  $\mathbb{C}^2$  by finite subgroups of  $SU_2(\mathbb{C})$ . The resolutions of these singularities lead to a connection with simple Lie algebras of type A, D and E. We can therefore speak about singularities of type A, D or E. After defining the singularities and explaining the aforementioned connection, I will present a construction due to P. Slodowy and H. Cassens who gave a description of the resolutions and deformations of Kleinian singularities in terms of quivers theory using a result called the McKay correspondence. I will conclude this presentation by an ongoing work consisting of generalizing P. Slodowy and H. Cassens results to other singularities linked to the inhomogeneous Lie algebras B, C, F and G.

**Yoshiyuki KIMURA (Kobe)**

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**Introduction to quiver varieties**

In this talk, I will introduce Nakajima quiver varieties, which are holomorphic symplectic varieties parametrizing (framed) representations of preprojective algebra, that is, they are defined as Hamiltonian reductions of cotangent bundle of quiver representations. They give an important large class of holomorphic symplectic varieties which naturally arise in geometric representation theory and related integrable systems. We provide some examples in finite types and affine types.

**Yuya TAKAYAMA (RIMS, Kyoto)**

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**Quivers and moduli spaces of instantons and sheaves.**

In this talk, I will illustrate some relations between quivers and moduli spaces of instantons and sheaves. In particular, I will explain them for (ordinary) quiver varieties, chainsaw quiver varieties and handsaw quiver varieties.

**Masa-Hiko SAITO (Kobe)**

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**Application of quiver varieties to the control theory**

TBA

**Nobuhiko TAHARA (Kobe)**

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**Explicit families of certain linear connections on  $\mathbb{P}^1 \setminus \{0, 1, \infty\}$**

Three spectral types are known each of which determine two-dimensional moduli space of Fuchsian system with three regular singularity on  $\mathbb{P}^1$ . I will explain an approach to construct explicit families of linear connections for these three cases, introducing the apparent singularity of the associated meromorphic connections as a coordinate of the moduli space.

**Kazunori MIYAZAKI (Kobe)**

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**On some examples of moduli spaces of meromorphic connections on  $\mathbb{P}^1$ .**

Painlevé equations can be derived from isomonodromic deformations of meromorphic connections over the projective line. In this talk, we will provide some examples of moduli spaces of meromorphic connections and phase spaces of Painlevé equations.

**Arata KOMYO (Kobe)**

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**Geometric description of the moduli space of parabolic connections on  $\mathbb{P}^1 \setminus \{t_1, \dots, t_5\}$  and the universal family.**

In this talk, I will explain a relationship between the moduli space of parabolic connections on  $\mathbb{P}^1 \setminus \{t_1, \dots, t_5\}$  and some Hilbert scheme of points on a surface. Moreover, we construct the universal family of the moduli space.

**Daisuke YAMAKAWA (Tokyo Institute of Technology)**

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**Applications of quiver varieties to moduli spaces of connections on  $\mathbb{P}^1$ , I**

In this lecture I will explain that some moduli spaces of meromorphic connections on the trivial bundle over the Riemann sphere can be identified with quiver varieties.

**Kazuki HIROE (Josai)**

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**Applications of quiver varieties to moduli spaces of connections on  $\mathbb{P}^1$ , II**

The additive Deligne-Simpson problem asks the existence of stable meromorphic connections with the prescribed local isomorphic classes on trivial bundles over the complex projective line. I will explain how the realization of the moduli spaces of meromorphic connections as quiver varieties are effectively applied to the additive Deligne-Simpson problem after the work of Crawley-Boevey.