Time Series Data

$y_t = \beta_0 + \beta_1 x_{t1} + \ldots + \beta_k x_{tk} + u_t$

2. Further Issues

Ch.11 Time Series Data: Further Issues

1. Stationary & Weakly Dependent Time Series
2. Asymptotic Properties of OLSE
3. Using Highly Persistent Time Series in Regression Analysis
4. Dynamically Complete Models & the Absence of Serial Correlation*
5. The Homoskedasticity Assumption for Time Series Models*

11.1 Stationary & Weakly Dependent

Stationary Stochastic Process

- A stochastic process is ***stationary*** if for every collection of time indices $1 \le t_1 < \ldots < t_m$, the joint distribution of $(x_{t1}, \ldots, x_{tm})$ is the same as that of $(x_{t1+h}, \ldots x_{tm+h})$ for $h \ge 1$.
  - Thus, stationarity implies that the x_t's are *identically distributed* and that the nature of any correlation between adjacent terms is the same across all periods.

Covariance Stationary Process

- A stochastic process is ***covariance stationary*** if
  1. $E(x_t)$ is constant,
  2. $Var(x_t)$ is constant and
  3. for any t, $h \ge 1$, $Cov(x_t, x_{t+h})$ depends only on h and not on t.
  - Thus, this weaker form of stationarity requires only that the mean and variance are constant across time, and the covariance just depends on the distance across time.

Weakly Dependent Time Series

- A stationary time series is ***weakly dependent*** if x_t and x_{t+h} are "almost independent" as h increases.
  - If for a covariance stationary process $Corr(x_t, x_{t+h}) \to 0$ as $h \to \infty$, we'll say this covariance stationary process is ***asymptotically uncorrelated*** (almost equivalently, weakly dependent).
  - So, we can apply the Law of Large Numbers (LLN) & CLT to sample averages.

MA(1) Process

- A ***moving average process of order one*** **[MA(1)]** can be characterized as one where

  $x_t = e_t + \alpha_1 e_{t-1}, t = 1, 2, \ldots$ (11.1)

  with e_t being an *i.i.d.* sequence with mean 0 and variance σ_e^2.
  - This is a stationary, weakly dependent sequence as variables 1 period apart are correlated, but 2 periods apart they are not.

AR(1) Process

- An ***autoregressive process of order one*** **[AR(1)]** can be characterized as one where

  $y_t = \rho_1 y_{t-1} + e_t, t = 1, 2,\ldots$ (11.2)

  with e_t being an *i.i.d.* sequence with mean 0 and variance σ_e^2.
  - For this process to be weakly dependent, it must be the case that $|\rho_1| < 1$.
  - $\text{Corr}(y_t, y_{t+h}) = \text{Cov}(y_t, y_{t+h})/(\sigma_y \sigma_y) = \rho_1^h$ (11.4) which becomes small as h increases.

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Trends Revisited

- A trending series
  - cannot be stationary, since the mean is changing over time.
  - can be weakly dependent.
- If a series is weakly dependent and is stationary about its trend, we will call it a ***trend-stationary process***.
  - As long as a trend is included, all is well.

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11.2 Asymptotic Properties of OLSE

Assumptions for Consistency

1. Linearity and Weak Dependence (TS.1')
2. A weaker zero conditional mean assumption: $E(u_t|\mathbf{x}_t) = 0$, for each t (TS.2')
3. No Perfect Collinearity (TS.3')

- Thus, for asymptotic unbiasedness (consistency), we can weaken the exogeneity assumptions somewhat relative to those for unbiasedness.

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Large-Sample Inference

Asymptotic Normality of OLSE

4. Weaker assumption of homoskedasticity: $\text{Var}(u_t|\mathbf{x}_t) = \sigma^2$, for each t (TS4')
5. Weaker assumption of no serial correlation: $E(u_t u_s| \mathbf{x}_t, \mathbf{x}_s) = 0$ for $t \neq s$ (TS5')

- With these assumptions, we have asymptotic normality and the usual standard errors, t statistics, F statistics and LM statistics are valid.

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11.3 Highly Persistent Time Series

- A ***random walk*** is an AR(1) model where $\rho_1 = 1$, meaning the series is not weakly dependent.

  $y_t = y_{t-1} + e_t \quad t = 1,2,\ldots$ (11.20)
  - With a random walk, the expected value of y_t is always y_0 – it doesn't depend on t.
  - $\text{Var}(y_t) = \sigma_e^2 t$, so it increases with t.
  - We say a random walk is highly persistent since $E(y_{t+h}|y_t) = y_t$ for all $h \geq 1$.

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Cont. Random Walks

- A random walk is a special case of what's known as a ***unit root process***.
- Note that trending and persistence are different things – a series can be trending but weakly dependent, or a series can be highly persistent without any trend.
- A ***random walk with drift*** is an example of a highly persistent series that is trending.

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Transforming Persistent Series

- In order to use a highly persistent series and get meaningful estimates and make correct inferences, we want to transform it into a weakly dependent process.
- We refer to a weakly dependent process as being ***integrated of order zero***, or I(0).
  - A random walk is integrated of order one, [I(1)], meaning a first difference will be I(0).

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Cont. Transforming Persistent Series

- In a random walk case, where $\{e_t\}$ is *i.i.d.* with zero mean and variance σ^2,

$$\Delta y_t = y_t - y_{t-1} = e_t \quad t = 2,3,\ldots \quad (11.24)$$

  - If $\{e_t\}$ is weakly dependent process, then $\{y_t\}$ is also weakly dependent.
- To decide whether a time series is I(1) or not, statistical tests can be used.
  - H_0: $\rho_1 = 1$ and H_1: $\rho_1 < 1$ (see ch.18)

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