# Algebraic Geometry and Integrable Systems, Kobe 2016

Date: December 5 - 9, 2016 Place: Room Z201, Graduate School of Science, Kobe Univ.

	9:30-10:30	10:45-11:45	13:30-14:30	14:45-15:45	16:00-17:00
Dec.5(Mon)			H. Nakajima	T. Hausel	S. Mukai
6(Tue)	H. Iritani	Y. Toda	B. Kim	T. Mochizuki	Y. Namikawa
$7(Wed)^*$	S. Kondo	C. Simpson	F. Loray	R. Donagi	MH. Saito
8(Thu)	K. Iwaki	A. Komyo	M. Mulase	S. Szabo	C. Sabbah
9(Fri)	A. Takahashi	K. Fukaya			

\* Dec.7(Wed) 18:00 – Party at Takigawa Memorial Hall

### Dec.5 (Mon)

- 13:15 Opening ceremony
  - Welcome speech of **President Takeda**
  - Practical Remarks
- 13:30 14:30 **Hiraku Nakajima** (RIMS, Kyoto University) Cherkis bow varieties
- 14:45 15:45 **Tamás Hausel** (EPFL, Switzerland/ IST, Austria) Equivariant Verlinde algebra for Higgs bundles
- 16:00 17:00 Shigeru Mukai (RIMS, Kyoto University)

Enriques surfaces: fundamental polarizations and their mirror families

# Dec.6 (Tue)

9:30 – 10:30 Hiroshi Iritani (Kyoto University)

On the Gamma conjecture associated with toric flips

- 10:45 11:45 **Yukinobu Toda** (Kavli IPMU, University of Tokyo) Gopakumar-Vafa invariants via vanishing cycles
- 13:30 14:30 **Bumsig Kim** (KIAS)

Quasimap Wall-crossings

- 14:45 15:45 **Takuro Mochizuki** (RIMS, Kyoto University) Curve test for enhanced ind-sheaves and holonomic D-modules
- 16:00 17:00 Yoshinori Namikawa (Kyoto University)A characterization of nilpotent orbit closures among symplectic singularities

### Dec.7 (Wed)

9:30 – 10:30 **Shigeyuki Kondo** (Nagoya University) Classification of Enriques surfaces with finite automorphism group

in characteristic  $2\,$ 

- 10:45 11:45 **Carlos Simpson** (Université Nice Sophia Antipolis) Higher direct images in nonabelian Hodge theory
- 13:30 14:30 Frank Loray (Université de Rennes1)Moduli of connections on curves: some examples
- 14:45 15:45 **Ron Donagi** (University of Pennsylvania) Hitchin's system and Geometric Langlands
- 16:00 17:00 Masa-Hiko Saito (Kobe University)
  Moduli spaces of connections and Higgs bundles: Algebraic geometry and differential equations of Painlevé type

# Dec.8 (Thu)

- 9:30 10:30 Kohei Iwaki (Nagoya University) Topological recursion, Painlevé equations and cluster algebras
- 10:45 11:45 **Arata Komyo** (Kobe University) Hamiltonian structures of isomonodromic deformations on moduli spaces

of parabolic connections

- 13:30 14:30 Motohico Mulase (University of California, Davis) Algebraic geometry of quantum curves and opers
- 14:45 15:45 **Szilard Szabo** (Budapest University of Technology and Economics) Elementary transformations of singular oper-like objects
- 16:00 17:00 Claude Sabbah (École Polytechnique) Irregular mixed Hodge structures and modules

Dec.9 (Fri)

9:30 – 10:30 Atsushi Takahashi (Osaka University)

On entropy of autoequivalences of smooth projective varieties

10:45 – 11:45 **Kenji Fukaya** (Simons Center for Geometry and Physics, Stony Brook) Lagrangian Floer theory in symplectic manifolds with divisors

# Abstract

### Hiraku Nakajima (RIMS, Kyoto University)

Title: Cherkis bow varieties

**Abstract:** Bow varieties are introduced by Cherkis as an ADHM type description of moduli spaces of U(n)-instantons on multi-centered Taub-NUT spaces. We will give their quiver description, and study their properties as complex symplectic algebraic varieties. In particular, they are integrable systems in Liouville sense, i.e., they have commuting hamiltonians whose level sets are algebraic tori. If time permits, I will explain expected relations to representation theory of affine Lie algebras. (Based on the joint work with Yuuya Takayama.)

