Nevanlinna theory Anal. functions Springer 193?
(新 '70)

Carlson - Griffiths A defect rela.

Ann. Math 95 ('72)
557 – 584

$f : \mathbb{C} \xrightarrow[\text{hol.}]{} \mathbb{P}^1 = \mathbb{C} \cup \{\infty\}$ non const

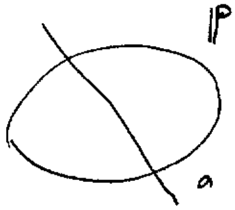
$a \in \mathbb{P}^1$ $f^{-1}(a)$

$\#(f^{-1}(a) \cap \Delta_r) = n(r, a)$

$n(r,a) \longrightarrow ?$ $r \to \infty$

$f : \mathbb{C}^n \longrightarrow \mathbb{P}^n$ non-const. hol.

$\mathbb{P}^n$



a : hyperplane

$f^{-1}(a)$: $(n-1)$-dim anal. set

$n(r, a) = f^{-1}(a) \cap \Delta r$ の面積

$n(r,a) \longrightarrow ?$ $r \to \infty$.

Carlson - Griffiths theory '72.

$f : \mathbb{C}^n \xrightarrow[\text{hol.}]{} W$ compact complex mfd.

$W \supset$ divisor $_{>0}$, f^{-1}(divisor)

line bdle F defined by $\{f_{jk}\}$

$\downarrow$ $\quad f_{jk}(w) : U_j \cap U_k \perp$ hol $\neq 0$

$W = \bigcup_j U_j \qquad U_j = (w_j^1, \dots, w_j^n)$

$\mathcal{O}(F)$: sheaf of hol. sections

$0 \neq \psi \in H^0(W, \mathcal{O}(F))$

ψ a divisor Ξ (ψ)

$*$ $\quad \psi = \{\psi_j(w)\} \qquad \psi_j(w)$: hol on U_j

$\psi_j(w) = f_{jk}(w)\, \psi_h(w)$ on $U_j \cap U_k$

$U_j \sqsubset \ni a_j(w) > 0$, C^∞-diff. s.t.

$a_j(w) = |f_{jk}(w)|^2 a_k(w)$

$\frac{\sqrt{-1}}{2\pi} \partial\bar\partial \log a_j(w) = \frac{\sqrt{-1}}{2\pi} \partial\bar\partial \log a_k(w)$ on $U_j \cap U_k$

$\therefore \frac{\sqrt{-1}}{2\pi} \partial\bar\partial \log a_j(w)$ is

$\odot$ Hermitian form $\quad \omega = \frac{\sqrt{-1}}{2} \sum g_{\lambda\mu}(w)\, dw_j^\lambda \wedge \overline{dw_j^\mu}$

$\sim c(F)$

$$|\psi|^2(w) \underset{\mathrm{def}}{=} \frac{|\psi_j(w)|^2}{a_j(w)} \qquad (j \text{ に indep})$$

$a_j = |f_{jk}|^2 a_k$ より a_k に適当な const をかけて

$|\psi|^2 < e^{-4}$ とできる.

<u>Notation</u>

$$z = x + iy$$

$$\frac{\sqrt{-1}}{2} dz \wedge d\bar{z} = dx \wedge dy$$

$$dV(z) = \left(\frac{\sqrt{-1}}{2}\right)^n dz_1 \wedge d\bar{z}_1 \wedge dz_2 \wedge d\bar{z}_2 \wedge \cdots \wedge dz_n \wedge d\bar{z}_n$$

volume

$$z = (z_1, \cdots, z_n)$$

$f : \mathbb{C}^n \longrightarrow W$ hol. f は not totally degenerate

(i.e. Jac. $f \equiv 0$ ではない)

$f^*(\omega)$: induced form on $\mathbb{C}^n$

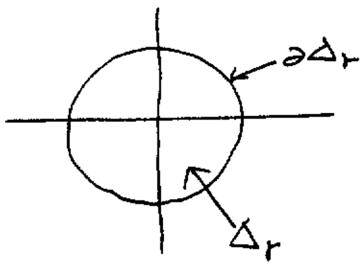
$$A(r) = \int_{\Delta_r} f^*(\omega) \wedge \sigma$$

$$\sigma = \sum_{\alpha=1}^{m} \sigma_\alpha \qquad \sigma_\alpha = dV(z_1, \cdots, z_{\alpha-1}, \overset{\alpha}{\widehat{\ \ }} z_{\alpha+1} \cdots, z_n)$$

$$= \left(\frac{\sqrt{-1}}{2}\right)^{n-1} dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge \cancel{dz_\alpha \wedge d\bar{z}_\alpha} \wedge \cdots \wedge dz_n \wedge d\bar{z}_n$$

$\mathbb{C}^n \supset D$ anal. set of codim 1

$$\int_{\Delta_r \cap D} \sigma = D \text{ の面積}$$



$$S(r) : \partial\Delta_r \text{ の面積} = \frac{2\pi^n}{(n-1)!} r^{2n-1}$$

<u>Def.</u> $$T(r) = \int_0^r \frac{A(t)}{s(t)} dt$$

$$\left(N(r,a) = \int_0^r \frac{n(t,a)}{2\pi t} dt \right)$$ (Nat: Nevanlinna の本)

$$A(t) = \int_{\Delta_r} f^*(\omega) \wedge \sigma$$

<u>Def</u> $$N(r,\psi) = \int_0^r \frac{n(t,\psi)}{s(t)} dt$$

$$n(t,\psi) = \int_{f^*\psi \cap \Delta_r} \sigma$$

$$u_\psi(z) = \frac{1}{4\pi} \log \frac{1}{|\psi|^2 (f(z))}$$

$$m(r,\psi) = \frac{1}{s(r)} \int_{\partial\Delta_r} u_\psi(z)\, dS_z$$

dS_z: $\partial\Delta_r$ 上の surface element

$= \mathcal{M}_r(u_\psi)$: $u_\psi(z)$ の $\partial\Delta_r$ 上の平均値

First Main theorem

Stoll:

$$T(r) = N(r,\psi) + m(r,\psi) - m(0,\psi)$$

provided

$$f(0) \notin (\psi) \quad \text{i.e.} \quad |\psi|^2(f(0)) \neq 0$$

Proof

$$d = \partial + \bar\partial$$

$$d^\perp = \sqrt{-1}(\bar\partial - \partial) \qquad \text{(Weyl : Merom. curve)}$$

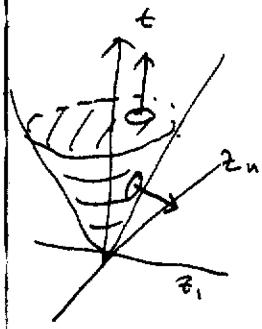
$$dd^\perp = 2\sqrt{-1}\,\partial\bar\partial$$

$$\partial\bar\partial \log \frac{1}{|\psi|^2(w)} = \partial\bar\partial \log \frac{a_j(w)}{\psi_j(w)\overline{\psi_j(w)}} = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial \log a_j(z) = w$$

$$\therefore\ dd^\perp u_\psi = 2\sqrt{-1}\,\partial\bar\partial u_\psi(z) = f^*\omega.$$

$$\frac{dt}{S(t)} = d\tau(t) \qquad \tau(t) := \frac{-(n-2)!}{4\pi^n}\frac{1}{t^{2n-2}}$$

$$T(r) = \int_0^r d\tau(t)\int_{\Delta_t} dd^\perp u_\psi \wedge \sigma$$



$$= \iint_{|z|<t<r} d\tau(t)\wedge dd^\perp u_\psi(z)\wedge\sigma(z) \qquad d\sigma = 0$$

$$\# = \tau(t)\cdot dd^\perp u_\psi \wedge \sigma(z)$$

$$= \int_{\bigtriangledown \text{の表面}} \# - \int \# + \int \# = \int_{\Delta r}[\tau(r) - \tau(|z|)]\,dd^\perp u_\psi \wedge \sigma$$

