

Lessons from
Low Dimensional
Topological Strings

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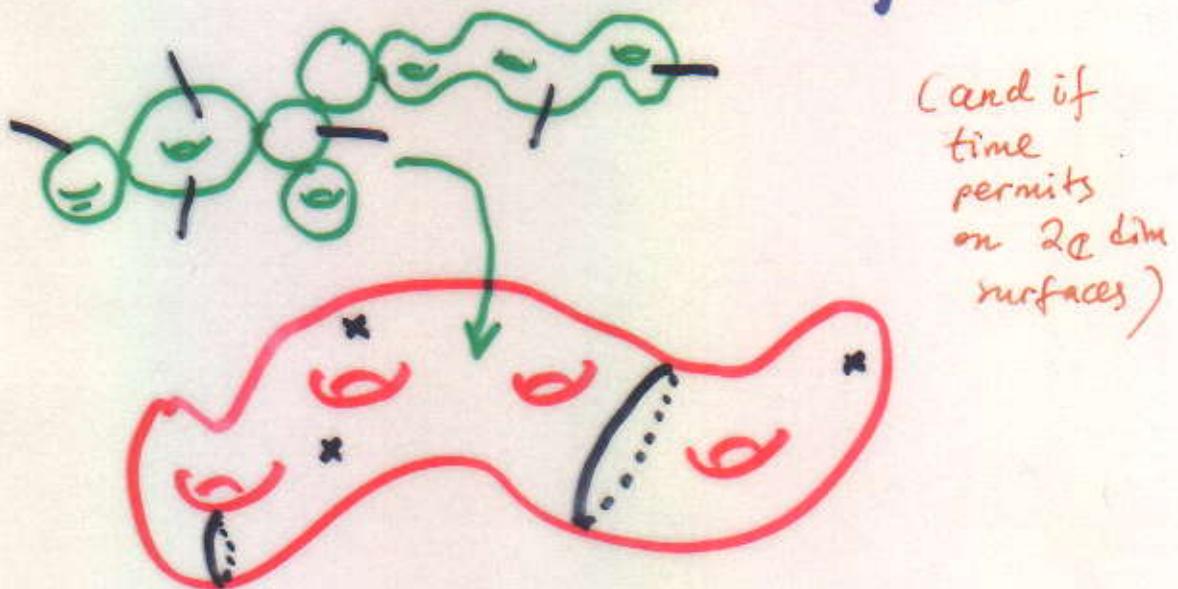
Kyoto

9 January 2008

We shall study topological
A type strings on

Σ - two dimensional

Riemann surface



We count holomorphic curves (stable maps)

landing some points on the source

to some submanifolds (0,1,2 dimensional)

in Σ , perhaps with some multiplicity
"tangency" condition

first
We are interested in the partition function

$$Z_{\Sigma}^{(t)} = \exp \sum_{g=0}^{\infty} t^{2g-2} F_g(t)$$

$$t = t(z) \in H^*(\Sigma) \otimes \mathbb{C}[z]$$

$$t(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!} (t_k^1 \cdot 1 + t_k^{\alpha_i} \cdot \alpha_i + t_k^{\omega} \cdot \omega)$$

$$1 \in H^0(\Sigma, \mathbb{Z})$$

$$\alpha_i, i=1, \dots, r_j(\Sigma) \in H^1(\Sigma, \mathbb{Z})$$

$$\omega \in H^2(\Sigma, \mathbb{Z})$$

$t_k^{\gamma} \leftrightarrow \sigma_k(\gamma)$ coupling of
the k 'th gravitational
descendent of
 $\gamma \in H^*(\Sigma)$

Mathematically :

$$F_g(t) = \sum_{\beta \in H_2(\Sigma, \mathbb{Z})} q^\beta \times$$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \sum_{k, \vec{\gamma}} t_{k_1}^{\gamma_1} t_{k_2}^{\gamma_2} \dots t_{k_n}^{\gamma_n} \times$$

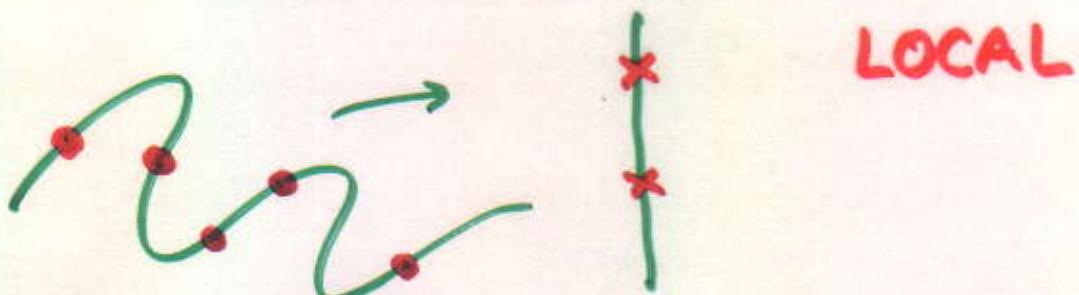
$$\times \int I_{GW}^{n, g, \beta} (\tau_{k_1}(z_1) \otimes \dots \otimes \tau_{k_n}(z_n))$$

$$\bar{\mathcal{M}}_{g, n}(\Sigma; \beta)$$

$$\tau_{k_i}(z_i) \sim \text{near } z_i \psi_i^{k_i}$$

moduli of stable maps of genus g
of degree β with n marked points

Generating function of enumerative
invariants



Mathematically the problem is
"solved" long time ago

Okounkov-Pandharipande, Givental
Dijkgraaf, Dubrovin-Zhang,
WDVV, Eguchi-Yang, Eguchi-Hori, etc.