### Tamás Hausel (EPFL, Switzerland/ IST, Austria)

**Title:** Equivariant Verlinde algebra for Higgs bundles

**Abstract:** Motivated by the work of Gukov and Du Pei we discuss a construction of a Frobenius algebra, which computes equivariant indices of line bundles on the moduli space of Higgs bundles. This is joint work with Andras Szenes.

### Shigeru Mukai (RIMS, Kyoto University)

**Title:** Enriques surfaces: fundamental polarizations and their mirror families **Abstract:** Among algebraic surfaces, Enriques' locate between K3 and rational surfaces. I first explain their 9 fundamental polarizations which correspond to the non-degenerate 9 vertices of the Dynkin diagram of type E10 (or T237). Each fundamental polarization has symmetry of the Weyl group of the complementary ADE diagram (of rank 9), and determines the "mirror", a 1-dimensional family of Enriques surfaces characterized by such an ADE diagram in a certain way. I will give explicit equations, degenerations and description of automorphism groups of the mirrors (with few exceptions).

### Hiroshi Iritani (Kyoto University)

**Title:** On the Gamma conjecture associated with toric flips

**Abstract:** Gamma conjecture in a broad sense predicts a relationship between decompositions of quantum differential equations and those of derived categories. In this talk, I will discuss a partially compactified Landau-Ginzburg mirror symmetry for toric stacks. This picture naturally leads to a formal decomposition of the quantum differential equations of toric stacks under flips. Part of this talk is based on joint work with Tom Coates, Alessio Corti and Hsian-Hua Tseng.

#### Yukinobu Toda (Kavli IPMU, University of Tokyo)

Title: Gopakumar-Vafa invariants via vanishing cycles

**Abstract:** I propose an ansatz for defining Gopakumar-Vafa invariants of Calabi-Yau threefolds, using perverse sheaves of vanishing cycles. The proposal is a modification of a recent approach of Kiem-Li, which is itself based on earlier ideas of Hosono-Saito-Takahashi. These invariants are conjectured to be equivalent to other curve-counting theories such as Gromov-Witten theory and Pandharipande-Thomas theory. The main theorem is that, for local surfaces, our GV invariants agree with PT invariants for irreducible curve classes. I also give a counter-example to the Kiem-Li conjectures, where our invariants match the predicted answer. This is a joint work with Davesh Maulik.

### Bumsig Kim (KIAS)

### Title: Quasimap Wall-crossings

**Abstract:** We introduce a wall-crossing formula for the virtual classes of epsilonstable quasimaps to GIT quotients and show it for complete intersections in products of projective spaces, with no positivity restrictions on their first Chern class. As a consequence, the wall-crossing formula relating the genus g descendant Gromov-Witten potential and the genus g epsilon-quasimap descendant potential is established. For the quintic threefold, the results may be interpreted as giving a rigorous and geometric interpretation of the holomorphic limit of the BCOV B-model partition function of the mirror family. This is a joint work with I. Ciocan-Fontanine.

### Takuro Mochizuki (RIMS, Kyoto University)

Title: Curve test for enhanced ind-sheaves and holonomic D-modules

**Abstract:** Recently, the Riemann-Hilbert correspondence was generalized in the context of general holonomic D-modules by A. D'Agnolo and M. Kashiwara. Namely, they proved that their enhanced de Rham functor gives a fully faithfully embedding of the derived category of cohomologically holonomic complexes of D-modules into the derived category of complexes of real constructible enhanced ind-sheaves.