Bott Chern.

$$\therefore\ T(r) = \int_{\Delta_r} [\tau(r) - \tau(|z|)]\, dd^\perp u_\psi \wedge \sigma$$

$$= \int_{\Delta_r} d\left([\tau(r) - \tau(|z|)]\, d^\perp u_\psi \wedge \sigma\right) + \int_{\Delta_r} d\tau(|z|) \wedge d^\perp u_\psi \wedge \sigma$$

一方

$$d\tau \wedge d^\perp u_\psi \wedge \sigma = \sqrt{-1}\,(\partial\tau - \bar\partial\tau) \wedge (\bar\partial u_\psi - \partial u_\psi) \wedge \sigma$$

$$= \sqrt{-1}\,(\partial\tau \wedge \partial u + \partial u \wedge \partial\tau) \wedge \sigma$$

$$= \int_{\Delta_r} d\left([\tau(r) - \tau(|z|)]\right) d^\perp u_\psi \wedge \sigma \longrightarrow \mathrm{I}$$

$$+ \int_{\Delta_r} du_\psi \wedge d^\perp \tau(|z|) \wedge \sigma \longrightarrow \mathrm{II}$$

$$\mathrm{II} = \int_{\Delta_r} d(u_\psi\, d^\perp\tau \wedge \sigma) - \int_{\Delta_r} u_\psi\, dd^\perp\tau \wedge \sigma$$

II の第2項 0.

$$dd^\perp\tau = \sqrt{-1}\; 2\partial\bar\partial\tau.$$

$$\mathrm{II} = \int_{\Delta_r} d(u_\psi\, d^\perp\tau \wedge \sigma) = \lim_{\varepsilon\to 0} \int_{\Delta_r - \Delta_\varepsilon} d(u_\psi\, d^\perp\tau \wedge \sigma)$$

$$= \int_{\partial\Delta_r} u_\psi \, d^\perp\tau \wedge \sigma - \lim_{\epsilon\to 0} \int_{\partial\Delta_\epsilon} u_\psi \, d^\perp\tau \wedge \sigma$$

$$d^\perp\tau(z) = -\frac{(n-2)!}{4\pi^2} \sqrt{-1}\,(\bar\partial - \partial)\left(\frac{1}{|z|^{2n-2}}\right)$$

$$= \frac{-(n-1)!}{4\pi^2}\sqrt{-1}\,\left(\Sigma z_\alpha d\bar z_\alpha - \Sigma \bar z_\alpha dz_\alpha\right)$$

⑨ $$d^\perp\tau \wedge \sigma \Big|_{\partial\Delta_r} = \frac{1}{S(r)} dS(z)$$

$$dr \wedge d^\perp\tau \wedge \sigma = \frac{1}{S(r)} dV(z)$$

$\therefore$

$$= \frac{1}{S(r)} \int_{\partial\Delta_r} u_\psi \, dS(z) - \lim_{\epsilon\to 0} =$$

$$= m_r(u_\psi) - \lim m_\epsilon(u_\psi)$$

$$= m(r, \psi) - m(0, \psi)$$

$$T(r) = \sum_{\alpha=1}^{n} T_\alpha(r) \qquad \sigma = \Sigma \sigma_\alpha$$

$$T_\alpha(r) = \int_{\Delta_r} d\left([\tau(r) - \tau(|z|)]\, d^\perp u_\psi \wedge \sigma_\alpha\right)$$

$$= \int_{\Delta_r} d_\alpha\left([\tau(r) - \tau(|z|)]\, d^\perp u_\psi \wedge \sigma_\alpha\right)$$

ただし $d_\alpha = \partial_\alpha + \bar{\partial}_\alpha$

$d_\alpha^\perp = \sqrt{-1}\,(\bar{\partial}_\alpha - \partial_\alpha)$

$\sigma_1 = dV(z'')$

10/23

W : compact complex manifold

$F = \{f_{jk}\}$ $\quad a_j = |f_{jk}|^2 a_k$ $\quad \omega = \frac{\sqrt{-1}}{2\pi}\partial\bar{\partial}\log a_j$

$\psi \in H^0(W, \mathcal{O}(F))$

$\{\psi_j(w)\}$ $\quad |\psi|^2(w) = \frac{|\psi_j(w)|^2}{a_j(w)}$

$f : \underset{z}{\mathbb{C}^n} \longrightarrow W$ hol. $\quad \Delta_r = \{z \in \mathbb{C}^n \,/\, |z| < r\}$

$u_\psi(z) = \frac{1}{4\pi} \log \frac{1}{|\psi|^2(f(z))}$

$T(r) := \int_0^r A(t) \frac{dt}{S(t)}$, $A(t) = \int_{\Delta_t} f^*\omega$

$N(r,\psi) := \int_0^r n(t,\psi) \frac{dt}{S(t)}$, $n(t,\psi) = \int_{(f^*\psi)\cap\Delta_r} \sigma$

$m(r,\psi) = \mathfrak{M}_r(u_\psi)$

$= \frac{1}{S(r)} \int_{\partial\Delta_r} u_\psi(z)\, dS(z)$

First main th.

$$T(r) = N(r,\psi) + m(r,\psi) - m(0,\psi)$$

証明

$$T(r) = \underbrace{\int_{\Delta_r} d\left(\left[\tau(r) - \tau(|z|)\right] d^{\perp} u_\psi \wedge \sigma\right)}_{I} + \underbrace{\int_{\Delta_r} d\tau \wedge d^{\perp} u_\psi \wedge \sigma}_{\substack{\shortparallel \\ m(r,\psi) - m(0,\psi)}}$$

$$\sigma = \sum_{\alpha=1}^{n} \sigma_\alpha \qquad \sigma_\alpha = dV(z_1, \cdots, \hat{z}_\alpha, \cdots, z_n)$$

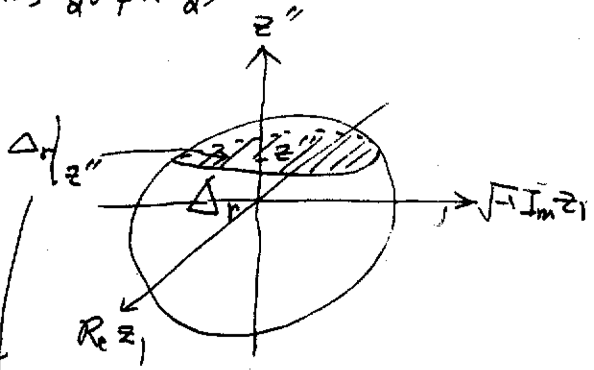
$$= \left(\frac{\sqrt{-1}}{2}\right)^{n-1} dz_1 \wedge d\bar{z}_1 \wedge \cdots \wedge \widehat{dz_\alpha \wedge d\bar{z}_\alpha} \wedge \cdots \wedge dz_n \wedge d\bar{z}_n$$

$$I = \sum_{\alpha=1}^{n} I_\alpha$$

$$I_\alpha = \int_{\Delta_r} d_\alpha\left(\left[\tau(r) - \tau(|z|)\right] d_\alpha^{\perp} u_\psi \wedge \sigma_\alpha\right)$$

