Conformal field theory, isomonodromy tau-functions and Painlevé equations

November 30 – December 2, 2017

B301, Graduate School of Science, Kobe University

PROGRAM

	9:30–10:20	10:40–11:30	13:00–13:50	14:00–14:50	15:20–16:10	16:20–17:10
Nov 30 (Thu)	Y. Haraoka		G. Rembado		A. Ikeda	
Dec 1 (Fri)	T. Suzuki		T. Mano		M. Cafasso	
Dec 2 (Sat)	K. Maruyoshi		O. Lisovyy			

Nov. 30 (Thu)

9:30–10:20 **Yoshishige Haraoka** (Kumamoto University, Japan)
Katz theory and KZ equation

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11:30–13:00 **Lunch**

13:00–13:50 **Gabriele Rembado** (University of Paris-Saclay, France)
Simply-laced quantum connections

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14:50–15:20 **Coffee Break**

15:20–16:10 **Akishi Ikeda** (Kavli IPMU, Japan)
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15:20–16:10 **Mattia Cafasso** (University of Angers, France)
Non-commutative Painlevé equations and systems of Calogero type

16:20–17:10 **Mattia Cafasso** (University of Angers, France)
Block Toeplitz determinants and integrable hierarchies

Dec. 2 (Sat)

9:30–10:20 **Kazunobu Maruyoshi** (Seikei University, Japan)
On Painlevé/Gauge Theory Correspondence

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Organizers

Hajime Nagoya (Kanazawa University), Masa-Hiko Saito (Kobe University)

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`http://www2.kobe-u.ac.jp/~mhsaito/events/1711kobe.html`

Abstracts

Mattia Cafasso (University of Angers, France)

1. Non-commutative Painlevé equations and systems of Calogero type

All Painlevé equations can be written as the motion of a particle under a time dependent potential, and as such they admit a natural generalisation to the case of several particles with an interaction of Calogero type (rational, trigonometric or elliptic). In this talk, I will show that these many-particles Hamiltonian systems admit an isomonodromic formulation, thus answering to a question raised by Takasaki. After explaining the general theory, I will focus on some examples and applications related to the second Painlevé equation.

This is a joint work with Marco Bertola and Vladimir Rubtsov.

2. Block Toeplitz determinants and integrable hierarchies

The aim of this talk is to explain how one can use block Toeplitz determinants, and in particular their asymptotics for large N, to effectively compute the tau function of integrable hierarchies.

This talk will be a summary of results I obtained in collaboration with Chaozhong Wu, Di Yang, Ann du Crest de Villeneuve, Pavlo Gavrylenko and Oleg Lisovyy.

Yoshishige Haraoka (Kumamoto University, Japan)

Katz theory and KZ equation

KZ equation is found by Knizhnik and Zamolodchikov as a differential equation satisfied by n-point correlation functions in CFT, and has been intensively studied from the viewpoint of representation theory. In this talk, we extend the Katz theory on rigid local systems to KZ equation. We define the additive and multiplicative middle convolutions for KZ equation and apply them to get several remarkable results.

Akishi Ikeda (Kavli IPMU, Japan)

On the monodromy representation of the confluent KZ equation

In this talk, we discuss the monodromy representation of the confluent KZ equation. First we see that the representation of the framed braid group, which is the semi-direct product of the braid group and a free abelian group, appears as the monodromy representation. Then, based on the integral representation of solutions, we construct the representations of the framed braid groups homologically by using integral cycles.

Oleg Lisovyy (University of Tours, France)

Tau functions as Widom's constants

I am going to explain how to associate a tau function to the Riemann-Hilbert problem set on a union of non-intersecting smooth closed curves with generic jump matrix. The main focus will be on the case of one circle, relevant to the analysis of Painlevé VI equation, its degenerations to Painlevé V and III as well as its extension to Fuji-Suzuki-Tsuda system. The tau functions in question will be defined as block Fredholm determinants of integral operators with integrable kernels. I will show that they can also be represented as combinatorial sums over tuples of Young

diagrams. For Riemann-Hilbert problems of isomonodromic origin, these sums coincide with dual Nekrasov-Okounkov partition functions of certain supersymmetric gauge theories.

Toshiyuki Mano (University of the Ryukyus, Japan)
Potential vector fields associated with solutions to Painlevé equations

In this talk, we introduce flat coordinates and potential vector fields on the spaces of variables of isomonodromic deformations of (generalized) Okubo systems and discuss its consequences. I would like to also treat algebraic and analytic studies on solutions to the sixth Painlevé equation in terms of potential vector fields if time permits.

Kazunobu Maruyoshi (Seikei University, Japan)
On Painlevé/Gauge Theory Correspondence

In this talk, I would like to overview the correspondence between Painlevé equations and four-dimensional rank-one $\mathcal{N} = 2$ theories, mostly from the viewpoint of the latter theories. In addition to the $SU(2)$ gauge theory with $N_f = 0, 1, 2, 3$ and 4 which correspond to Painleve III_3, III_2, III_1, V, and VI, we will find that three $\mathcal{N} = 2$ rank-one superconformal theories of Argyres-Douglas type correspond to Painlevé I, II and IV by studying the Seiberg-Witten curves of the former and the connection associated with the latter isomonodromic problem. Based on this correspondence we provide long-distance expansions at various canonical rays for all Painlevé functions in terms of magnetic and dyonic Nekrasov partition functions for $\mathcal{N} = 2$ gauge theories and Argyres-Douglas theories at self-dual Omega background or equivalently in terms of $c = 1$ irregular conformal blocks.

Gabriele Rembado (University of Paris-Saclay, France)
Simply-laced quantum connections

In this talk, we construct a family of flat connections generalising the KZ connection. This family is obtained via deformation quantisation of certain time-dependent Hamiltonian systems controlling the isomonodromy deformations of meromorphic connections on the Riemann sphere: the simply-laced isomonodromy systems.

In a first talk we will describe the construction of the simply-laced quantum connections, and in a second one we will compare them with other quantum connections already known in the literature.

Takao Suzuki (Kindai University, Japan)
A higher order generalization of the Painlevé VI equation with $W(\boldsymbol{A}^{(1)}_{2n+1})$ symmetry

In this talk, we propose a higher order generalization of the Painlevé VI equation from a viewpoint of an affine Weyl group symmetry of type A. Firstly we derive a generalized P_{VI} system from a Drinfeld-Sokolov type integrable hierarchy by a similarly reduction. Next we state some properties of our system: Lax pairs, an affine Weyl group symmetry and a particular solution in terms of the generalized hypergeometric function. If time permits, we discuss its q-analogue and associated τ-functions.