Kobe Summer School

(Advanced Science and Technology, Graduate Course) 2014

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Date : July 28 -- August 1 (5days), 2014
Time : 13:20--14:50, 15:10--16:40, 17:00--18:30 (every day)
Place: B314, Building B, Graduate School of Science

A mathematical theory of quantum curves

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A mathematical theory of quantum curves : what they are, and what they do

The theme of the course is to present recent exciting developments around the notion of “quantum curves” that relates many research frontiers of mathematics and theoretical physics. The goal is to develop a mathematical theory of this emerging field of research invariants.
After reviewing classical theory, we will explain the theoretical background of this new field of mathematics.

The quantum curves connect classical theory of differential equations and modern theory of topological invariants, such as Gromov-Witten, Seiberg-Witten, and quantum knot invariants.
The course develops a universal structure of these invariants, originated by various groups of physicists, based on geometry and analysis of Riemann surfaces.

1. How do we see the moduli spaces of curves?
2. The parabola $x = y^2$ knows the Witten-Kontsevich theory
3. Quantum invariants give the exact WKB analysis of classical ODEs
4. The simplest example of the topological recursion is Catalan numbers
5. The mathematical framework of the theory is the Hitchin fibrations
6. Geometry of ruled surfaces, log Calabi-Yau spaces, and the Hilbert scheme of points
7. From Gromov-Witten to Seiberg-Witten via a spectral curve - a glimpse of KP equations