I_1 の計算

$$(z_1, \underbrace{z_2, \cdots, z_n}) = (z_1, \quad z'' \quad)$$



$$I_1 = \int_{|z''|<r} dV(z'') \int_{\Delta_r|z''} d_1\left(\left[\tau(r) - \tau(|z|)\right] d_1^{\perp} u_\psi\right)$$

$$= \left\{ (z_1, z'') \;\middle|\; |z_1| < \sqrt{r^2 - |z''|^2} \right\}$$

$$\sigma_1 = dV(z'')$$

$$\int_{\Delta_r | z''} d_1\left([\tau(r) - \tau(|z_1|)\right) d_1^{\perp} u_\psi$$ の計算

$$u_\psi(z_1, z'') = \frac{1}{4\pi} \log \frac{1}{|\psi|^2 (f(z_1, z''))} = \frac{1}{4\pi} \log \frac{a_j(z)}{|\psi_j(f(z_1, z''))|}$$

divisor $(f^*\psi)$: $\psi_j(f(z_1, z'')) = 0$.

z_1 に関して $\psi_j(f(z_1, z'')) = 0$ の根を

$z_1 = \zeta_h = \zeta_h(z'')$, $h = 1, 2, 3, \cdots$ とする。

$(z'') \ni a''$ $a'' \longrightarrow r_0$ — 円周と divisor が交わらない.

$|z'' - a''| < \varepsilon$, $|z_1| < r^\circ$ のとき

$$\frac{a_j(z)}{|\psi_j(f(z_1, z''))|^2} = \frac{\alpha(z)}{\prod_{|\zeta_h(z'')| < r_0} |z_1 - \zeta_h(z'')|^2}$$

$\alpha(z) \neq 0$, C^∞-diff. function

$\therefore$

$$u_\psi(z_1, z'') = \frac{1}{2\pi} \sum_{|\zeta_h| < r^\circ} \log \frac{1}{z_1 - \zeta_h} + \beta \qquad \beta : C^\infty$$

$\therefore$

$$\int_{\Delta_r | z''} d_1\left([\tau(r) - \tau(|z_1|)]\, d_1^{\perp} u_\psi(z_1, z'')\right)$$

$$= \underbrace{\int_{円周\ |z|=r}}_{=0} + 留数$$

$X(f\cdot g) = X(f)\cdot g + f\cdot X(g)$ $\qquad X(\mathrm{const}) = 0.$

$$= + \sum_{|\zeta_h|<r} \frac{1}{2\pi}\oint_{\zeta_h} [\tau(r)-\tau(|z|)]\, d_1^{\perp} \log |z_1-\zeta_h|$$

ただし, $\displaystyle\oint_{\zeta} = \lim_{\varepsilon\to 0}\int_{|z-\zeta|=\varepsilon}.$

$$= \sum \frac{1}{2\pi}\lim_{\varepsilon\to 0}\int [\tau(r)-\tau(|z|)]\, d\theta \qquad z = (\zeta_h + \varepsilon e^{i\theta}, z'')$$

$$= \sum_{|\zeta_h|<r} [\tau(r) - \tau(|(\zeta_h, z'')|)]$$

$$\therefore \quad I_1 = \int_{(f^*\psi)\cap\Delta r} [\tau(r)-\tau(|z|)]\,\sigma_1(z)$$

$$\therefore \quad I = \sum I_\alpha = \int_{(f^*\psi)\cap\Delta r} [\tau(r)-\tau(|z|)]\,\sigma$$

$$= \iint_{\substack{|z|<t<r \\ z\in(f^*\psi)}} d\tau(t)\wedge\sigma(z)$$

$$= \int_0^r d\tau(t)\underbrace{\int_{(f^*\psi)\cap\Delta r}\sigma(z)}_{\substack{\parallel \\ n(t,\psi)}} = \int_0^r \frac{n(t,\psi)}{S(t)}\,dt$$

$$= N(r,\psi)$$

q.e.d.

$$dz \qquad dw_k = \left(\frac{\partial w_k}{\partial w_j}\right) dw_j \qquad jk.$$

§ Second main theorem (Wu ?) ↑Weyl 名前のつけかえ

Theorem (Nevanlinna)

W上任意の volume form $v = e_j(w)\, dV_j(w)$,

C^∞ 2n-form $\qquad e_j(w) > 0$

$$f : \mathbb{C}^n \longrightarrow W$$

$$f^*(v) = e_j(f(z))\, dV_j(f(z)) = e_j(f(z))\, |J_j(z)|^2\, dV(z)$$

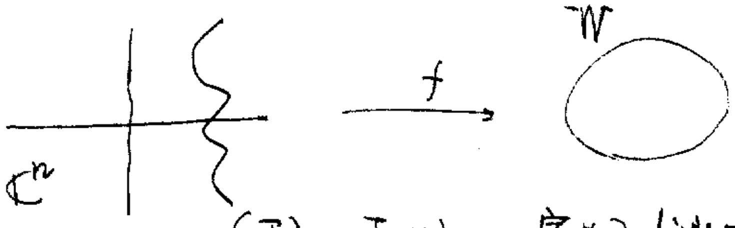
J_j: Jacob. f

$$= \xi(z)\, dV(z)$$ とおく.

$$\omega_K = \frac{\sqrt{-1}}{2\pi}\, \partial\bar\partial \log e_j(w) \sim c(K) = -c_1(W)$$

(K : canonical line bundle)

$$T_K(r) = \int_0^r \frac{A_K(t)}{S(t)}\, dt, \qquad A_K(t) = \int_{\Delta_t} f^*(\omega_K)$$



(J) : $J_j(z) = 0$ の定める divisor

$$N_1(r) = \int_0^r n_1(t)\, \frac{dt}{S(t)} \qquad n_1(t) = \int_{(J)\cap\Delta_t} \sigma$$

$$M(r) = \mathfrak{M}_r(\eta) \qquad \eta(z) = \frac{1}{4\pi} \log \xi(z)$$

Thm

$$T_K(r) + N_1(r) = M(r) - M(0)$$

$$u_\psi(z) = \frac{1}{4\pi}\log\frac{a_j(f(z))}{|\psi_j(f(z))|^2} \longleftrightarrow \eta(z) = \frac{1}{4\pi}\log\left(b_j(f(z))\,|J_j(z)|^2\right)$$

$$\omega = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log a_j \longleftrightarrow \omega_K = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log b_j.$$

$$n(t,\psi) = \int_{(f^*\psi)\cap\Delta_r}\sigma \longleftrightarrow \int_{(J)\cap\Delta_r}\sigma = n_1(t).$$

q.e.d.

Remark ① Jac f ~~is~~ not identically zero.

f : fixed.

Not.

$$\begin{array}{c} F \\ \downarrow \\ W \end{array} \qquad \omega = \omega_F \leftrightarrow c(F)$$

$$T(r) = T_F(r)$$

$$a_j = |f_{jk}|^2 a_k \longmapsto \omega_F = \frac{\sqrt{-1}}{2\pi}\partial\bar\partial\log a_j$$

- f_{jk} をかえる: $f_{jk} \to f_{jk}\dfrac{f_k}{f_j}$ non-zero.

  ω_F は inv.