In this talk, we discuss a condition when a complex of real constructible enhanced ind-sheaves is induced by a cohomologically holonomic complex of D-modules. We characterize such complexes in terms of their restriction to holomorphic curves.

# Yoshinori Namikawa (Kyoto University)

**Title:** A characterization of nilpotent orbit closures among symplectic singularities **Abstract:** A singular affine symplectic variety  $(X, \omega)$  is conical if its coordinate ring R is positively graded and  $\omega$  is homogeneous. A conical symplectic variety plays an important role in algebraic geometry and geometric representation theory. The maximal degree of the minimal homogeneous generator of R is called the maximal weight of X.

There are only finitely many conical symplectic varieties with a fixed dimension 2d and a fixed maximal weight N.

In this talk we will give a classification of conical symplectic varieties with maximal weight 1. They are affine spaces with standard symplectic forms or nilpotent orbit closures of complex semisimple Lie algebras with Kirillov-Kostant forms.

### Shigeyuki Kondo (Nagoya University)

**Title:** Classification of Enriques surfaces with finite automorphism group in characteristic 2

**Abstract:** I will discuss classification of Enriques surfaces with finite automotphism group in characteristic 2. This is joint work with Toshiyuki Katsura and Gebhard Martin.

### Carlos Simpson (Université Nice Sophia Antipolis)

**Title:** Higher direct images in nonabelian Hodge theory

**Abstract:** We report on joint work with Ron Donagi and Tony Pantev on the Dolbeault calculation of the higher direct image of a parabolic Higgs bundle. In order to obtain the parabolic structure on the higher direct image, which is the expected one, we need to analyze the nearby cycles construction near a normal crossing point in the theory of Hodge modules-twistor D-modules.

### Frank Loray (Université de Rennes1)

**Title:** Moduli of connections on curves: some examples **Abstract:** We will describe some examples of moduli spaces of parabolic bundles and logarithmic connections on curves of low genus.

#### Ron Donagi (University of Pennsylvania)

**Title:** Hitchin's system and Geometric Langlands **Abstract:** We discuss an approach to the Geometric Langlands Conjecture based on Hitchin's system combined with non Abelian Hodge theory.

### Masa-Hiko Saito (Kobe University)

**Title:** Moduli spaces of connections and Higgs bundles: Algebraic geometry and differential equations of Painlevé type

Abstract: In the first half of this talk, I will recall the known results on the algebraic constructions on the moduli spaces of stable parabolic connections and Higgs bundles on a smooth projective curve. I will continue to explain about the Riemann-Hilbert correspondence from a family of moduli spaces of singular connections to the corresponding moduli spaces of (generalized) monodromy data, then I will explain how we can prove the geometric Painlevé property of isomonodromic differential equations, which include the classical 2nd order Painlevé equations. In the second half of the talk, I will investigate explicit geometric structures of moduli spaces of parabolic connections and Higgs bundles. We show that on these moduli spaces we can define the apparent singularity map globally, and we can also define dual coordinates at least Zariski locally for these moduli spaces. The spectral curves for Higgs bundles play essential roles for this explicit geometry. We will also mention about two different Lagrangian fibrations on moduli spaces.

#### Kohei Iwaki (Nagoya University)

**Title:** Topological recursion, Painlevé equations and cluster algebras **Abstract:** Eynard-Orantin's topological recursion is a remarkable algorithm which associates a hierarchy of invariants to a given spectral curve. It is known that many geometric invariants, such as Gromov-Witten invariants, Hurwitz numbers, Mrizakhani's hyperbolic volumes etc., are obtained in the context. In this talk, generalizing a result of Eynard et.al, I'll show that the tau-function of Painlevé equations arise form topological recursion for semi-classical spectral curves of corresponding isomonodromy systems. If time allows, I'll also discuss a relation between (the most simplest) cluster algebra and free energy of topological recursion via the exact WKB analysis.

