Workshop on
Calabi-Yau Varieties and Related Topics

Dates: Aug. 8 (Thu) – Aug. 9 (Fri), 2019

Venue: Event-space B, Hakodate Community Plaza "G-Square"
（北海道函館市本町 24 1 シエスタハコダテ 4階;　https://www.sharestar.jp/）

This workshop is supported by JSPS KAKENHI Grant numbers JP18K03200 (Yasuhiro Goto) and JP16K05105 (Shinobu Hosono).

August 8 (Thu.)

10:00–11:00 Masaru Nagaoka (University of Tokyo)

Compactifications of affine homology 3-cells into quadric fibrations

11:20–12:20 Katsuhisa Furukawa (University of Tokyo)

Cubic hypersurfaces with positive dual defects

14:00 – 15:00 Hiromichi Takagi (Gakushuin University)

Examples of key varieties of prime $\mathbb{Q}$-Fano threefolds of codimension 4

15:20 – 16:20 Akihiro Kanemitsu (Kyoto University)

Mukai pairs and simple K-equivalence

16:40 – 17:40 Shinobu Hosono (Gakushuin University)

$K3$ analogues of the elliptic lambda function

18:30 – Dinner (discussion)

August 9 (Fri.)

10:00–11:00 Atsushi Kanazawa (Kyoto University)

Stability spaces with a view toward period maps and attractor geometry

11:20 – 12:20 Yukiko Konishi (Tsuda University)

Almost duality for Saito structure and complex reflection groups

14:00 – 15:00 Yasuhiro Goto (Hokkaido Univ. of Education)

Calculations on the formal groups of Calabi-Yau fourfolds

15:10 – 16:10 Noriko Yui (Queen's University)

Siegel modularity of certain Calabi–Yau threefolds over $\mathbb{Q}$

16:30 – Free discussion

Organizers: Y. Goto (Hokkaido Edu.), S. Hosono (Gakushuin), N. Yui (Queen's Univ.)

Abstracts

Aug. 8

Masaru Nagaoka (University of Tokyo)

Title: Compactifications of affine homology 3-cells into quadric fibrations

Abstract: Hirzebruch raised the problem to classify all the compactifications of the affine space $\mathbb{A}^n$ into smooth complete complex manifolds with the second Betti number $B_2 = 1$. A trivial example of such compactifications is the pair of the projective space and its hyperplane $(\mathbb{P}^n, \mathbb{P}^{n-1})$. Non-trivial examples appear in dimension three, and when the ambient spaces are projective, the complete classification of compactifications was achieved by Furushima, Nakayama, Peternell and Schneider. When $B_2 = 2$, non-trivial compactifications appear even in dimension two, which was studied by Mori.

In this talk, I will deal with compactifications of affine homology 3-cells, which have the same homologies as $\mathbb{A}^3$, into quadric fibrations such that the boundary divisors contain fibers. I will explain that all such affine homology 3-cells are isomorphic to $\mathbb{A}^3$ and all such compactifications can be connected by explicit elementary links preserving $\mathbb{A}^3$ to $\mathbb{P}^3$.

Katsuhisa Furukawa (University of Tokyo)

Title: Cubic hypersurfaces with positive dual defects

Abstract: The dual defect of a projective variety X in $\mathbb{P}^N$ is the difference between $N-1$ and the dimension of the dual variety X^* of X. Zak classified cubic hypersurfaces X with positive dual defects in the case when X^* is smooth. Hwang gave a characterization of secants of Severi varieties in terms of cubic hypersurfaces with nonzero Hessians and nonzero prolongations.

I will talk about structure of contact loci of cubic hypersurfaces X with positive dual defect in general settings, without the assumption that X^* is smooth.