- a_j をかえる.

$$\hat a_j = |f_{jk}|^2\,\hat a_k$$

$$\frac{\hat a_j}{a_j} = \frac{\hat a_k}{a_k} = a(w) > 0.$$

w の C^∞-diff. function

$$\hat{\omega}_F = \frac{\sqrt{-1}}{2\pi} \partial\bar{\partial} \log \hat{a}_j$$

$$\bullet\ \hat{\omega}_F - \omega_F = \frac{\sqrt{-1}}{2\pi} \partial\bar{\partial} \log a(w)$$

$\therefore$

$$\hat{A}_F(t) - A_F(t) = \frac{1}{4\pi}\int_{\Delta_t} dd^{\perp} \log a(f(z))$$

$$\hat{T}_F(r) - T_F(r) = \frac{1}{4\pi}\mathfrak{M}_r\left(\log a(f(z))\right)$$

$$0 < {}^{\exists}M_0 \leq a(w) \leq {}^{\exists}M_1$$

$\therefore$

$$\log M_0 \leq \hat{T}_F(r) - T_F(r) \leq \log M_1$$

Theo $T_F(r)$ は有界な函数を除いて unique.

2 Assume F: positive line bundle $\underset{\text{def}}{\Longleftrightarrow}$ ω_F: pos. def.

Theo $\varliminf_{r\to\infty} \dfrac{T_F(r)}{\log r} > 0$

1変数のとき trivial.

$$T(r) = \int_0^r \frac{A(t)}{t}\,dt.$$

$$A(t) = \int_{\Delta_t} f^*\omega. \quad : t について単増大.$$

n 変数のとき $S(t) \sim t^{2n-1}$ で同様には行かない.

<u>Lemma</u> $\begin{cases} \Omega \ (= f^*(\omega_F)) \text{ を } \text{Hermitian Form} \\ d\Omega = 0 \text{ とする.} \end{cases}$

$$\int_{\Delta_r} \Omega \wedge \sigma = \frac{r^{2n-2}}{4^{n-1}(n-1)!} \int_{\Delta_r} \Omega \wedge (dd^{\perp} \log |z|^2)^{n-1}$$

これから

$$A_F(t) = \int_{\Delta_t} f^*(\omega_F) \wedge \sigma = \frac{1}{c} t^{2n-2} \Lambda(t)$$

$$\Lambda(t) = \int_{\Delta_r} \underbrace{f^*(\omega_F) \wedge (dd^{\perp} \log |z|^2)}_{\geq 0}{}^{n-1} \quad \text{単調増大.}$$

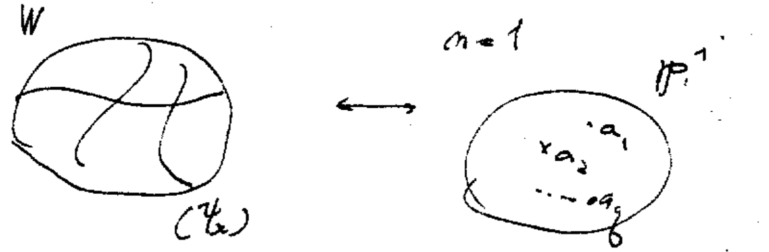
o.K.

§2. Second main theorem, defect relation.

$$T(r) = N(r, \psi) + m(r, \psi) - m(0, \psi)$$

$(f^*\psi) \cap \Delta_r$ の面積の大きくなり方を表わす.

$$H^0(W, \mathcal{O}(F)) \ni \psi_\lambda \qquad \lambda = 1, \cdots, q.$$



Second main theorem. $0 < \beta < 1$ と定めると.

$$\sum_{\lambda=1}^{q} m(r, \psi_\lambda) + N_1(r) \leqq -T_K(r) + O(\log T(r))$$

for $r \longrightarrow +\infty$, $r \notin E$

except. set

$$\int_E d(r^\beta) < \infty$$

$m(r, \psi)$ は一般には補正項.

談話会 .... 簡単な(つまらない)注意

10/24. complex structure の変形 の理論

pseudo-group structure への拡張

Kodaira: On deformation of complex pseudo-group structures
Ann Math '60

Kodaira - Spencer: Multifoliate structures
Ann. Math. '61

Spencer: いくつか Ann Math '62 306~445
〃 '65 389~490.

* 一般論はあるが example の知られる余り例がない.

今日は example を.

C^∞-diff のとき.

foliation: 昔は誰も. 今は盛ん. (つまらないと思ってたが)

——— o ———

*

① C^∞-foliation の変形

② C^∞-symplectic str. の変形

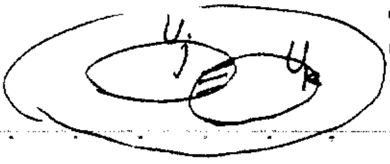
$C^\infty \equiv$ diff.

Van de Ven 現代数学は2種類.
① 定理があって応用例がない
② 例. 定理がない.

compact diff manifold. M の structure.

$X = M = \bigcup_j U_j \qquad U_j : (x_j^1, \ldots, x_j^n)$

diff. str は一定



$g_{jk} : (x_k^\alpha) \longrightarrow (x_j^\alpha)$

$x_j^\alpha = g_{jk}^\alpha (x_k)$

g_{jk} : 条件 Γ をみたすとき $\longrightarrow$ M上に Γ-structure

合理的条件 Γ $\rightarrow$ (g_{jk}) : pseudo group

$$M = \{U_j, g_{jk}\}$$

Mの変形 —— g_{jk} をかえる.

$$M_t = \{U_j, g_{jk}(z_k, \underline{\underline{t}})\}$$

$\downarrow$

X

tについて diff. なら

$\{M_t \mid t \in \Delta\}$

conn. mfd.

を diff. family.

まず Infinitesimal deformation

$$\frac{\partial M_t}{\partial t}$$

を考える。

Γ − infinitesimal — vector field $\sum_{\alpha=1}^{n} \theta_j^{\alpha} \frac{\partial}{\partial x_j^{\alpha}}$

こういうものの sheaf : Θ

$$\theta_{jk}(t) = \sum \frac{\partial g_{jk}^{\alpha}(z_k, t)}{\partial t} \frac{\partial}{\partial x_j^{\alpha}} \quad \text{on } U_j \cap U_k$$

$$\theta_{ij} + \theta_{jk} + \theta_{ki} = 0$$

これの cohomology class $\{\theta_{jk}\} \in H^1(M_t, \Theta_t)$

これを $\dfrac{\partial M_t}{\partial t}$ とする。

$H^1(M_t, \Theta_t)$

$H^q(M, \Theta)$ を調べる.

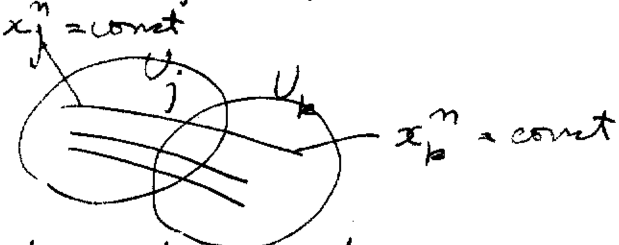
Θ の分解

$$0 \to \Theta \to \Phi^0 \to \Phi^1 \to \cdots\cdots$$

complex str. の場合 $\begin{cases} \Phi^q : \text{vector } (0,q)\text{-form} \\ \text{map} = \bar{\partial} \end{cases}$

X 上の codim 1 の foliation M

def
$$\begin{cases} x_j^\alpha = g_{jk}^\alpha(x_k), \quad \alpha = 1, \ldots, n-1 \\ x_j^n = g_{jk}^n(x_k^n) \end{cases}$$



inf. transf.
$$\Theta = \sum_{\alpha=1}^{n-1} \theta_j^\alpha(x_j) \frac{\partial}{\partial x_j^\alpha} + \theta_j^n(x_j^n) \frac{\partial}{\partial x_j^n}$$

このとき $H^1(M, \Theta)$ がどうなるか.

