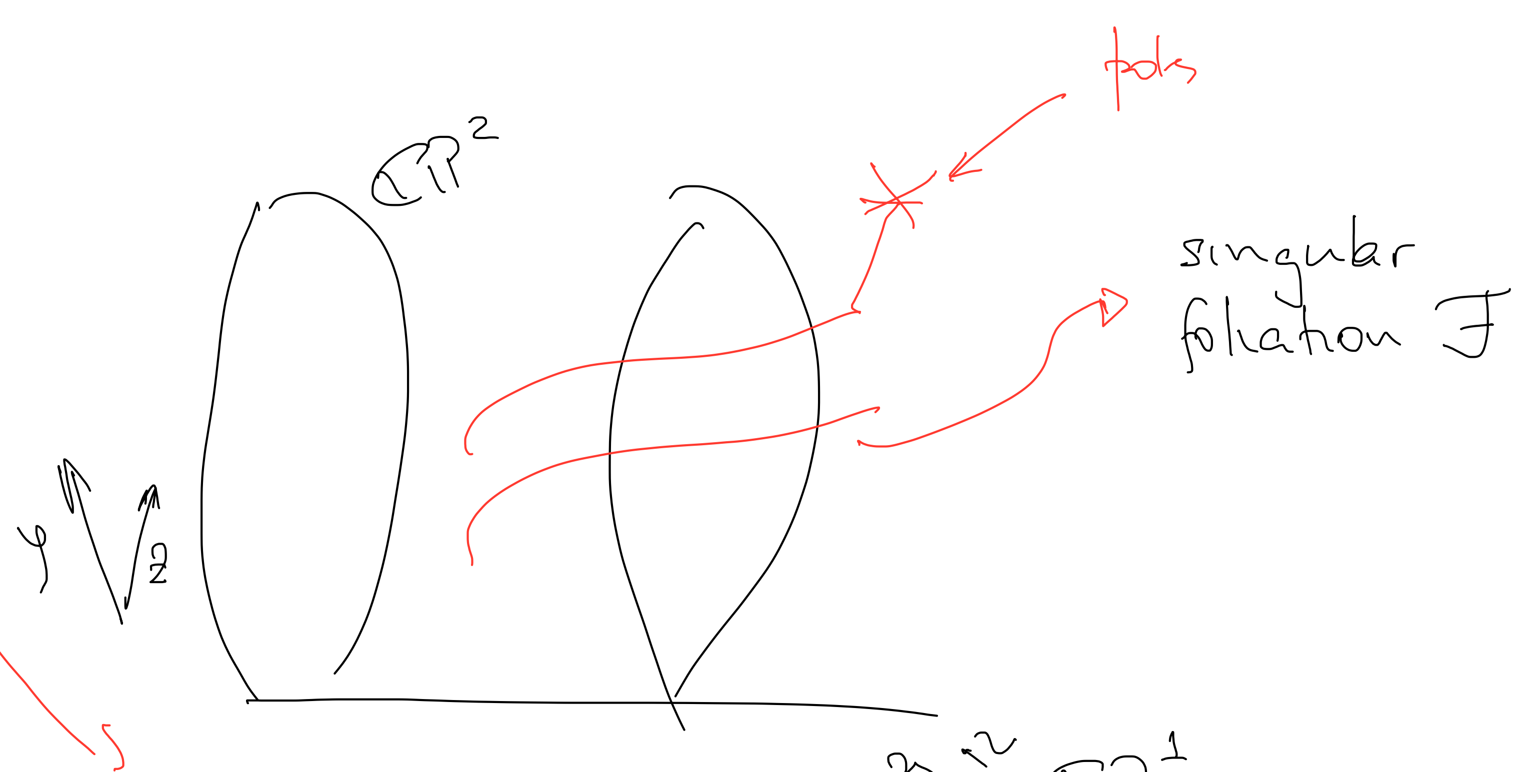
$$Z_{VI} = \frac{\partial}{\partial x} + z\frac{\partial}{\partial y} + H_{\alpha\beta\gamma\delta}(x,y,z)\frac{\partial}{\partial z}$$