More precisely, there is
enough literature on the subject
so that in principle one can
write the answer

We need something much more
EXPLICIT to learn smth about
the physics of the model

Physically

$$t^{-2} \text{ (red diagram)} + \text{ (red diagram)} + t^2 \text{ (red diagram)} +$$

$$\sim \text{ (green diagram)} + \text{ (green diagram)} + \text{ (green diagram)} + \dots$$

$$= \log \int D\Phi \exp(-t^{-2} S(\Phi))$$

What are $S(\Phi)$, Φ ?

$$S(\Phi) = \int_{\Sigma} Z(\Phi)$$

We are after some theory on Σ

- complex structure independent
- depends on symplectic structure
- $\text{Diff}(\Sigma)$ - invariant (closed strings)

Essentially - topological theory

Reminder on TFTs in 2d

Observables \mathcal{O}_α \mathcal{Q} operator

$$\{ Q, \mathcal{O}_\alpha^{(0)} \} = 0$$

$$d \mathcal{O}_\alpha = \{ Q, \mathcal{O}_\alpha^{(1)} \}$$

$$d \mathcal{O}_\alpha^{(1)} = \{ Q, \mathcal{O}_\alpha^{(2)} \}$$

$$\langle e^{T_\alpha^{(0)} \mathcal{O}_\alpha^{(0)} + T_\alpha^{(1)} \oint_{\partial i} \mathcal{O}_\alpha^{(1)} + T_\alpha^{(2)} \int_{\Sigma} \mathcal{O}_\alpha^{(2)}} \rangle$$

$i=1, \dots, 2g(\Sigma)$

More refined definition uses generalized
GW classes

$$f: \overline{\mathcal{M}}_{g(\Sigma), n} \longrightarrow \overline{\mathcal{M}}_{g(\Sigma), 0}$$

TFT (as opposed to the topological string)
integrates over cycles in the fiber of
forgetting map

TFT

For fixed $T_\alpha^{(2)}$ the correlation functions
of O -observables are easy to

Compute :

ALGEBRA

$$O_\alpha^{(o)} O_\beta^{(o)} = C_{\alpha\beta}^\gamma O_\gamma^{(o)}$$

$$= C_{\alpha\beta\gamma}$$

$$O_O = 1$$

$$C_{\alpha\beta} \circ = \gamma_{\alpha\beta}$$

$$= \text{Tr } V^{g-1} O_{\alpha_1} \dots O_{\alpha_K}$$

$$V = \gamma^{\alpha\beta} C_{\alpha\beta}^\gamma O_\gamma$$

handle gluing operator

$$O_\gamma$$

Counting holomorphic maps

$$\phi: C \rightarrow \Sigma$$

GW theory

Can be related to the counting
of ramified coverings

Hurwitz theory

Representations of

$$\pi_1(\Sigma) \rightarrow S_N$$

N - degree of the map

The translation GW/H

is now available thanks
to Okounkov and Pandharipande

Stationary sector

$$t_k^1 = 0, \quad t_k^{\alpha_i} = 0$$

RANDOM PARTITIONS!

irreps
of S_N

$$Z_{\Sigma}(t^{\omega}) = \sum_{\substack{\lambda \in \text{partitions} \\ \text{irreps } (\mu_{\lambda})}} \left(-\frac{q}{t^{\omega}}\right)^{|\lambda|} \times \mu_{\lambda} \times {}^{(-g(\Sigma))}$$

$$\times \exp \sum_{K=0}^{\infty} \frac{1}{K!} t_K^{\omega} t^K p_{K+1}(\lambda)$$

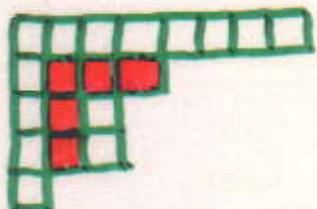
μ_{λ} - Plancherel measure

$p_k(\lambda)$ - k'th Casimir

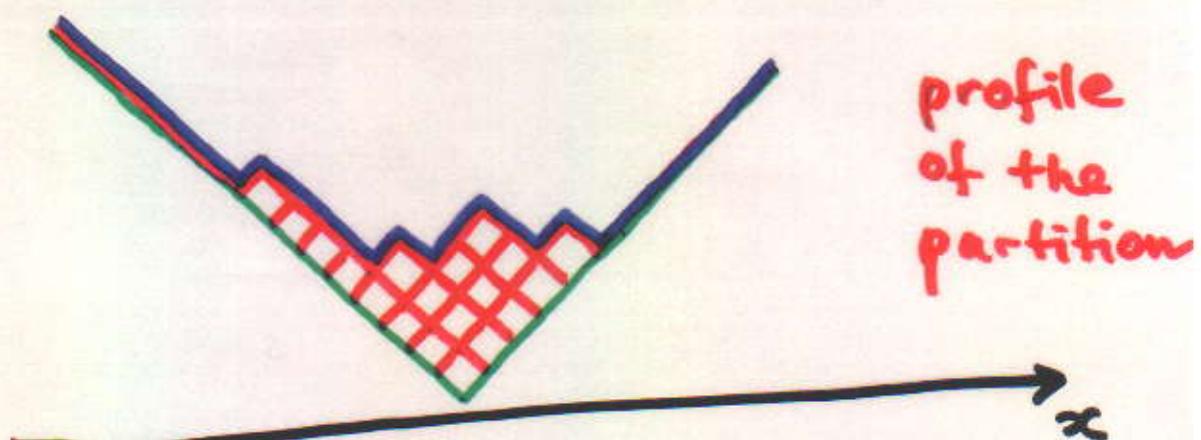
The Plancherel measure

$$\mu_\lambda = \left(\frac{\dim \lambda}{|\lambda|!} \right)^2 =$$

$$= \prod_{\square \in \lambda} \frac{1}{h_\square^2}$$



$$= \exp \frac{1}{4} \iiint dx_1 dx_2 \delta(x_1 - x_2) f_\lambda''(x_1) f_\lambda''(x_2)$$



$$f_\lambda(x) = |x| + \sum_{i=1}^{\infty} \frac{|x - \frac{1}{n}(\lambda_i - i + 1)| - |x - \frac{1}{n}(\lambda_i - i)|}{-|x - \frac{1}{n}(-i + 1)| + |x - \frac{1}{n}(-i)|}$$