* complex の場合　　　Θ : hol.

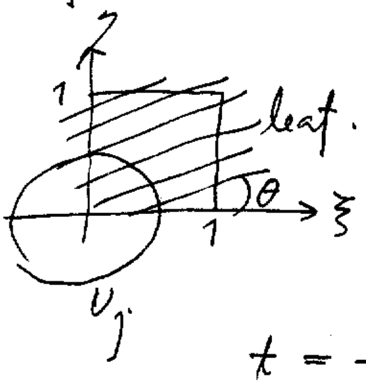
$$\dim_{\mathbb{C}} H^1(M, \Theta) = \text{moduli 数}$$

だいたいわかるもの.

これが変形を考える意味.

例 次元をみよう.

$$X = \mathbb{R}^2 / \mathbb{Z}^2$$



$t = -\tan\theta$

$$\begin{cases} x_j = \xi \\ y_j = \eta + t\xi \end{cases}$$

$y_j = \text{const} \longleftrightarrow$ leaf (foliation)

これのさだめる str. : M_t とする.

$$\begin{cases} \dfrac{\partial}{\partial x} = \dfrac{\partial}{\partial x_j} = \dfrac{\partial}{\partial x_k} \\[2ex] \dfrac{\partial}{\partial y} = \dfrac{\partial}{\partial y_j} = \dfrac{\partial}{\partial y_j} \end{cases}$$

Θ :

$$\theta^1(x,y)\frac{\partial}{\partial x} + \theta^2(y)\frac{\partial}{\partial y}$$

Θ の分解

$$0 \longrightarrow \Theta \longrightarrow V \xrightarrow{\frac{\partial}{\partial x}} \Phi^1 \longrightarrow$$

tensor.

vector field.

$$v^1(x,y)\frac{\partial}{\partial x} + v^2(x,y)\frac{\partial}{\partial y} \longmapsto \frac{\partial v^2}{\partial x}\frac{\partial}{\partial y}dx$$

実は

$$F \xrightarrow[\text{diff. funct}]{\partial/\partial x} F$$

$$\therefore\ H^1(M_t, \Theta_t) = \frac{H^0(M, F)}{\frac{\partial}{\partial x} H^0(M, F)} = \frac{H^0(X, F)}{\frac{\partial}{\partial x} H^0(X, F)}$$

- $$\frac{\partial}{\partial x_j} = \frac{\partial}{\partial \xi} - t\frac{\partial}{\partial \eta} = \frac{\partial}{\partial x}$$

$$\dim_{\mathbb{R}} H^1(M_t, \Theta_t) = \begin{cases} \infty & t:\text{有理数} \\ 1 & t:\text{代数的無理数} \\ \infty \text{ or } 1 & t:\text{超越数} \end{cases}$$

例えば $t = \sum_{k=1}^{\infty} \frac{1}{n_k}$ $\begin{cases} n_1 = 2 \\ n_{k+1} = n_k \cdot n_k! \end{cases}$ 超越数

のときは $\underline{\infty}$

$\psi \in H^0(X, F)$ がいつ $\frac{\partial}{\partial x}\varphi$ とかけるか

$$\psi = \sum a_{mn} e^{2\pi i(m\xi + n\eta)} \quad (\text{Fourier 級数})$$

$$\varphi = \sum c_{mn} e^{2\pi i(\quad \cdot \quad)}$$

とすると

$$\frac{\partial\varphi}{\partial x} = \left(\frac{\partial}{\partial\xi} - t\frac{\partial}{\partial\eta}\right)\varphi$$

$$= \sum 2\pi i\,(m - tn)\, c_{mn}\, e^{2\pi i(m\xi + n\eta)}$$

問題は $\underbrace{a_{mn}}_{\text{given}} = 2\pi i\,(m - tn)\underbrace{c_{mn}}_{?}$ を求める。

τ: 無理数のとき

$$a_{00} = 0.$$

$$C_{mn} = \frac{a_{mn}}{2\pi i(m - \tau n)}$$

$\ast$ C^∞になる条件 $\Longleftrightarrow$ $\sum (m^2+n^2)^k |C_{mn}|^2 < \infty \quad \forall k$

$\ast$ は

$$\sum (m^2+n^2)^k |a_{mn}|^2 < \infty \quad \text{given.}$$

のとき

$$\sum \frac{(m^2+n^2)^k |a_{mn}|^2}{|m - \tau n|^2} < +\infty$$ がいえるか.

algebraic のときは.

$$\left|\frac{m}{n} - \tau\right| \geqq \frac{c}{|n^8|} \quad \text{(Diophantos 近似)}$$

$\dim H^1$ 有理数 compact leaf
無理数 non-compact leaf.

〜 代数的判断

↓

ダメか.

Symplectic structure.

$\frac{1}{2}\sum \varepsilon_{\alpha\beta} dx^\alpha \wedge dx^\beta = dx^1 \wedge dx^2 + dx^3 \wedge dx^4 + \cdots + dx^{2n-1} \wedge dx^{2n}$

$$\varepsilon_{\alpha\beta} = \begin{pmatrix} \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & & 0 \\ & \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & \\ 0 & & \ddots \end{pmatrix}$$

X 上の symplectic structure M, f

$$\begin{cases} X = \bigcup_j U_j \quad (x_j^\alpha \\ g_{jk} \text{の条件} \Gamma : f = \sum \varepsilon_{\alpha\beta} dx_j^\alpha \wedge dx_j^\beta = \sum \varepsilon_{\alpha\beta} dx_k^\alpha \wedge dx_k^\beta \end{cases}$$

で定まる。

$M = (X, f)$

Lie-deriv.

$\Theta = \{\theta\} \qquad \theta \cdot f = 0.$

$$\theta \cdot f = \sum \varepsilon_{\alpha\beta} d\theta_j^\alpha \wedge dx_j^\beta$$

ただし

$$\theta = \sum \theta_j^\alpha \frac{\partial}{\partial x_j^\alpha}$$

◎ $\dim_{\mathbb{R}} H^1(M, \Theta) = b_2$ (M の Betti 数)

$$0 \to \Theta \to V \xrightarrow{} \quad \text{vect. field の sheaf}$$

$$V \ni \sum v_j^\alpha \frac{\partial}{\partial x_j^\alpha} \mapsto \varphi = \sum \varepsilon_{\alpha\beta} v_j^\alpha dx_j^\beta$$

$$\sum v_j^{\alpha} \frac{\partial}{\partial x_j^{\alpha}} \qquad \underline{v \circ f = d\varphi.}$$

$$\updownarrow$$

tensor

$$\varphi = \sum \varepsilon_{\alpha\beta}\, v_j^{\alpha}\, dx_j^{\beta}$$

$\therefore$

$$\begin{array}{ccccccccc} 0 \longrightarrow & \Theta & \longrightarrow & V & \xrightarrow{\circ f} & \Phi^2 & \xrightarrow{d} & \Phi^3 & \longrightarrow \cdots \\ & & & \cong & \nearrow_{d} & & & & \\ & & & \Phi^1 & & & & & \end{array}$$

(de-Rham が1つずれている.)

