Basic Mathematical Tools

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1. Summation Operator & Descriptive Statistics
2. Properties of Linear Functions
3. Proportions and Percentages
4. Some Special Functions & Their Properties
   - Quadratic Functions
   - The Natural Logarithm
5. Differential Calculus

1 Summation & Descriptive Stat.

- The summation operator:

$$\sum_{i=1}^{n} x_i \equiv x_1 + x_2 \cdots + x_n \qquad [A.1]$$

  - Property 1: $\sum_{i=1}^{n} c = nc.$ [A.2]
  - Property 2: $\sum_{i=1}^{n} cx_i = c\sum_{i=1}^{n} x_i.$ [A.3]
  - Property 3: $\sum(ax_i + by_i) = a\sum x_i + b\sum y_i$ [A.4]
  - $\sum(x_i/y_i) \neq (\sum x_i)/(\sum y_i)$
    - $x_1/y_1 + x_2/y_2 \neq (x_1 + y_1)/(x_2 + y_2)$
  - $\sum x_i^2 \neq (\sum x_i)^2$
    - $x_1^2 + x_2^2 \neq (x_1 + x_2)^2 = x_1^2 + 2x_1x_2 + x_2^2$

Descriptive Statistics

- Average:

$$\bar{x} = (1/n)\sum x_i \qquad [A.5]$$

  - $\sum(x_i - \bar{x}) = 0$ [A.6]
  - $\sum(x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$ [A.7]
  - $\sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i(y_i - \bar{y})$

$$= \sum(x_i - \bar{x})y_i \qquad [A.8]$$

- Median: the central value
  - {-4, 0, 2, 4, 8} mean=2, median=2
  - {-4, 0, 2, 4, 18} mean=4, median=2

2. Properties of Linear Functions

- Linear Function 1:

$$y = \beta_0 + \beta_1 x \qquad [A.9]$$

  - e.g. *housing* = 164 + 0.27 *income* (See Fig.A1)

- Linear Function 2:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \qquad [A.12]$$

  - Partial effect & ceteris paribus

$$\beta_1 = \frac{\Delta y}{\Delta x_1} \quad \text{if } \Delta x_2 = 0$$

  - e.g. *quantity* = 120 – 9.8 *price* + 0.3 *income* (See Fig.A2)

FIGURE A.1

Graph of *housing* = 164 + .27 *income*.

