

A TOPOLOGICAL APPROACH TO CHARACTERISING HYPERSTATIONARY SETS ON $\mathcal{P}_\kappa(A)$

M. CATALINA TORRES

ABSTRACT. Given a topological space (X, τ) , we can define a sequence of topologies *derived topologies* $(X, \tau_0, \tau_1, \dots, \tau_\xi, \dots)$, where $\tau_0 = \tau$ and $\tau_\zeta \subseteq \tau_\xi$ for $\zeta < \xi$. This might be done by declaring $d_{\tau_\xi}(T)$ to be open in $\tau_{\xi+1}$, and taking unions at limit stages. In the paper “Derived Topologies on Ordinals and Stationary Reflection” Bagaria characterised the non-isolated points in the ξ -th derived topology on ordinals as those ordinals that satisfy a strong iterated form of stationary reflection, which he called ξ -simultaneous-reflection.

The generalization of combinatorial properties of ordinals to the setting of $P_\kappa(A) := \{X \subseteq \kappa : |X| < \kappa\}$, where κ is an uncountable regular cardinal and $A \subseteq \kappa$ has been widely studied. In this context, we extend the notion of higher stationarity and construct a sequence of topologies $\langle \tau_0, \tau_1, \dots \rangle$ on $P_\kappa(A)$, characterising the simultaneous reflection of “high”-stationary subsets of $\mathcal{P}_\kappa(A)$ in terms of elements in the base of a derived topology on $\mathcal{P}_\kappa(A)$.

DEPARTAMENT DE MATEMÀTIQUES I INFORMÀTICA, UNIVERSITAT DE BARCELONA. GRAN
VIA DE LES CORTS CATALANES, 585, 08007 BARCELONA, CATALONIA.
Email address: mactorrespa@gmail.com