(Φ^q: q-form の sheaf)

$$\therefore \dim_{\mathbb{R}} H^q(M, \Theta) = \dim H^{q+1}(\text{de-Rham}) = b_{q+1}$$

$$\therefore \quad \frac{\partial M_t}{\partial t} \in H^2(M, \mathbb{R}) \cong \frac{H^0(M_t, d\Phi^1)}{d\, H^0(M_t, \Phi^1)}$$

M_t $\qquad x_j^{\alpha} - x_j^{\alpha}(\xi, t)$ diff.

$\downarrow$

$X \ni \xi$ fixed.

$$f(\xi, t) = \frac{1}{2} \sum \varepsilon_{\alpha\beta}\, dx_j^{\alpha} \wedge dx_j^{\beta}$$

$$\frac{\partial f}{\partial t} \in H^0(M_t, d\Phi^1)$$

$$(\text{mod } d\, H^0(M_t, \Phi^1))$$

$$\updownarrow$$

周期を考える.

$H_2(X, \mathbb{R})$ の base $\{ Z_\lambda \mid \lambda = 1, 2, \cdots, b_2 \}$

$$\int_{Z_\lambda} f(\xi, t) = \omega_\lambda(t)$$

$$\frac{\partial M_t}{\partial t} \Longleftrightarrow \left(\frac{\partial \omega_1(t)}{\partial t}, \cdots\cdots, \frac{\partial \omega_{b_2}(t)}{\partial t} \right)$$

$\Updownarrow$

str. の moduli.

$$M = (X, f)$$
$$\downarrow$$
$$X$$

$$\int_{Z_\lambda} f = \omega_\lambda$$

$$\omega(M) = (\omega_1, \cdots, \omega_{b_2}) \in R^{b_2}$$

$$M \longrightarrow \omega(M) \quad \text{moduli.}$$

$|t| < \varepsilon$ （小さな変形） に対しては

存在 $\omega(M) \longrightarrow M$. がいえる。

$\because$

Darboux.

$$\begin{cases} 2\text{-form} \quad \psi(t) \text{ on } X \quad |t| < \varepsilon. \\ \qquad\qquad\quad \text{diff.} \\ d\psi(t) = 0 \\ \underbrace{\psi(t) \wedge \psi(t) \wedge \cdots \wedge \psi(t)}_{n\text{回}} \neq 0 \ \cdots \ X \text{の各点で} \end{cases}$$

$\Longrightarrow$

$\Rightarrow \quad X = \bigcup_j U_j$ covering $\quad x_j^\alpha(\xi, t)$

$$\psi(t) = \frac{1}{2} \sum \varepsilon_{\alpha\beta}\, dx_j^\alpha(\xi, t) \wedge dx_j^\beta(\xi, t)$$

とできる。

f : given

γ_λ : 2-forms of base

$$d\gamma_\lambda = 0$$

$$\int_{Z_\nu} \gamma_\lambda = \delta_{\lambda\nu}$$

$$\psi(t) = f + \sum_{\lambda=1}^{b_2} t_\lambda \gamma_\lambda$$: Thの条件をみたす.

$\downarrow$

$M_t \qquad \omega(M_t) = (\omega_1 + t_1, \omega_2 + t_2, \cdots, \omega_{b_2} + t_{b_2})$

$\forall \omega \in \mathbb{R}^{b_2}$ を与えて M がつくれるか

i.e. ω_λ given から $\int_{Z_\lambda} \psi = \omega_\lambda \quad d\psi = 0$

となる ψ の存在は de Rham.

$\left\{ \begin{array}{l} \psi^n = \psi \wedge \psi \wedge \cdots \wedge \psi \text{ が至るところ non-zero} \\ \text{とできるか.} \end{array} \right.$

条件がいる

M^4のとき <u>example</u> torus.

K3-surface (complex dim 2)

orientation を $\psi \wedge \psi > 0$ にきめる.

$$\psi = \sum \omega_\lambda \gamma_\lambda$$

$$A_{\lambda\mu} = \int_M \gamma_\lambda \wedge \gamma_\mu$$

条件 $\sum A_{\lambda\nu} \omega_\lambda \omega_\nu > 0$. 必要条件

torus の場合は十分条件.

γ_λ $dx^\alpha \wedge dx^\beta$ $(\alpha < \beta)$ 6 個

$$\psi \wedge \psi = \left(\sum A_{\lambda\nu} \omega_\lambda \omega_\mu\right) dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4$$

K3-surface の場合 $b_2 = 22$ で うまくいかない.

$$\psi = \sum c_{\alpha\beta} dx^\alpha \wedge dx^\beta \qquad c_{\alpha\beta} : \text{const}$$

問題 — M^4 torus

$\left.\begin{matrix}\psi \\ \varphi\end{matrix}\right)$ d-closed two form $= \sum c_{\alpha\beta} dx^\alpha \wedge dx^\beta$
$= \sim$

$$\int_{Z_\lambda} \varphi = \int_{Z_\lambda} \psi$$

$$\Rightarrow ?\ M \xrightarrow{\text{diffeo}} M \qquad \varphi \longmapsto \psi.$$

* complex symplectic manifold の例

$$f = \sum \varepsilon_{\alpha\beta}\, dz^\alpha \wedge dz^\beta$$

わかっている例

torus

K3-surface

この直積（変形にも直積）

この場合 submanifold の概念がない.

10/25. Second M. Thm の証明

$$u_\psi(f(z)) = \frac{1}{4\pi} \log \frac{1}{|\psi|^2(f(z))}$$

を修正して

$$\rho_\psi(w) = \frac{\kappa}{[\log|\psi|^2(w)]^2\, |\psi|^2(w)} \qquad \kappa > 0 : \text{small constant.}$$

を考える (Nevanlinna)

$\forall$ (ψ) simple divisor

$\to w_j^1$

$$|\psi|^2(w) = |w_j^1|^2$$

$$\int \frac{dw_j^1 \wedge d\bar{w}_j^1}{|w_j^1|^2} \sim \int \frac{r\,dr\,d\theta}{r^2} \qquad r=0 \text{で発散}$$

一方.

$$\int \frac{dw \wedge d\bar{w}}{[\log|w|^2]^2 |w|^2} \sim \int \frac{r\,dr\,d\theta}{(\log r^2)^2 r^2} \qquad r=0 \text{で収束.}$$

$$\tilde{u}_\psi(z) = \frac{1}{4\pi} \log \rho_\psi(f(z)) = u_\psi(z) - \underbrace{\frac{1}{2\pi} \log\log \frac{1}{|\psi|^2(f(z))}}_{= \mathcal{L}_\psi(z)} + \kappa_1$$

$*$ $$\tilde{T}_\psi(r) = \int_0^r \tilde{A}_\psi(t)\, d\tau(t)$$

$$\tilde{A}_\psi(t) = \int_{\Delta_t} dd^c \tilde{u}_\psi \wedge \sigma$$

- $\tilde{m}(r,\psi) = \mathcal{M}_r(\tilde{u}_\psi)$

- $\tilde{T}_\psi = T(r) - T_\psi(r)$

ただし

$$T_\psi = \int_0^r a_\psi(t)\, d\tau(t)$$

$$a_\psi(t) = \int_{\Delta_t} dd^\perp \mathcal{L}_\psi \wedge \sigma$$

- $\tilde{m}(r,\psi) = m(r,\psi) - \mu_\psi(r) + \kappa_1$

$$\mu_\psi(r) = \mathcal{M}_r(\mathcal{L}_\psi)$$

FMT : $T(r) = N(r,\psi) + m(r,\psi) - m(0,\psi)$

同じ計算をして

$$T_\psi(r) = \mu_\psi(r) - \mu_\psi(0)$$

∵ $T_\psi(r) = \displaystyle\int_{\Delta_r} d(\tau(r) - \tau(|z|))\, d^\perp \mathcal{L}_\psi \wedge \sigma + \underbrace{\int_{\Delta_r} d\mathcal{L}_\psi \wedge d^\perp \tau(|z|) \wedge \sigma}_{\mathcal{M}_r(\mathcal{L}_\psi) - \mathcal{M}_0(\mathcal{L}_\psi)}$

第1項 $= \sum \oint [\quad] d^{\perp} \mathcal{L}_\psi$

一方. $$d^{\perp} \log \log \frac{1}{|\psi|^2} = \underbrace{\frac{1}{4\pi \log \frac{1}{|\psi|^2}}}_{\substack{\downarrow \\ 0. \\ (\psi = 0).}} u_\psi$$

$(f^*\psi)$ での留数は 0.