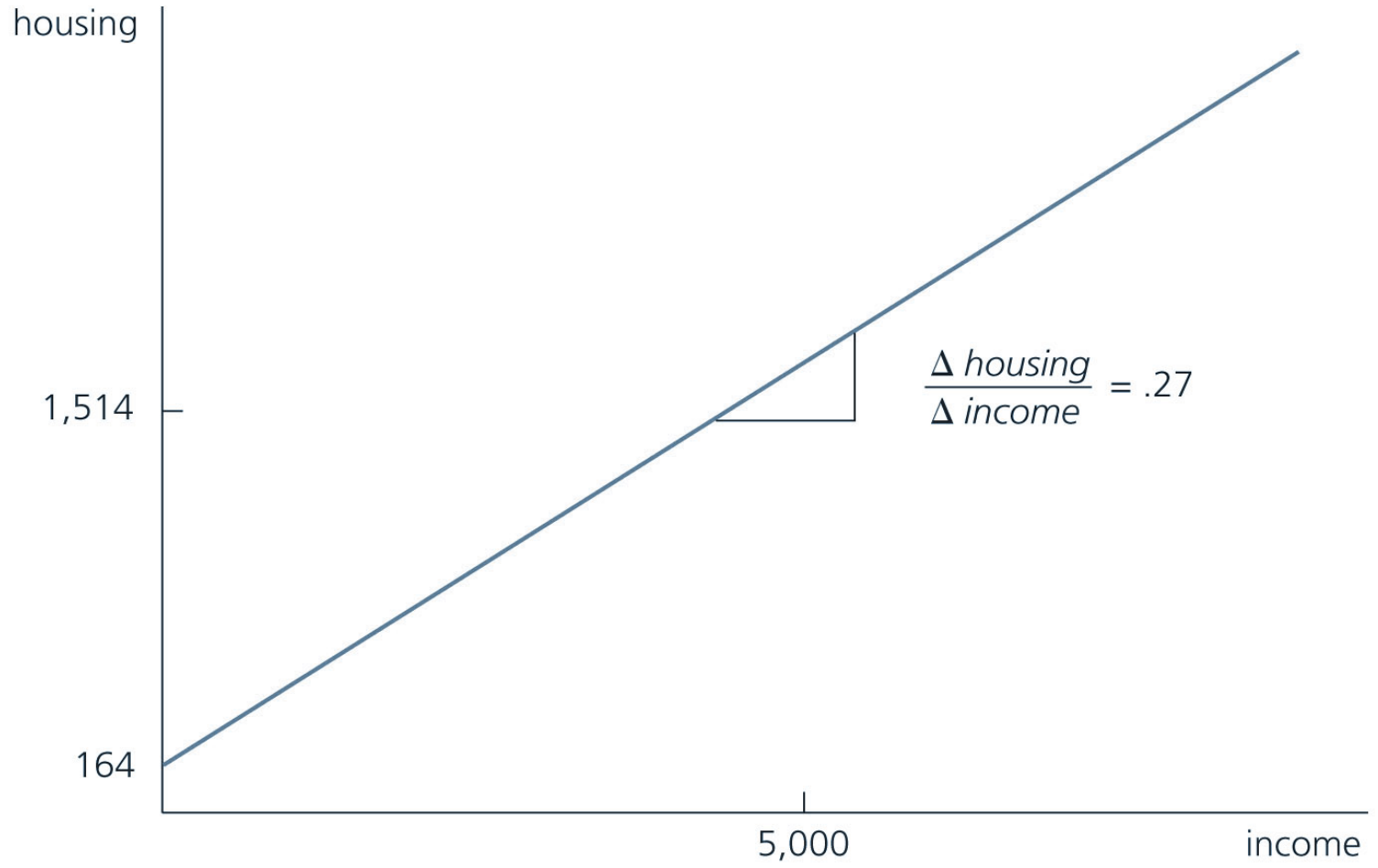
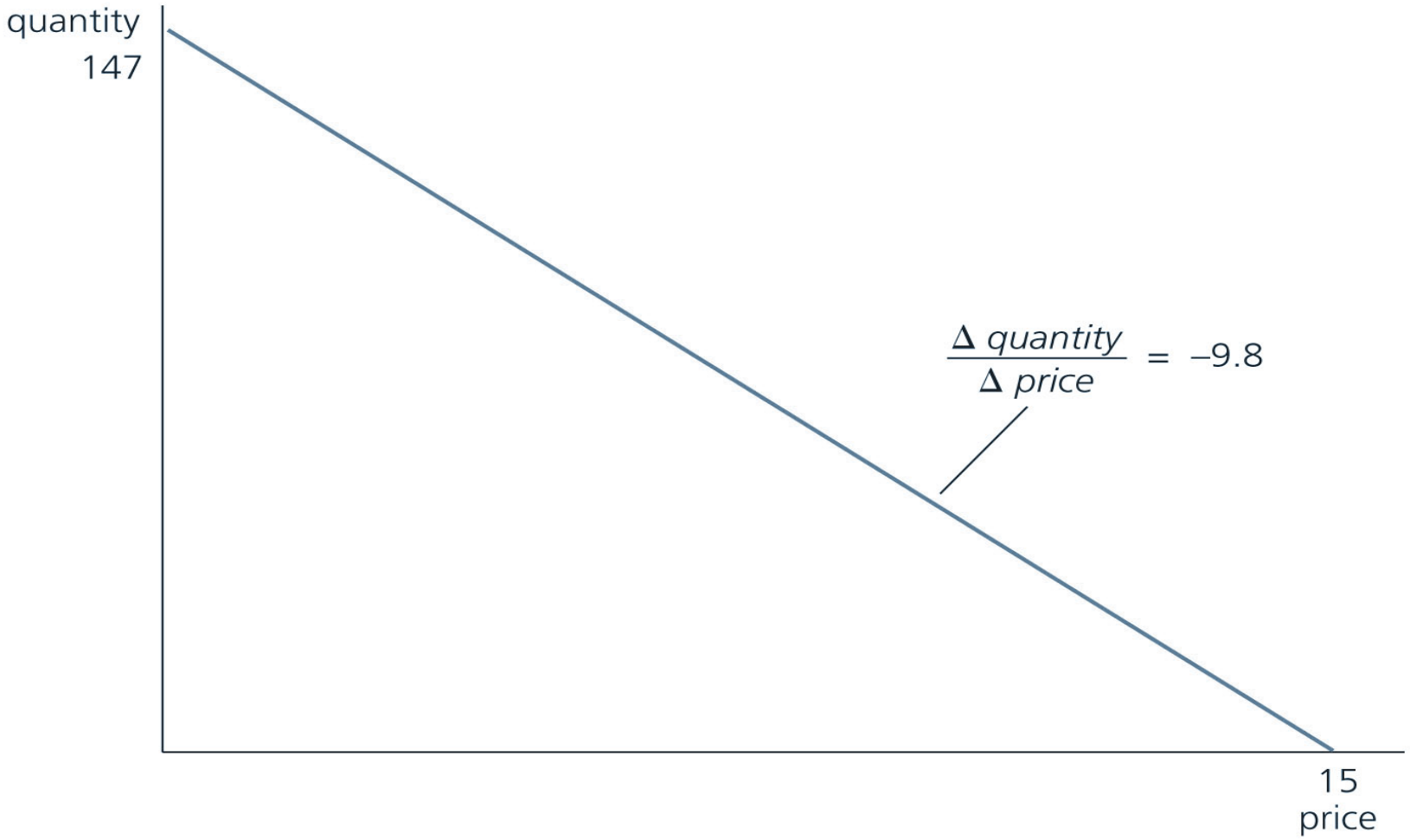