The Casimirs

$$\frac{P_K(\lambda)}{k!} = \text{Coeff}_{u^K} \sum_{i=1}^{\infty} e^{u(\lambda_i - i + \frac{1}{2})}$$

$$= \int f_\lambda''(x) x^{k+1} dx$$

$$\lambda_1 - \frac{1}{2} > \lambda_2 - \frac{3}{2} > \lambda_3 - \frac{5}{2} > \dots$$

$$P_K(\lambda) \sim \sum_i (\lambda_i - i + \frac{1}{2})^K$$

$$\gamma(x) = \frac{x^2}{2\pi^2} \left(\log x - \frac{3}{2} \right) + \frac{1}{12} \log x +$$

$$+ \sum_{g=2}^{\infty} t_g^{2g-2} \frac{B_{2g}}{2g(2g-2)} x^{2-2g}$$

$$x \rightarrow \infty$$

$$\gamma(x+t) + \gamma(x-t) - 2\gamma(x) = \log \frac{1}{x}$$

the kernel

$$\gamma(x) = \frac{d}{ds} \Big|_{s=0} \frac{1}{\Gamma(s)} \int_0^\infty \frac{dt}{t} t^s \frac{e^{-tx}}{(1-e^{-xt})(1-e^{-\bar{x}t})}$$

Compare with matrix model

$$\int dM e^{-Tr V(M)}$$

$$\rho_M(x) = Tr \delta(x-M)$$

$$\int e^{2 \int_{x < y} \rho(x) \rho(y) \log(x-y) dx dy} - \int \rho(x) V(x)$$

[Dg]



$$-1 \quad \overbrace{\quad}^{+1}$$

$$\rho_M \sim f'_>$$

The partition function in the stationary sector

looks similar to that of

generalized two dimensional Yang-Mills
theory

Witten'92

$$Z_{\Sigma}^G(t^{\omega}) = \int \frac{d\phi d\Lambda d\Psi}{\text{vol}(g)} \exp(-S_{YM})$$

$$S_{YM} = i \sum \text{Tr}(\Phi F + \frac{1}{2} \Psi \Psi) + \sum \omega_k t_k^{\omega} \text{Tr} \frac{\Phi^{k+1}}{(k+1)!}$$

$$Z_{\Sigma}^G = \sum_{R \in \text{Rep}(G)} (\dim R)^{2-2g(\Sigma)} \exp(-C_{k+1}(R) t_k^{\omega})$$

Integrability and fermions

For $\Sigma = \mathbb{CP}^1$ in the stationary sector
the partition function has a free
fermion representation (τ -function)

$$Z_{\mathbb{CP}^1} = \langle 0 | e^{-\frac{J_1}{\pi}} e^{t_k^\omega W_{k+2}^{(0)}} e^{\frac{qJ-1}{\pi}} | 0 \rangle^{\text{OP}}$$

$$W_k^{(0)} \sim \oint : \tilde{\Psi} \partial^{k-1} \Psi : \quad \partial = \pm \partial_z$$

$$J(z) = : \tilde{\Psi}(z) \Psi(z) :$$

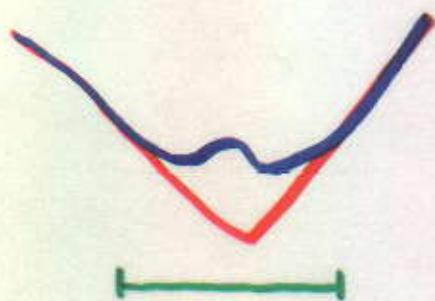
$$\Psi(z) = \sum_{r \in \mathbb{Z}_L + \frac{1}{2}} \Psi_r z^{-r} \left(\frac{dz}{z}\right)^{\frac{1}{2}}$$

$$\tilde{\Psi}(z) = \sum_{r \in \mathbb{Z}_L + \frac{1}{2}} \tilde{\Psi}_r z^r \left(\frac{dz}{z}\right)^{\frac{1}{2}}$$

$$\{\Psi_r, \tilde{\Psi}_s\} = \delta_{r,s}$$

In the limit $\hbar \rightarrow 0$

The PARTITION FUNCTION IS DOMINATED
by the limit shape $f_{\infty}(x)$



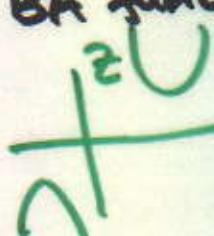
f''_* has finite support

70's
Vershik
Kerov
Logan
Schepp
 $t_k^{**} = 0$
 $k > 0$
 $NN +$
Marsha
-kov

dispersionless Toda
lattice

$$S(z) = \int (z-x) \log(z-x) f''_*(x) dx + \\ + \sum t_k^\omega t^k =$$

BA function



$$\frac{z}{\sqrt{\omega}} = \sum_k t_k^\omega \Omega_k + 2z(\log z - 1)$$

Hamiltonian

$$\Omega_k = z^k_+ - z^k_-$$

$$z = v + \lambda(\omega + \omega')$$

Spectral curve

(v, λ) determined from

$$\frac{ds}{dw} = 0, w = \pm 1$$

$z \leftrightarrow$ Lax operator

To determine the t_K^1 dependence t_K^1 ?