### Arata Komyo (Kobe University)

**Title:** Hamiltonian structures of isomonodromic deformations on moduli spaces of parabolic connections

**Abstract:** We give a Hamiltonian description of the vector field determined by the isomonodromic deformation on a moduli space of parabolic connections. The moduli space of parabolic connections is constructed by Inaba-Iwasak-Saito and Inaba. Hamiltonian description of the isomonodromic deformation on moduli spaces of certain connections was essentially given by Krichever and Hurtubise. We apply their ideas to the moduli space of parabolic connections. First, we give a description of the vector field determined by the isomonodromic deformation as an element of the hypercohomology of a certain complex. Second, we recall the natural symplectic structure of the moduli space of parabolic connections, and we define Hamiltonian functions on affine open sets of the moduli space of parabolic connections. By the description of the isomonodromic deformation, we can show that this symplectic structure and these Hamiltonian functions give a Hamiltonian description of the isomonodromic deformation.

#### Motohico Mulase (University of California, Davis)

### Title: Algebraic geometry of quantum curves and opers

Abstract: Quantum curves are conceived as generating 'functions' of holomorphic B-model theory of genus g for all values of g. The quantization of the holomorphic geometry on the B-model side is the mirror dual corresponding to Gromov-Witten theory of all genera on the A-model side. The talk will focus on a particular quantization process of Hitchin spectral curves via solving a conjecture of Gaiotto on the construction of opers by scaling limit. The algebro-geometric implications of the conjectural formula, now a theorem, will be discussed. The talk is based on my joint work with Olivia Dumitrescu of Central Michigan University.

#### Szilard Szabo (Budapest University of Technology and Economics)

**Title:** Elementary transformations of singular oper-like objects

Abstract: We study the geometry of some singular de Rham and Dolbeault moduli spaces over the projective line. We start by showing that in the logarithmic case the Fuchsian locus is tangent to the forgetful map to the underlying quasi-parabolic bundle. Next, we present some results on the relationship between Garnier equations and Beauville–Mukai type integrable systems, that we then use the to discuss the relationship between quasi-parabolic bundles and apparent singularities. Time permitting, we carry out stability analysis in some 2-dimensional irregular Dolbeault moduli spaces.

### Claude Sabbah (École Polytechnique)

Title: Irregular mixed Hodge structures and modules

**Abstract:** In the talk, I will define the (neutral Tannakian) category of irregular mixed Hodge structures, starting from the notion of mixed twistor structure (Simpson) and using the notion of mixed twistor D-module (T. Mochizuki). Applications will be given to a Künneth formula for the irregular Hodge filtration attached to a Thom-Sebastaini sum of meromorphic functions. This is a joint work with Jeng-Daw Yu (Taipei).

# Atsushi Takahashi (Osaka University)

**Title:** On entropy of autoequivalences of smooth projective varieties

**Abstract:** Entropy for endofunctors of triangulated categories is defined by Dmitrov-Haiden-Katzarkov-Kontsevich. Based on the joint work with Kohei Kikuta, the categorical entropy of an surjective endomorphism of a complex smooth projective variety is shown to be equal to its topological entropy, which is done by DHKK under a certain technical condition.

It is natural to expect a generalization of the fundamental theorem by Gromov-Yomdin: the entropy of an autoequivalence of a complex smooth projective variety should be given by the logarithm of the spectral radius of the induced automorphism of the numerical Grothendieck group. This conjecture holds for elliptic curves (Kikuta's result) and if the canonical or anti-canonical sheaf is ample.

## Kenji Fukaya (Simons Center for Geometry and Physics, Stony Brook)

**Title:** Lagrangian Floer theory in symplectic manifolds with divisors **Abstract:** Relative Gromov-Witten invariant is defined and studied for projective algebraic variety with given divisors (and its symplectic analogue). In this talk I will explain its analogue in Lagrangian Floer theory.