FIGURE A.2

Graph of *quantity* = 120 − 9.8 *price* + .03 *income*, with *income* fixed at $900.



3. Proportions & Percentages

- Proportionate change:

$$\frac{x_1 - x_0}{x_0} = \frac{\Delta x}{x_0} \qquad [A.14]$$

- Percentage change:

$$\%\Delta x = 100\frac{\Delta x}{x_0} \qquad [A.15]$$

- Percentage point change:
  - The change in a variable that is measured as a %.

Cont. Proportions & Percentages

- Example: Michigan Sales Tax Increase
  - In March 1994, Michigan voters approved a sales tax increase from 4% to 6%.
  - Someone referred to this as a two percentage point increase, or an increase of two cents on the dollar.
  - Others called it a 50% increase in the tax rate.
  - Both claims are correct; they are simply different ways of measuring the increase in the sales tax.

4. Special Functions

- Quadratic Functions:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 \qquad [A.16]$$

  - The slope:

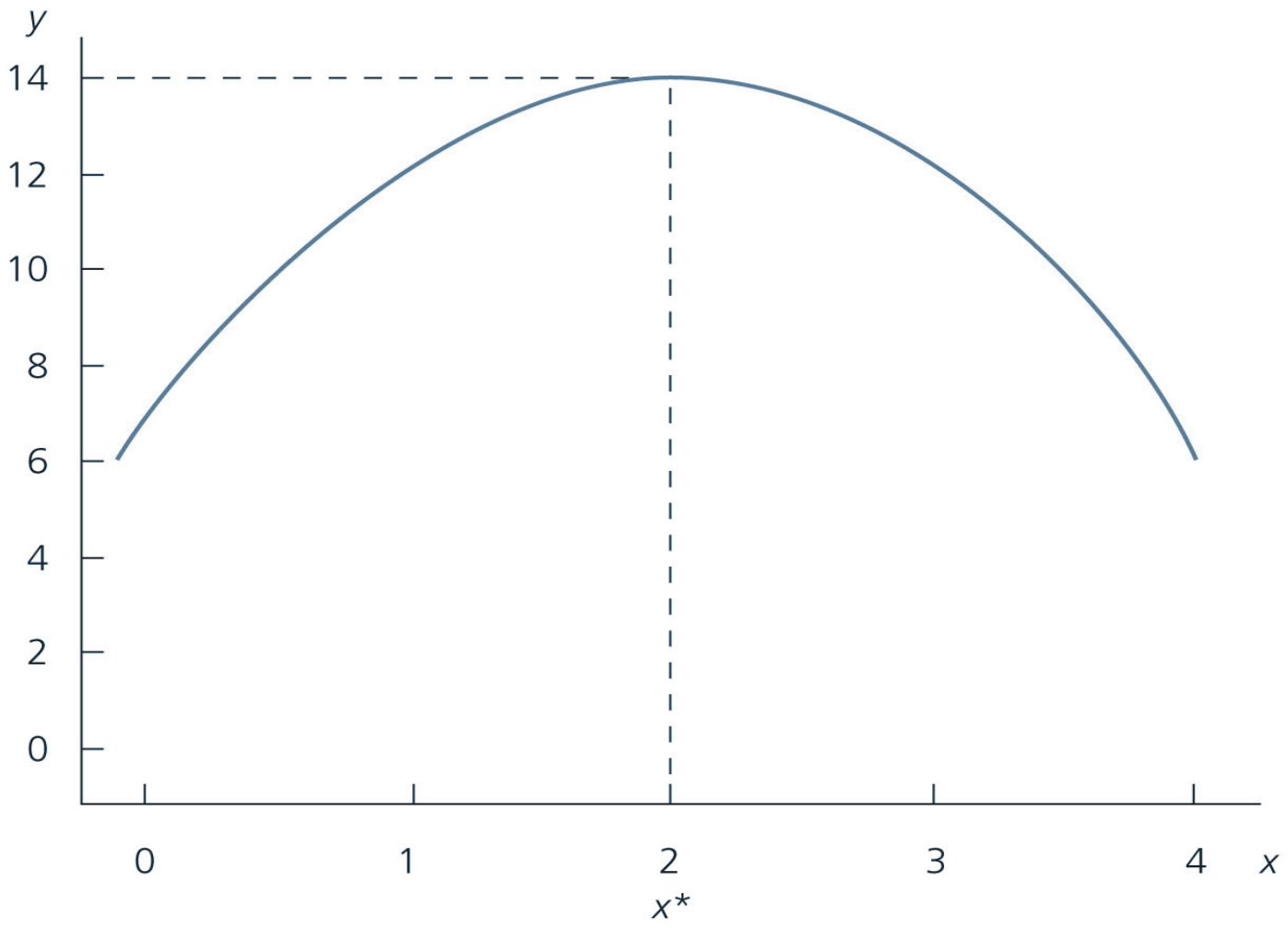
$$\frac{\Delta y}{\Delta x} \approx \beta_1 + 2\beta_2 x \qquad [A.18]$$