We shall solve the Virasoro constraints

Conjectured by Eguchi-Yang, Hori
proven by Okounkov - Pandharipande

$$L_k Z = 0 \quad , \quad k \geq -1$$

$$L_k \sim \frac{\partial}{\partial t_{k+1}^2} + t^1 \frac{\partial}{\partial t^1} + t^\omega \frac{\partial}{\partial t^\omega} + \frac{\partial^2}{\partial t_\omega \partial t^\omega}$$

schematically

Thus Z obeys some kind of heat kernel equation

$$\frac{\partial}{\partial t} Z + \frac{\partial^2}{\partial x^2} Z = 0 \quad \xrightarrow{\text{plane waves}}$$
$$Z \sim \int dk e^{ikx - k^2 t}$$

remains a sum of plane waves

The solution is

$$Z_{CP^1} = \sum_{\lambda} e^{2r(\lambda; t')} x$$

$$\times \exp \sum_{k=0}^{\infty} t_k^{\omega} \cdot \frac{1}{k!} \frac{p_k(\lambda; t')}{(k+1)!}$$

Evolved Casimirs

$$S_{\lambda}(u; t')^+ = \sum_{k=0}^{\infty} p_k(\lambda; t') \frac{u^{k+1}}{(k+1)!}$$

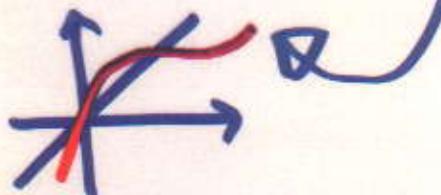
= positive part of

Euler
MacLauren

$$\sum_{i=1}^{\infty} e^{u \tau(\pm(\lambda_i - i + \frac{1}{2}))}$$

$$t'(y) = \sum t_k' \frac{y^k}{k!}$$

$$\tau(x) - t^1(\tau(x)) = x$$



$$\mu_\lambda = \prod_{i < j} \left(\frac{\lambda_i - \lambda_j + j - i}{j - i} \right)^2$$

$$e^{r(\lambda; 0)} //$$

$$e^{r(\lambda; t')} = \prod_{i < j} \left(\frac{y_i - y_j}{j - i} \right)^2$$

$$y_i = \tau \left(t \left(\lambda_i - i + \frac{1}{2} \right) \right)$$

The Plancherel measure

$$e^{r(\lambda; t')} = \text{regularized Vandermonde}$$

of $y_i = \tau(t_h(\lambda_i - i + \frac{1}{2}))$

$$\text{Explicitly : } r(\lambda; t') = \frac{1}{8} \iint dx_1 dx_2 f_\lambda''(x_1) f_\lambda''(x_2) G(x_1, x_2)$$

$$+ \frac{1}{2t_h} \int dx f_\lambda''(x) I[\Psi](x)$$

$$G(x_1, x_2) = I_1, I_2 \left[\log \left(\frac{\tau(\tilde{x}_1) - \tau(\tilde{x}_2)}{\tilde{x}_1 - \tilde{x}_2} \right) \right] + \\ + \gamma(x_1 - x_2)$$

$$\Psi(x) = \sum_{l=0}^{\infty} \frac{t'_0 e^{-\delta_{l,1}} \tau(x)^l}{l!} \left[\log \tau(x) - \sum_1^l \frac{1}{k} \right]$$

$$+ (x + t'_0) (\log(x + t'_0) - 1) - t'_0 \log \tau(x)$$

$$I[f](x) = \frac{1}{\pi} \int_{-t'_0}^x f(x') dx' + \sum_{g=1}^{\infty} \frac{2^{1-2g}-1}{(2g)!} B_{2g} f^{(2g-1)}(x)$$

Higher genus Σ

$$Z_\Sigma = \sum_{\lambda - \text{partitions}} \exp \chi(\Sigma) r(\lambda; t)$$

$$\times \exp \sum_k \frac{t_k^\omega}{(k+1)!} P_{k+1}(\lambda; t)$$

$$\times \exp \sum_{i,j=1}^{z_g(\Sigma)} \eta_{ij} \sum_{k,e} \frac{t_k^{\alpha_i} t_e^{\alpha_j}}{k! e!} x_{k+e}(\lambda; t)$$

$$P_{k+1} \sim \sum_i y_i^{k+1}$$

$$y_i = t'(y_i) =$$

$$x_k \sim \sum_i \frac{y_i^k}{1 - t''(y_i)}$$

$$= t(\lambda_i - i + \frac{1}{2})$$

$$t'(z) = \sum_{k=0}^{\infty} t_k^1 \frac{z^k}{k!}$$

?

Two dimensional Yang-Mills theory

Again

On Σ we study 2d YM theory now

Let G denotes a Lie group

W, S^I, R - G -inv. functions on
 $\mathfrak{g} = \text{Lie } G$

$I = 1, \dots, r_g(\Sigma)$

$$Z_\Sigma^G = \int \frac{DAD\psi D\phi}{\text{Vol}(g)} \exp(-S)$$

$$S = \sum_{\Sigma} (W + \alpha_I \wedge S^I + \omega R)(F)$$

$$F = F_A + \Psi + \phi$$

curvature
2-form

$$\Omega_{\Sigma}^2 \otimes g$$

$$\Omega_{\Sigma}^1 \otimes g$$

$$\Omega_{\Sigma}^0 \otimes g$$

The partition function of 2d YM

$$Z_\Sigma^G = \sum \left(\det \frac{\partial^2 W}{\partial \phi \partial \phi} \right)^{2g(\Sigma)-2} (\dim R_\lambda)^{2-2g(\Sigma)} \times$$

