Chapter 11

Time Series Data

\[ y_t = \beta_0 + \beta_1 x_{t1} + \ldots + \beta_k x_{tk} + u_t \]

2. Further Issues

Ch. 11 Time Series Data: Further Issues

1. Stationary & Weakly Dependent Time Series
2. Asymptotic Properties of OLSE
3. Using Highly Persistent Time Series in Regression Analysis
4. Dynamically Complete Models & the Absence of Serial Correlation*
5. The Homoskedasticity Assumption for Time Series Models*

11.1 Stationary & Weakly Dependent

Stationary Stochastic Process

- A stochastic process is \textit{stationary} if for every collection of time indices \(1 \leq t_1 < \ldots < t_m\), the joint distribution of \((x_{t_1}, \ldots, x_{t_m})\) is the same as that of \((x_{t_1+h}, \ldots, x_{t_m+h})\) for \(h \geq 1\).
- Thus, stationarity implies that the \(x_t\)'s are \textit{identically distributed} and that the nature of any correlation between adjacent terms is the same across all periods.

Weakly Dependent Time Series

- A stationary time series is \textit{weakly dependent} if \(x_t\) and \(x_{t+h}\) are “almost independent” as \(h\) increases.
  - If for a covariance stationary process \(\text{Cov}(x_t, x_{t+h}) \to 0\) as \(h \to \infty\), we’ll say this covariance stationary process is \textit{asymptotically uncorrelated} (almost equivalently, weakly dependent).
  - So, we can apply the Law of Large Numbers (LLN) & CLT to sample averages.

Covariance Stationary Process

- A stochastic process is \textit{covariance stationary} if
  1. \(E(x_t)\) is constant,
  2. \(\text{Var}(x_t)\) is constant and
  3. for any \(t, h \geq 1\), \(\text{Cov}(x_t, x_{t+h})\) depends only on \(h\) and not on \(t\).
- Thus, this weaker form of stationarity requires only that the mean and variance are constant across time, and the covariance just depends on the distance across time.

MA(1) Process

- A \textit{moving average process of order one} is characterized as one where
  \[ x_t = e_t + \alpha x_{t-1}, \quad t = 1, 2, \ldots \] (11.1)
  with \(e_t\) being an \textit{i.i.d.} sequence with mean 0 and variance \(\sigma^2\).
- This is a stationary, weakly dependent sequence as variables 1 period apart are correlated, but 2 periods apart they are not.
AR(1) Process

An autoregressive process of order one [AR(1)] can be characterized as one where
\[ y_t = \rho_1 y_{t-1} + e_t, \quad t = 1, 2, \ldots \] (11.2)
with \( e_t \) being an i.i.d. sequence with mean 0 and variance \( \sigma_e^2 \).

- For this process to be weakly dependent, it must be the case that \( |\rho_1| < 1 \).
- \( \text{Corr}(y_t, y_{t+h}) = \frac{\text{Cov}(y_t, y_{t+h})}{\sqrt{\text{Var}(y_t) \text{Var}(y_{t+h})}} = \rho_1^h \) (11.4)
  which becomes small as \( h \) increases.

Trends Revisited

- A trending series
  - cannot be stationary, since the mean is changing over time.
  - can be weakly dependent.
- If a series is weakly dependent and is stationary about its trend, we will call it a trend-stationary process.
  - As long as a trend is included, all is well.

11.2 Asymptotic Properties of OLSE

Assumptions for Consistency
1. Linearity and Weak Dependence (TS.1')
2. A weaker zero conditional mean assumption: \( E(u_t|x_t) = 0 \), for each \( t \) (TS.2')
3. No Perfect Collinearity (TS.3')
   - Thus, for asymptotic unbiasedness (consistency), we can weaken the exogeneity assumptions somewhat relative to those for unbiasedness.

Large-Sample Inference

Asymptotic Normality of OLSE
4. Weaker assumption of homoskedasticity: \( \text{Var}(u_t|x_t) = \sigma^2 \), for each \( t \) (TS4')
5. Weaker assumption of no serial correlation: \( E(u_t u_s|x_t, x_s) = 0 \) for \( t \neq s \) (TS5')
   - With these assumptions, we have asymptotic normality and the usual standard errors, \( t \) statistics, \( F \) statistics and LM statistics are valid.

11.3 Highly Persistent Time Series

- A random walk is an AR(1) model where \( \rho_1 = 1 \), meaning the series is not weakly dependent.
  \[ y_t = y_{t-1} + e_t, \quad t = 1, 2, \ldots \] (11.20)
  - With a random walk, the expected value of \( y_t \) is always \( y_0 \) – it doesn’t depend on \( t \).
  - \( \text{Var}(y_t) = \sigma_e^2 t \), so it increases with \( t \).
  - We say a random walk is highly persistent since \( E(y_{t+h}|y_t) = y_t \) for all \( h \geq 1 \).

Cont. Random Walks

- A random walk is a special case of what’s known as a unit root process.
- Note that trending and persistence are different things – a series can be trending but weakly dependent, or a series can be highly persistent without any trend.
- A random walk with drift is an example of a highly persistent series that is trending.
In order to use a highly persistent series and get meaningful estimates and make correct inferences, we want to transform it into a weakly dependent process. We refer to a weakly dependent process as being *integrated of order zero*, or I(0).

- A random walk is integrated of order one, [I(1)], meaning a first difference will be I(0).

In a random walk case, where \{\varepsilon_t\} is i.i.d. with zero mean and variance \(\sigma^2\),

\[
\Delta y_t = y_t - y_{t-1} = \varepsilon_t, \quad t = 2, 3, \ldots \tag{11.24}
\]

If \{\varepsilon_t\} is weakly dependent process, then \{y_t\} is also weakly dependent.

To decide whether a time series is I(1) or not, statistical tests can be used.

- \(H_0: \rho_1 = 1\) and \(H_1: \rho_1 < 1\) (see ch.18)