  - The turning point:

$$x^* = \frac{b_1}{-2b_2} \qquad [A.17]$$

FIGURE A.3

Graph of $y = 6 + 8x - 2x^2$.



Cont. Special Functions

- Natural Logarithm: $y = \ln(x)$ [A.21]
  - Some algebraic facts:

    $\log(x_1 x_2) = \log(x_1) + \log(x_2)$, $x_1, x_2 > 0$

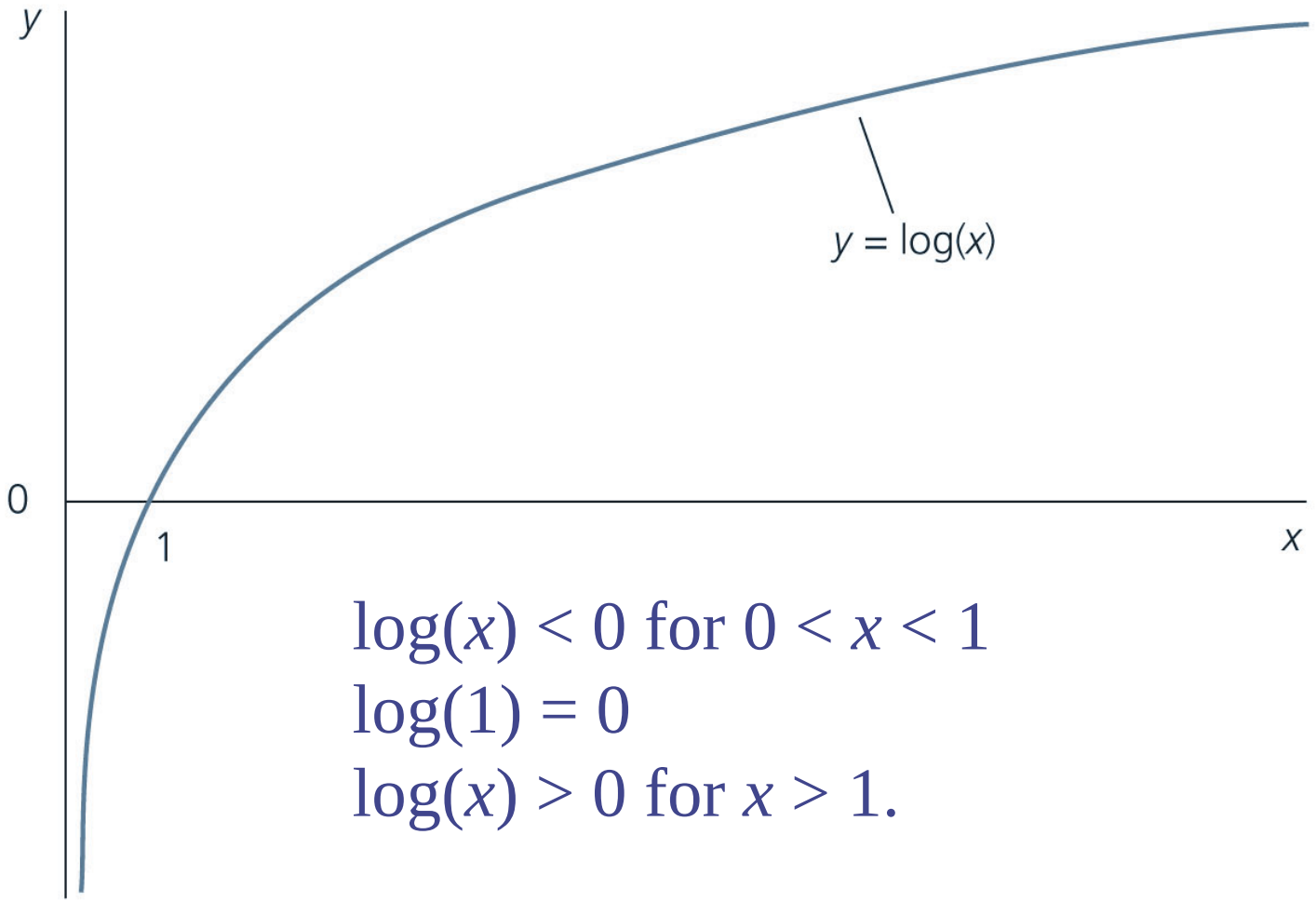
    $\log(x_1 / x_2) = \log(x_1) - \log(x_2)$, $x_1, x_2 > 0$