λ - dominant weights
of G
 $\leftrightarrow \text{Reps}(G)$

$$\times \exp R(\hat{\phi}_\lambda) + \eta_{IJ} \frac{\partial S^I}{\partial \phi^a} \frac{\partial S^J}{\partial \phi^b} g^{ab}_{\langle \hat{\phi} \rangle}$$

$$g_{ab} = \frac{\partial^2 W}{\partial \phi_a \partial \phi_b}$$

$$G = U(N)$$

$$\lambda_i - i + \frac{1}{2} + \frac{N}{2}$$

$$\frac{\partial W}{\partial \phi_a} (\hat{\phi}_\lambda) = \lambda + \rho$$

Comparing with Z_Σ we got we identify

$$t W = \frac{1}{2} \text{Tr} \phi^2 - \sum_{k=0}^{\infty} \frac{t_k^1}{(k+1)!} \text{Tr} \phi^{k+1}$$

$$t S^I = \sum t_k^I \frac{\text{Tr} \phi^{k+1}}{(k+1)!}, \quad t R = \sum t_k^\omega \frac{\text{Tr} \phi^{k+1}}{(k+1)!}$$

ϕ is $\infty \times \infty$ matrix

"close" to

$\text{diag}(\tau(i - \frac{1}{2}))_{i=1,2,\dots}$

$$\frac{1}{(k+1)!} \text{Tr } \phi^{k+1} := \text{Coeff}_{u^{k+1}} \text{Tr } e^{\underset{\uparrow}{u}\phi}$$

well-defined
for $\text{Re}(\tau u) < 0$
analytic cont.
 $u \in \mathbb{C} \setminus 0$

Note, that W, S^I, R are not
the most general G -invariants

Single trace operators only

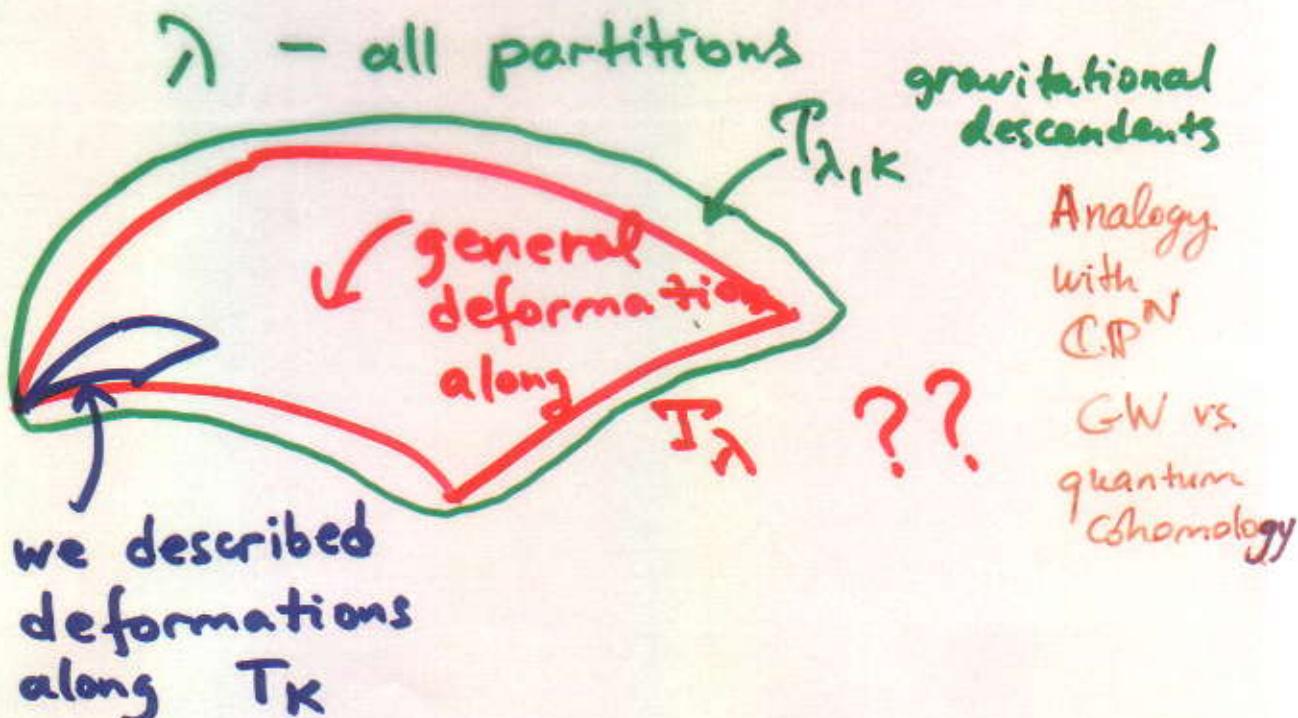
Thus the partition function
of the topological string on Σ
is that of the TFT on Σ
in the "single-trace" sector

More abstractly

The algebra of observables is
generated by

$$\mathcal{O}_k \quad k=0,1,2,\dots$$

while its spectrum is labelled by



Two dimensional YM theory

$$(\phi, \psi, A)$$

is a limit of supersymmetric
 $\mathcal{N}=2$ twisted 2d YM theory

$$(\phi, \psi, A; \bar{\phi}, \bar{\psi}, \chi, H)$$

$$S = S_{YM} + Q \int_{\Sigma} \text{Tr} \left(\chi F_A + \psi^* d_A \bar{\phi} + \right. \\ \left. + \eta [\phi, \bar{\phi}] + \gamma^* \chi H \right)$$