$\mu_\psi(r)$ の評価

∘ $$\frac{1}{b-a}\int_a^b \log \varphi \leqq \log\left(\frac{1}{b-a}\int_a^b \varphi\right)$$

(相乗平均 ≦ 相加平均)

∴

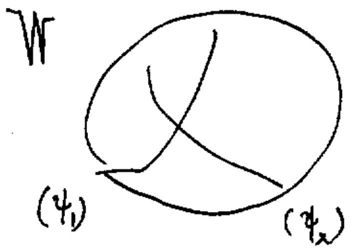
$$\mu_\psi(r) = \mathfrak{M}_r\left(\mathcal{L}_\psi(z)\right) = \mathfrak{M}_r\left(\frac{1}{2\pi}\log\log\frac{1}{|\psi|^2}\right)$$

$$\leqq \frac{1}{2\pi} \log\left(\mathfrak{M}_r\Big(\underbrace{\log\frac{1}{|\psi|^2}}_{4\pi u_\psi}\Big)\right)$$

$$= \frac{1}{2\pi} \log\left(m(r,\psi) \cancel{+\log 4\pi}\right) + \underbrace{\frac{1}{2\pi}\log 4\pi}_{\text{const.}}$$

最初 $|\psi|^2 < e^{-4}$ にとったから

$$\frac{1}{2\pi}\log 4 \leqq \mu_\psi(r) \leqq \frac{1}{2\pi}\log m(r,\psi) + \frac{1}{2\pi}\log 4\pi.$$



F は >0. i.e. $\omega = \omega_F > 0$

$\psi_\lambda \in H^0(W, \mathcal{O}(F))$, $\lambda = 1, 2, \cdots, q$

<u>Assume</u> (ψ_λ) は simple

$\sum_{\lambda=1}^{q} (\psi_\lambda)$ は normal crossing 以外の singularity をもたない.

(本質上の仮定ではない.
(これがないとウソになる: delicate な条件)

main theo. $0 < \beta < 1$

$$\sum_{\lambda=1}^{q} m(r, \psi_\lambda) + N_1(r) \leqq -T_K(r) + O(\log T(r))$$

for $r \longrightarrow +\infty$, $r \notin E$ except. set

$$\int_E d(r^\beta) < +\infty$$

<u>Proof</u> $$\rho(w) = \prod_{\lambda=1}^{q} \rho_{\psi_\lambda}(w) = \kappa^q \prod_{\lambda=1}^{q} \frac{1}{[\log|\psi|^2]^2 |\psi|^2},$$

where $|\psi|^2 = |\psi|^2(w)$

<u>Lemma</u> (Clemens - Griffiths) $\Leftarrow$ $\boxed{\text{Essential !!}}$

$$\left(\frac{1}{4\pi} dd^{\perp} \log \rho(w)\right)^n \geqq \rho(w)\,\omega^n, \quad \omega = \omega_F,$$

for suffic. small $\kappa > 0$.

($n = 1$: Nevanlinna ではない)

$$\frac{1}{4\pi} dd^{\perp} \log \rho(w) = \sum_{\lambda=1}^{g}\left(1 - \frac{2}{\log \frac{1}{|\psi_\lambda|^2}}\right)\omega$$

$$+ \sum_{\lambda=1}^{g} \frac{2}{\left[\log|\psi_\lambda|^2\right]^2} \underbrace{\frac{\sqrt{-1}}{2\pi} \partial \log \frac{1}{|\psi_\lambda|^2} \wedge \bar{\partial} \log \frac{1}{|\psi_\lambda|^2}}_{\geqq 0}$$

$$\geqq \frac{g}{2}\omega$$

$$\therefore \left(\frac{1}{4\pi} dd^{\perp} \log \rho(\omega)\right)^n \geqq \left(\frac{g}{2}\right)^n \omega^n$$

$\rho(w)$ は $W - \bigcup_\lambda (\psi_\lambda)$ で連続, > 0

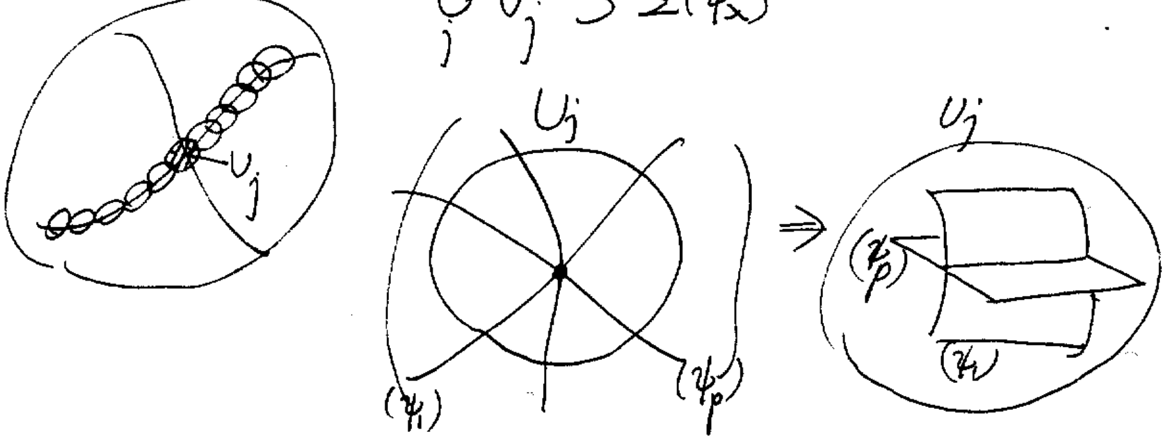
・$dd^{\perp} \log \rho$ は κ に無関係.