$$H_{\alpha\beta\gamma\delta} = f/g \quad \text{rational}$$

$$Y = g\frac{\partial}{\partial x} + zg\frac{\partial}{\partial y} + f\frac{\partial}{\partial z}$$

vanishes at 0, 1, ∞ "

$$\mathbb{C}^3 \hookrightarrow \mathbb{CP}^1 \times \mathbb{CP}^2$$

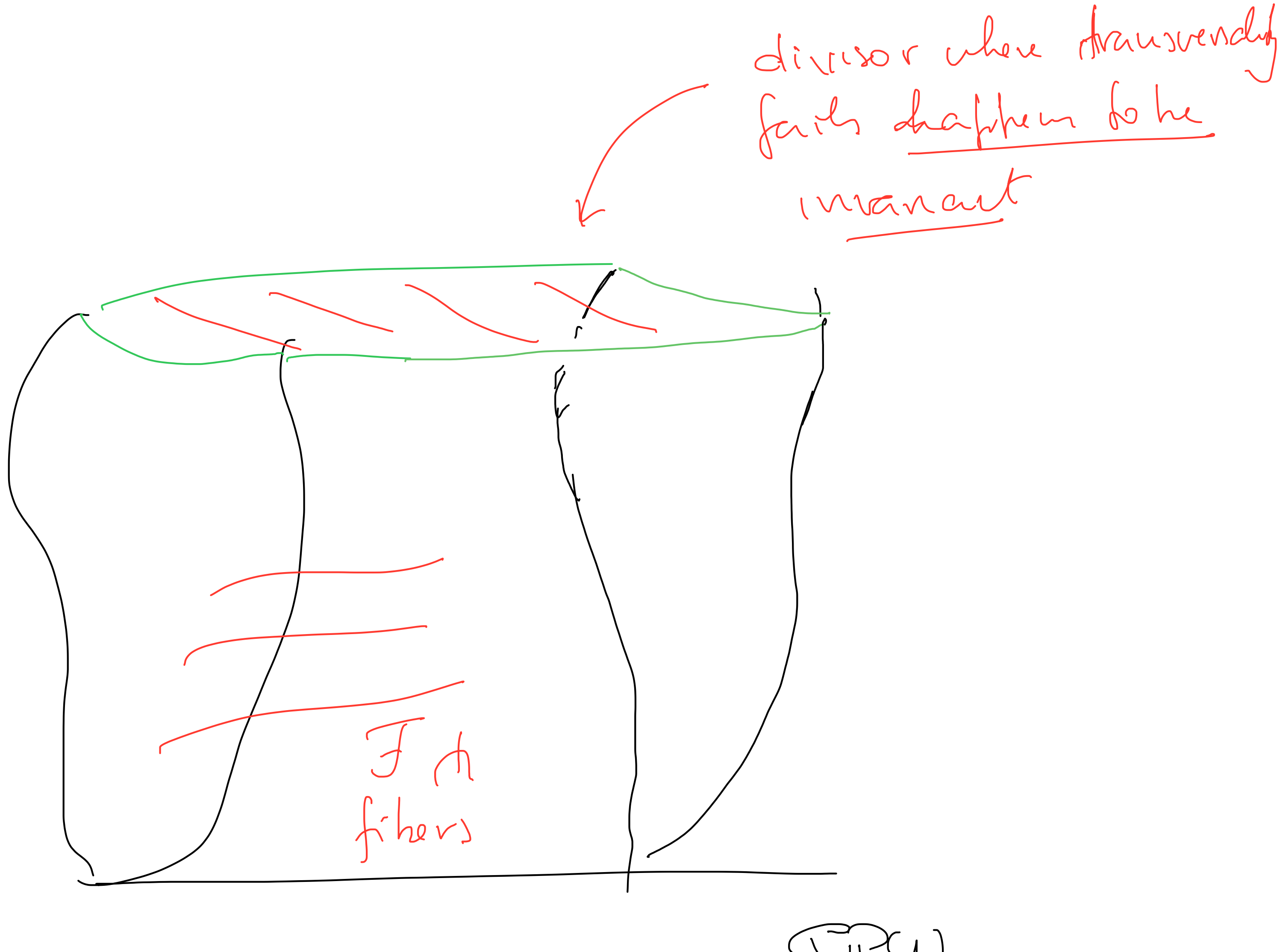


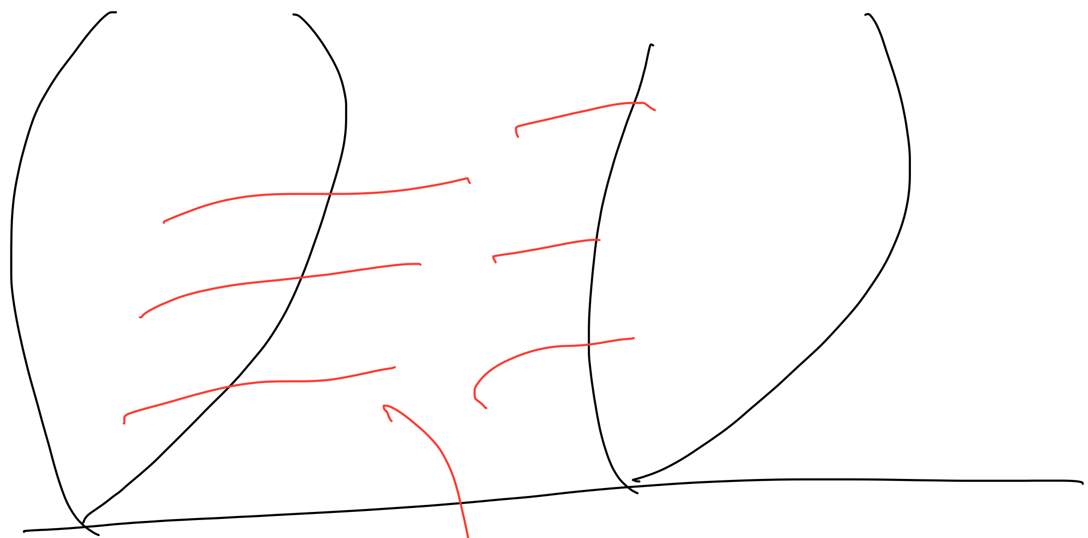

3 INV. FIBERS

$\mathcal{F}$ is "badly behaved" $\left\{ \begin{array}{l} \text{plenty of singularities} \\ \text{lack of transversality} \end{array} \right.$

$\Rightarrow$ Okamoto: get rid of "poles" by using blowing up (more precisely get rid of "dicritical" singular points)

$\Rightarrow$ He went all the way and a nicer picture emerges.





fibration with <u>open</u> surfaces

foliation transverse to the fibers away from the sing. pts

## HOLONOMY REPRESENTATION ?

$$\rho : \pi_1(\mathbb{CP}^1 \setminus \{0,1,\infty\}) \xrightarrow{\;!\;} \mathrm{Aut}(\text{Fiber})$$

Yes thanks to Painlevé property

(in the original sense of Painlevé)

In fact, let $\pi$ be the projection and $L$ a leaf of $\mathcal{F}$. Then

$$\pi : L \longrightarrow \mathbb{CP}^1 \setminus \{0,1,\infty\}$$

is <u>local diffeomorphism</u> (transversality condition)

<u>BUT</u>

it also lifts paths (solutions of P6 can always be locally continued away from the "singular points" $0,1,\infty$

<u>Conclusion</u>: $\pi$ is a local diffeomorphism having the <u>path-lifting property</u>

$$\Rightarrow \pi : L \longrightarrow \mathbb{CP}^1 \setminus \{0,1,\infty\}$$

is a <u>covering map</u>

The holonomy representation

$$\rho : \pi_1(\mathbb{CP}^1 \setminus \{0,1,\infty\}) \longrightarrow \mathrm{Aut}(\text{Fiber})$$

is well defined

The dynamics of $\rho(\pi_1(\mathbb{CP}^1 \setminus \{0,1,\infty\})) \subset \mathrm{Aut}(\text{Fiber})$

contains basically all of the information on the dynamics of P6 itself (i.e, the transverse dynamics of P6 is fully encoded in this holonomy group