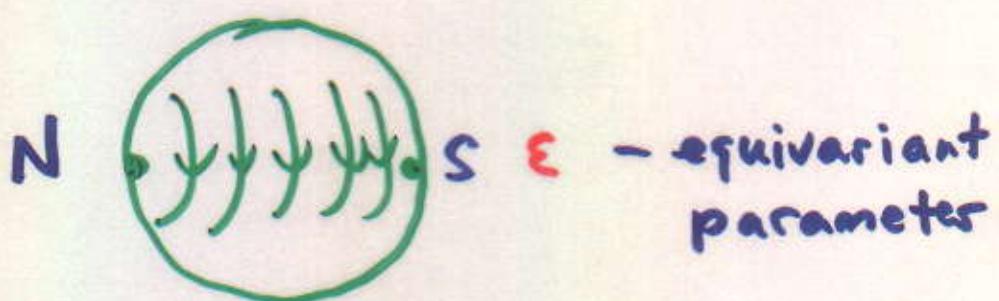
$$\sim \frac{1}{g^2} \int \text{Tr} F_A \wedge *F_A + \dots$$

Cohomological Field Theory
(path integral representation
of $\delta(F_A)$)

$\Sigma = \mathbb{CP}^1$ has $SU(2)$ isometry

by working equivariantly w.r.t. $U(1)$

we can generalize the 2d YM theory
on \mathbb{CP}^1



$$\int \Omega_W^{(2)} + \int \omega R \rightarrow$$

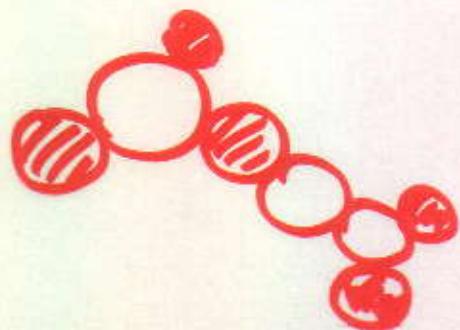
$$\rightarrow \frac{W^+(N)}{\epsilon} - \frac{W^-(S)}{\epsilon}$$

generalized
NORMAL
MATRIX
MODEL

↙
Z-Toda

$$Z_{\mathbb{CP}^1} \sim \int d\phi^+ d\phi^- \Delta(\phi^+) \Delta(\phi^-) e^{W^+(\phi^+)/\epsilon - W^-(\phi^-)/\epsilon} \sum_{\lambda} \delta(\phi^+ - \phi^- - \lambda)$$

Genus zero $F_0(\varepsilon, x^+, x^-)$



$$x^+ = \frac{t'}{\varepsilon}$$

$$x^- = -\frac{t'}{\varepsilon} + t^\omega$$

\sum over fixed points of C^* action
on $\bar{M}_{0,n}(\mathbb{C}\mathbb{P}^1, \rho)$

Sum over trees =

Critical points of some
functional

\approx

Kontsevich
94

Analogous to the
limit shape equations

$$S = \sum_{d=1}^{\infty} (-1)^{d+1} \phi_d \phi_d^* \frac{d!}{d^{d-1}} \left(\frac{\varepsilon^2}{q}\right)^d$$

$$+ \frac{1}{2\varepsilon} \int_{-\infty}^{\xi} (t - \varepsilon x^+(t) - \phi(t))^2 - (t - \varepsilon x^+(t))^2$$

$$+ \frac{1}{2\varepsilon} \int_{\xi^*}^{+\infty} (t + \varepsilon \bar{x}(t) - \phi^*(t))^2 - (t + \varepsilon \bar{x}(t))^2$$

$$+ \frac{1}{2\varepsilon} \int_0^{\xi} (t - \varepsilon x^+(t))^2 + \frac{1}{2\varepsilon} \int_0^0 (t + \varepsilon \bar{x}^*(t))^2$$

$$\phi(t) = \sum_{d=1}^{\infty} d \cdot \phi_d \cdot e^{d \cdot t / \varepsilon}$$

$$x^\pm(t) = \sum x_k^\pm \frac{t^k}{k!}$$

$$\phi^*(t) = \sum_{d=1}^{\infty} d \cdot \phi_d^* e^{-d \cdot t / \varepsilon}$$

We extremize w.r.t. $(\phi_d, \phi_d^*, \xi, \xi^*)$

Spectral curve

$$ze^{-\frac{z}{\epsilon}} = \lambda w e^{-\frac{\lambda(w+\bar{w}^*)+v^*}{\epsilon}}$$

(w, z, z^*)

$$z^* e^{+\frac{z^*}{\epsilon}} = \lambda w^{-1} e^{\frac{1}{\epsilon}(\lambda(w+\bar{w}^*)+v)}$$