κ を充分小さくすれば

$\rho(w) = \infty$ になる近傍をあらかじめ除いておくこと.

$$\left(\frac{1}{4\pi} dd^{\perp} \log \rho\right)^n \geqq \rho\, \omega^n \quad : \text{on } W - \bigcup_j U_j$$

$$\bigcup_j U_j \supset \Sigma(\psi_\lambda)$$



U_j の座標系 : $(w_j^1, \dots, w_j^n)$ を

$\psi_{\lambda j}(w) = w_j^{\lambda}$ $(1 \le \lambda \le p)$ にとる。

↑ normal crossing を essential に使う。

$$|\psi_\lambda|^2(w) = \frac{|w_j^\lambda|^2}{a_j(w)}$$

∞ での比較なので

$$\partial \log \frac{1}{|\psi|^2} \wedge \bar{\partial} \log \frac{1}{|\psi|^2}$$

を $\partial \log \dfrac{1}{|w_j^\lambda|^2} \wedge \bar{\partial} \log \dfrac{1}{|w_j^\lambda|^2}$ におきかえる。

∴

$$\frac{1}{4\pi} dd^{\perp} \log \rho \ge \frac{\delta}{2} \cdot \omega + \sum_{\lambda=1}^{p} \frac{2}{(\log|\psi_\lambda|^2)^2} \frac{\sqrt{-1}}{2\pi} \frac{dw^\lambda}{w^\lambda} \wedge \frac{d\bar{w}^\lambda}{\bar{w}^\lambda}$$

∴

$$\left(\frac{1}{4\pi} dd^{\perp} \log \rho\right)^n \ge \left(\frac{\delta}{2}\right)^{n-p} \omega^{n-p} \wedge \prod_{\lambda=1}^{p} \frac{a_j}{\underbrace{\pi[\log|\psi_\lambda|^2]^2|\psi_\lambda|^2}} dw_j^1 \wedge d\bar{w}_j^1 \wedge \cdots \wedge dw_j^p \wedge d\bar{w}_j^p$$

∴ Kを小さくとれば

$$\ge \rho(w)\, \omega^n \quad \text{on } U_j.$$

q. e. d.

$v = e_j(w)\, dV(w_j)$ volume form.

$$\left(\frac{1}{4\pi} dd^{\perp} \log \rho\right)^n \ge \rho(w) v$$

- $\displaystyle\int_W \rho(w) v < +\infty$ (ρ のつくり方)

$$\therefore \int_{\Delta r} f^*(\rho(w)v) < +\infty \qquad f : \mathbb{C}^n \longrightarrow W.$$

ここで

$$f^*(\rho(w)v) = \rho(f(z))\, \xi(z)\, dV(z)$$

$$\xi(z) = e_j(f(z))\, |J_J(z)|^2$$

i.e.

$$\int_{\Delta r} \underbrace{\rho(f(z))\, \xi(z)}_{\substack{\text{W}\\ 0.}}\, dV(z) < +\infty$$

$\therefore$

$$\Psi(r) \underset{def}{=} \int_{\Delta r} \left(\rho(f(z))\,\xi(z)\right)^{\frac{1}{n}} dV(z) < +\infty$$

$$= \int_0^r dt \int_{\partial \Delta_t} (\quad)\, dS = \int_0^r S(t)\, \Phi(t)\, dt$$

$$\Phi(t) = \frac{1}{S(t)} \int_{\partial \Delta_t} \rho(f)\, \xi\, dS(z)$$

<u>Lemma</u> (Nevanlinna) 一般に $\Phi(t) \geqq 0$, 連続

$$\int_0^r S(t)\Phi(t)\, dt = \Psi(r), \quad \int_0^r \frac{\Psi(t)}{S(t)}\, dt = \Xi(r)$$

とおく。 $0 < \beta < 1$ を与えたとき (本をみよ)

$$\Phi(r) \leqq \Xi(r)^\nu \qquad \text{for } r \notin E$$

where E open $\displaystyle\int_E d(r^\beta) < +\infty$

$$\left(\frac{n}{n-1}\right)^2 > \nu = \left(\frac{4n-2}{2n-2+\beta} - 1\right)^2 > 1$$

$$T_K(r) + N_1(r) = M(r) - M(0)$$

より

$$\sum_1^{8} m(r, \psi_\lambda) + M(r) \leqq O(\log T(r)) \quad r \notin E$$

といえばよい.

$N(r, \psi) \geqq 0$ より

$$T(r) + m(0, \psi) \geqq m(r, \psi)$$

$$\therefore \quad \mu_\psi(r) \leqq O(\log T(r))$$

$$\therefore \quad \sum_{\lambda=1}^{8} \tilde{m}(r, \psi_\lambda) + M(r) \leqq O(\log T(r)) \quad r \notin E$$

といえばよい.

$$\underset{\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim\sim}{\sum \tilde{m}(r, \psi_\lambda) + M(r)} = \mathfrak{M}_r\Big(\sum_\lambda \underset{\frac{1}{4\pi}\log \rho_{\psi_\lambda}}{\underset{\shortparallel}{\tilde{u}_\psi}} + \eta \Big)$$

$$= \frac{1}{4\pi} \mathfrak{M}_r\Big(\log\big(\prod \rho_{\psi_\lambda} \cdot \xi\big)\Big)$$

$$= \frac{1}{4\pi} \mathfrak{M}_r\big(\log(\rho(f(z)) \cdot \xi)\big)$$

$$= \frac{n}{4\pi} \mathfrak{M}_r\Big(\log(\rho(f(z)) \cdot \xi)^{\frac{1}{n}}\Big)$$

$$\leqq \frac{n}{4\pi} \log \mathfrak{M}_r(\rho(f) \cdot \xi)^{\frac{1}{n}} = \frac{n}{4\pi} \log \Phi(r)$$

Lemma (N) より

$$\leqq \frac{n\nu}{4\pi} \log \Xi(r) \qquad \text{for} \quad r \notin E.$$

Lemma (C-G) より

$$\rho(f(z))\,\xi(z)\,dV(z) \leqq \Big(\underbrace{\frac{1}{4\pi} dd^{\perp} \log \rho(f(z))}_{\shortparallel}\Big)^{n}$$

$$\frac{\sqrt{-1}}{2} \sum_{\alpha,\beta=1}^{n} h_{\alpha\beta}\, dz_{\alpha} \wedge d\bar{z}_{\beta}.$$

$$= n!\, \det(h_{\alpha\beta})\, dV(z).$$

$\therefore \quad \rho(f(z))\,\xi(z) \leqq n!\, \det(h_{\alpha\beta})$

一方 $\quad (\det h_{\alpha\beta})^{\frac{1}{n}} \leqq \frac{1}{n} \sum_{\alpha=1}^{n} h_{\alpha\alpha}$

$\therefore$

$\circ \quad \big(\rho(f(z))\,\xi(z)\big)^{\frac{1}{n}} \leqq \sum_{\alpha=1}^{n} h_{\alpha\alpha}$

$$\Big(\frac{\sqrt{-1}}{2} \sum h_{\alpha\beta}\, dz_{\alpha} \wedge d\bar{z}_{\beta}\Big) \wedge \sigma = \sum h_{\alpha\alpha}\, dV(z)$$

$\therefore \quad (\rho(f)\,\xi)^{\frac{1}{n}}\, dV \leqq \frac{1}{4\pi} dd^{\perp} \log \rho(f) \wedge \sigma$

一方 $\quad \Phi(r) = \int_{\Delta_r} (\rho(f)\,\xi)^{\frac{1}{n}}\, dV(z) = \sum_{\lambda=1}^{q} \int \frac{1}{4\pi} dd^{\perp} \log \rho_{\psi_\lambda} \wedge \sigma$

$$= \sum_{\lambda=1}^{q} \int \frac{1}{4\pi} dd^{\perp} \tilde{u}_{\psi_\lambda} \wedge \sigma$$

$$= \sum_{\lambda=1}^{q} \tilde{A}_{\psi_\lambda}$$

$$\therefore \quad \Xi(r) = \int_0^r \Psi(t)\, d\tau(t) = \sum_{\lambda=1}^{q} \widetilde{T}_{\psi_\lambda}(r)$$

$$\leqq q\,T(r) + \text{const.}$$

$\therefore$

$$\sum_{\lambda=1}^{q} \widetilde{m}(r, \psi_\lambda) + M(r) \leqq O(\log T(r))$$

$$\text{for } r \notin E.$$

QED.