Hiromichi Takagi (Gakushuin University)

Title: Examples of key varieties of prime $\mathbb{Q}$-Fano threefolds of codimension 4

Abstract: To understand a projective variety X (Fano, Calabi-Yau, etc), a key variety for X has been constructed for several cases. Here a key variety Z for X is a bigger projective variety containing X as a weighted complete intersection (in weighted projective spaces). A famous example is due to Professor Mukai; X is a prime Fano threefold of degree 12 and Z is the 10 dimensional orthogonal Grassmannian.

In my talk, I will explain my constructions of key varieties for prime $\mathbb{Q}$-Fano threefolds of codimension 4 and explain its group actions and relations with classical Fano

varieties $\mathbb{P}^2 \times \mathbb{P}^2$ and $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$. Moreover, I will describe birational geometry of X using its key variety.

Akihiro Kanemitsu (Kyoto University)

Title: Mukai pairs and simple K-equivalence

Abstract: A birational map between two smooth projective varieties is called a simple K-equivalent map if (1) the map preserves the canonical divisors, and (2) the map can be resolved in both sides by single smooth blow-ups. In this talk, I will show a structure theorem of simple K-equivalent maps, which relates those maps to a special kind of Fano manifolds. As an application, I will give the classification of simple K-equivalent maps in some cases.

Shinobu Hosono (Gakushuin University)

Title: $K3$ analogues of the elliptic lambda function

Abstract: Studying a family of $K3$ surfaces given as double covers of $\mathbb{P}^2$ branched along six lines in general positions, we will obtain explicit forms of $K3$ analogues of the elliptic lambda functions. These are given as ratios of some genus two theta functions, and define modular functions on the period domain (with level two structure). We find these functions by making suitable resolutions of the period domain (which is also given as configuration space of six lines on $\mathbb{P}^2$). This is based on recent works with B.Lian, H.Takagi and S.-T.Yau (arXiv:1903.09373 and arXiv:1810.00606).

Aug. 9

Atsushi Kanazawa (Kyoto University)

Title: Stability spaces with a view toward period maps and attractor geometry

Abstract: The space of stability conditions for the derived category of a projective manifold is a Kahler analogue of the period domain in the Hodge theory. I will introduce some new structures on the stability spaces and discuss their potential applications, in particular the period maps and attractor geometry.

Yukiko Konishi (Tsuda University)

Title: Almost duality for Saito structure and complex reflection groups

Abstract: It is known that the orbit spaces of the finite Coxeter groups and the Shephard groups admit two types of Saito structures without metric. One is the underlying structures of the Frobenius structures constructed by Saito and Dubrovin, respectively. The other is the natural Saito constructed by Kato-Mano-Sekiguchi and by Arsie-Lorenzoni. We study the relationship between these two Saito structures from the viewpoint of almost duality. This is a joint work with Minabe Satoshi.

Yasuhiro Goto (Hokkaido Univ. of Education)

Title: Calculations on the formal groups of Calabi-Yau fourfolds

Abstract: Calabi-Yau varieties are associated with formal groups of dimension 1 and such formal groups in positive characteristic are classified with the height. We consider several examples of Calabi-Yau fourfolds in weighted projective spaces and compute the height of their formal groups.

Noriko Yui (Queen's University)

Title: Siegel modularity of certain Calabi–Yau threefolds over $\mathbb{Q}$

Abstract: We will consider a number of examples of Calabi–Yau threefolds defined over $\mathbb{Q}$ having the Hodge numbers $h^{p,q} = 1$ for all pairs p, q with $p + q = 3$. Two of these Calabi–Yau threefolds are equipped with real multiplication by some real quadratic fields $K = \mathbb{Q}(\sqrt{d})$ with square-free integers $d > 1$, and satisfy the Hilbert modularity over K. Starting with the Hilbert modularity over K, we will establish the Siegel modularity over $\mathbb{Q}$ of such Calabi–Yau threefolds that their (cohomological) L-functions coincide with the Andrianov L-functions of Siegel modular forms of weight 3, genus 2 on paramodular subgroups of level N of $Sp(4, \mathbb{Q})$.