$$\Omega = \sum T_K d\Omega_K + \bar{T}_K d\bar{\Omega}_K$$

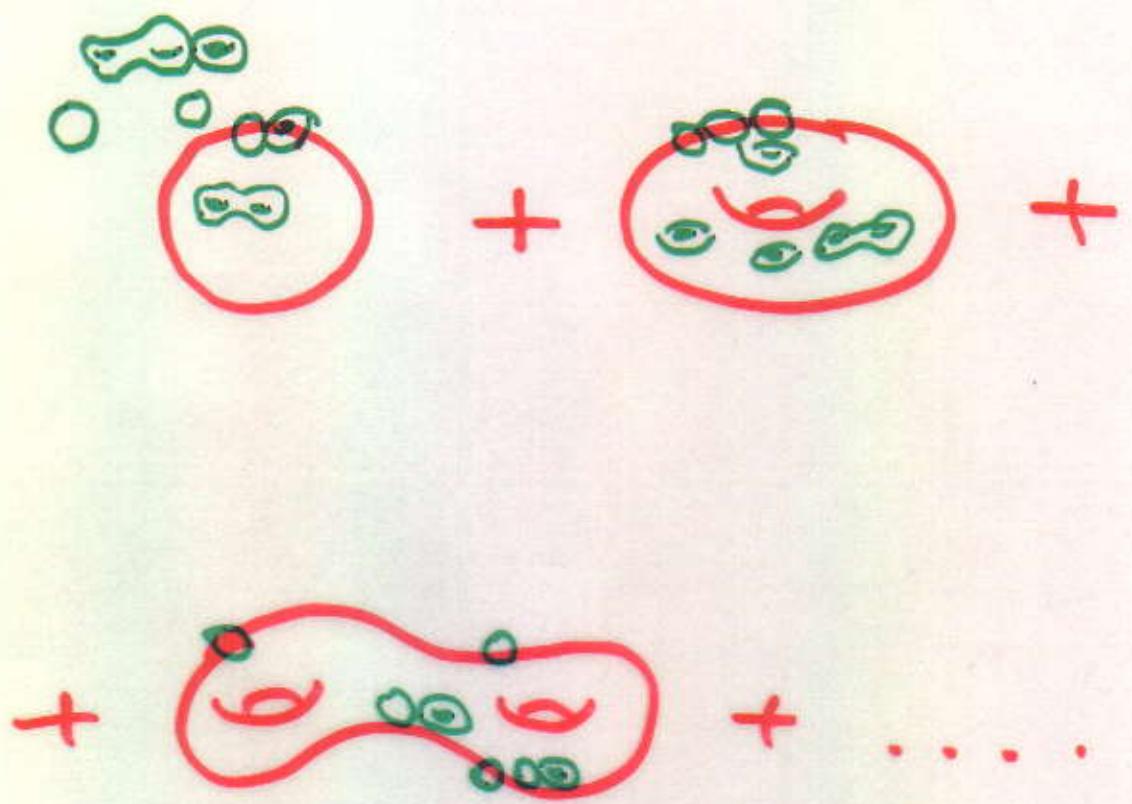
Hamiltonians

$$\Omega_K = z^K_+ , \quad z = \lambda w + v_+ + O(\frac{1}{w})$$

$$\bar{\Omega}_K = \bar{z}^K_- , \quad \bar{z} = \lambda \bar{w}^* + v_- + O(w)$$

Solution

$$\phi_d = \frac{d^{d-1}}{d!} \left(\frac{\lambda}{\epsilon} \right)^d \phi \left(w e^{-\frac{\lambda(w+\bar{w}^*)+v_+}{\epsilon}} \right)^d \Omega$$



M. Green
early 80's

At the same time on the string side

We are dealing with Equivariant GW theory of $\mathbb{C}\mathbb{P}^1$

Okounkov-Pandharipande have shown that the partition function is the τ -function of Toda hierarchy

We need to determine more precisely what $GL(\infty)$ element does it correspond to

$$\langle e^{T_{k,j_k}} e^{\bar{T}_{k,\bar{j}_k}} \rangle$$

Ueno
Takasaki

Mirror map

Toda times $T_k, \bar{T}_k \neq$ k>1

GW times x_k^+, x_k^- k>0

Getzler - Okounkov - Pandharipande
polynomials $P_n(t)$

$$\varepsilon X(\varepsilon) - t = \sum_{n=1}^{\infty} n \varepsilon^{n-1} T_n P'_n(t)$$

$$P_n(t) = \frac{t^n}{n} + \varepsilon t^{n-1} h_{n-1} + \dots + t$$

$$\sum_{e=0}^{n-1} \frac{(n-1)!}{(n-e)!} h_{n-1}^{(e)} \varepsilon^e t^{n-e}$$

$$\sum_{\substack{1 \leq k_1 < k_2 < \dots < k_e \leq n-1}} \frac{1}{k_1 k_2 \dots k_e} = h_{n-1}^{(e)}$$

generalized harmonic numbers

Higher dimensions

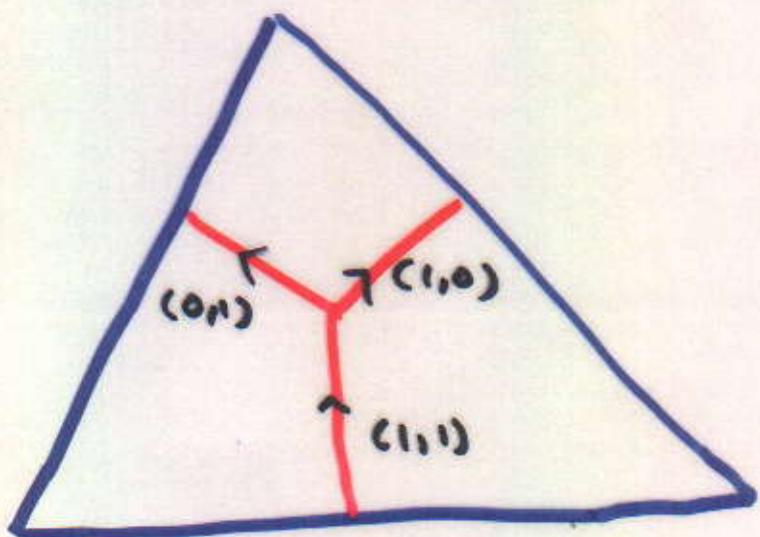
$\mathbb{C}\mathbb{P}^2, \mathbb{P}^1 \times \mathbb{P}^1, \dots$