    $\log(x^c) = c\log(x)$, $x > 0$, c any number.
  - Proportionate change:

    $\log(x_1) - \log(x_0) \approx (x_1 - x_0) / x_0 = \Delta x / x_0$
  - Elasticity: $\frac{\Delta \log(y)}{\Delta \log(x)} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y}$ [A.24]

FIGURE A.4

Graph of $y = \log(x)$.



$\log(x) < 0$ for $0 < x < 1$

$\log(1) = 0$

$\log(x) > 0$ for $x > 1$.

5. Differential Calculus

- Functions & the derivatives:

  $y = \beta_0 + \beta_1 x + \beta_2 x^2$; $dy/dx = \beta_1 + 2\beta_2 x$

  $y = \beta_0 + \beta_1 / x$; $dy/dx = -\beta_1/(x^2)$

  $y = \beta_0 + \beta_1 \ln(x)$; $dy/dx = \beta_1/x$

- Partial derivative
  - If $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, then

$$\frac{\partial y}{\partial x_1} = \beta_1, \qquad \frac{\partial y}{\partial x_2} = \beta_2.$$

Quiz 1

Family	Monthly Housing Expenditures (Dollars)
1	300
2	200
3	700

i. Find the average monthly housing expenditure.

ii. Find the median monthly housing expenditure.

iii. Suppose that family number 2 increases its monthly housing expenditure to $500, but the expenditures of other families remain the same. Compute the average and median housing expenditures.

Quiz 2

Suppose the equation below describes the relationship between the average number of classes missed during a semester (missed) and the distance from school (distance, measured in miles):

$$missed = 3 + 0.2\ distance.$$

i. Sketch this line, being sure to label the axes. How do you interpret the intercept in this equation?

ii. What is the average number of classes missed for someone who lives five miles away?

iii. What is the difference in the average number of classes missed for someone who lives 10 miles away and someone who lives 20 miles away?

Quiz 3

Suppose that quantity of compact discs is related to price and income by

$$quantity = 120 - 10\ price + 0.05\ income.$$

What is the demand for CDs if *price* = 15 and *income* = 1000? What does this suggest?

Quiz 4

Suppose that the return from holding a particular firm's stock goes from 15% in one year to 18% in the following year. The majority shareholder claims that "the stock return only increased by 3%," while the chief executive officer claims that "the return on the firm's stock increased by 20%." Reconcile their disagreement.

Quiz 5

Let *grthemp* denote the proportionate growth in employment, at the county level, from 1990 to 1995, and let *salestax* denote the county sales tax rate, stated as a proportion. Interpret the intercept and slope in the equation

$$grthemp = 0.043 - 0.78\, salestax.$$