Time Series Data

$y_t = \beta_0 + \beta_1 x_{t1} + \ldots + \beta_k x_{tk} + u_t$

3. Serial Correlation

Ch.12 Serial Correlation in Time Series

1. Properties of OLS with Serially Correlation
2. Testing for Serial Correlation
3. Correcting for Serial Correlation with Strictly Exogeneous Regressors
4. Differencing & Serial Correlation*
5. Serial Correlation-Robust Inference
6. Heteroskedasticity in Time Series Regression*

12.1 Properties of OLS with S.C.

- With serial correlation, OLSE is
  - Unbiasedness (under TS1-TS3) & Consistency (under TS1'-TS3')
  - Not efficiency & invalid t-stat, etc.

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} + 2\frac{\sigma^2}{SST^2}\sum_{t}^{n-1}\sum_{j=1}^{n-t}\rho^j x_t x_{t+j} \quad (12.4)$$

  - valid R-squared unless $\{y_t\}$ is I(1)
  - Inconsistency with lagged dependent variables

12.2 Testing for Serial Correlation

Testing for AR(1) Serial Correlation

- We can test the null hypothesis:

  H_0: $\rho = 0$ in $u_t = \rho u_{t-1} + e_t$, $t = 2,\ldots, n$,

  where u_t is the error term and e_t is *i.i.d.*
- With strictly exogenous regressors, the test is very straightforward – simply regress the residuals on lagged residuals and use a t-test.
  - See (12.14)

Cont. Testing for AR(1)

- An alternative is the Durbin-Watson statistic (D.W.), which is calculated by many packages.
  - See (12.15) and (12.16)
  - If the DW statistic is around 2, then we can reject serial correlation, while if it is significantly less than 2, we cannot reject.
  - Critical values are difficult to calculate, making the t test easier to work with.

Testing for AR1 without Strictly Exogenous

- If the regressors are not strictly exogenous, then neither the t or DW test will work.
- Regress the residual (or y) on the lagged residual and all of the x's like (12.18).
  - The inclusion of the x's allows each x_{tj} to be correlated with u_{t-1}, so we don't need assumption of strict exogeneity.

Testing for Higher Order S.C.

- We can test for AR(q) serial correlation in the same basic manner as AR(1).
- Just include q lags of the residuals in the regression and test for joint significance.
- We can use F test or LM test, where the LM version is called a Breusch-Godfrey test and is $(n-q)R^2$ using R^2 from residual regression.
  - We can also test for seasonal forms.

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12.3 Correcting for Serial Correlation

- Start with case of strictly exogenous regressors, and maintain all Gauss-Markov assumptions except no serial correlation.
- Assume errors follow AR(1):

$$u_t = \rho u_{t-1} + e_t,\ t = 1,2,\ldots, n \quad (12.26)$$

  - $\mathrm{Var}(u_t) = \sigma_e^2/(1-\rho^2)$ (12.27)
  - We need to try and transform the equation so we have no serial correlation in the errors.

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Cont. Correcting for S.C.

- Consider that since $y_t = \beta_0 + \beta_1 x_t + u_t$, then $y_{t-1} = \beta_0 + \beta_1 x_{t-1} + u_{t-1}$
- If you multiply the second equation by ρ, and subtract it from the first, we get

$$y_t - \rho y_{t-1} = (1-\rho)\beta_0 + \beta_1(x_t - \rho x_{t-1}) + e_t \quad (12.28)$$

where $e_t = u_t - \rho u_{t-1},\ t \geq 2$.

  - This quasi-differencing results in a model without serial correlation.

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Feasible GLS Estimation

- Problem with this method is that we don't know ρ, so we need to get an estimate first.
  - We can just use the estimate obtained from regressing residuals on lagged residuals.
- Depending on how we deal with the first observation, this is either called ***Cochrane-Orcutt*** or ***Prais-Winsten*** estimation.

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Cont. Feasible GLS

- Often both ***Cochrane-Orcutt*** and ***Prais-Winsten*** are implemented iteratively.
- This basic method can be extended to allow for higher order serial correlation, AR(q).
  - Most statistical packages will automatically allow for estimation of AR models without having to do the quasi-differencing by hand.

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12.5 Serial Correlation-Robust Inference

- What happens if we don't think the regressors are all strictly exogenous?
- It's possible to calculate ***serial correlation-robust standard errors***, along the same lines as heteroskedasticity robust standard errors.
  - Idea is scaling the OLS standard errors to take into account serial correlation.

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Serial Correlation-Robust S.E.

1. Estimate normal OLS to get residuals, root MSE.
2. Run the auxiliary regression of x_{t1} on $x_{t2}, \ldots, x_{tk}$
3. Form $\hat{a}_t$ by multiplying these residuals with $\hat{u}_t$
4. Choose g – say 1 to 3 for annual data, then

$$\hat{v} = \sum_{t=1}^{n} \hat{a}_t^2 + 2\sum_{h=1}^{g} [1 - h/(g+1)] \left(\sum_{t=h+1}^{n} \hat{a}_t \hat{a}_{t-h} \right)$$

and $se\left(\hat{\beta}_1\right) = [SE/\hat{\sigma}]^2 \sqrt{\hat{v}}$, where SE is the usual OLS standard error of $\hat{\beta}_j$

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