Two dimensional toric varieties

Tropical Approach

Mikhailov,
Sethi,
'98

counting graphs



Again, we get Feynman
rules of some field theory

$$(a, b) \in H_2(T^2, \mathbb{Z})$$

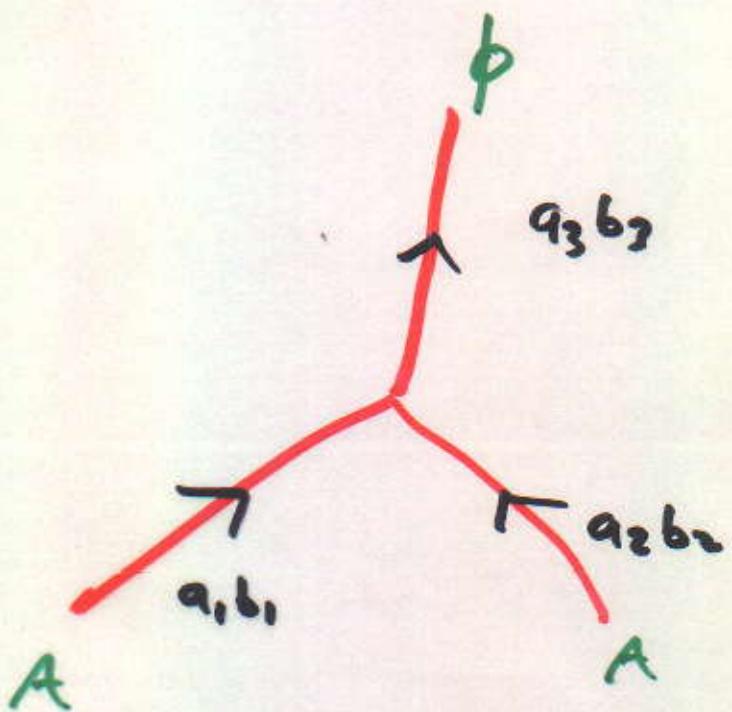
$$v_{a,b} = a \partial_{x^1} + b \partial_{x^2}$$

$$\langle \phi_{a,b}(x) A_{a',b'}(y) \rangle =$$

$$= \int_0^\infty dt e^{-t L_{V_{a,b}}} \langle v_{a,b} \rangle$$

$$\delta^{(2)}(x-y) (dx - dy) \wedge (d\bar{x} - d\bar{y}) \sim \frac{1}{d}$$

gauge $\langle v_{a,b} A_{a,b} \rangle = 0$



$$\delta_{a_3, a_1+a_2} \delta_{b_3, b_1+b_2} \times (a_1 b_2 - a_2 b_1)$$

To summarize, we have

the Feynman rules of 2d YM
theory

$$Tr \Phi F + \sum t_k \frac{\omega^2}{(k+2)!} Tr \frac{\Phi^{k+2}}{(k+2)!}$$

$G = SDiff(T^2)$

Boundary conditions
complicated

$$\Delta e^{SA}$$

Summary

- Full partition function of A type top. string on Σ can be computed , as a sum over partitions
- The result is 2d YM theory with ∞ -dim. gauge group
STRING THEORY GIVES THE DEFINITION OF THIS THEORY
(LESSON FOR NONCOMMUTATIVE GEOMETRY à la A.Connes)
- YANG-MILLS THEORY is TFT
STRING THEORY GIVES THE SINGLE TRACE SECTOR

- Multi-trace deformations ?

$$T_\lambda (\phi_\lambda, \phi_{\lambda_2}, \dots)$$

AIS/CFT
STRING
INTERPRETATION
?
Ahar
Berkooz
Silvest
Witten

- Coupling to gravity

Classes on $\overline{\mathcal{M}}_{g(\Sigma), n}$

II
CYxK3

STRING
INTERPRETATION
?

- Adding D-branes

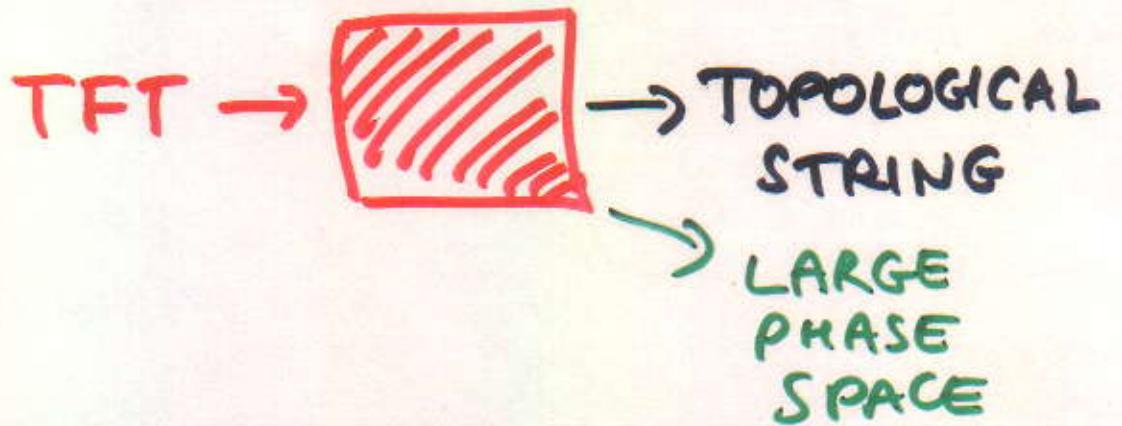
Wilson loop observables

- HIGHER DIMENSIONS ? TROPICAL
GEOMETRY

- INTEGRABLE HIERARCHY

- EQUIVARIANCE

with
SASHA
ALEXANDRO



QUANTUM GRAVITY IN TARGET SPACE

- NEW INTEGRABLE SYSTEM

$$T_K \rightarrow T_{\lambda, n}$$

- IN HIGHER DIMENSIONS
results are preliminary

$$2d \text{ YM}, G = \text{SDiff}(T^2)$$

- IN 3D, IN PROGRESS

$$\text{CS}, G = \text{Volume-preserving Diff}$$

$$\text{of } T^3, \approx \text{Kodaira-Spencer}$$

FROM **A** MODEL ON Σ

TO 4d GAUGE THEORY

$\mathbb{CP}^1 \times \mathbb{Z}_N$ IN STATIONARY
SECTOR \leftrightarrow

$N=2$ 4d SUPER-YANG-MILLS
(SEIBERG - WITTEN)

CHALLENGE: full equivariant
theory (ies)

